

Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2011 Paper 2: Calculator
1 hour 20 minutes

The total number of marks available is 49.

You must write down all the stages in your working.

1. Olga normally runs a total distance of 28 miles per week. (3)
She decides to increase her distance by 10% a week for the next four weeks.
How many miles will she run in the fourth week?

Solution

$$\begin{aligned}\text{Distance} &= 28 \times 1.1^4 \\ &= \underline{\underline{40.9948 \text{ miles}}}.\end{aligned}$$

2. Expand and simplify (3)

$$(3x + 1)(x^2 - 5x + 4).$$

Solution

	x^2	$-5x$	$+4$
$3x$	$3x^3$	$-15x^2$	$+12x$
$+1$	$+x^2$	$-5x$	$+4$

$$(3x + 1)(x^2 - 5x + 4) = \underline{\underline{3x^3 - 14x^2 + 7x + 4}}.$$

3. Solve the equation (4)

$$2x^2 + 3x - 7 = 0.$$

Give your answers **correct to 2 significant figures**.

Solution

$a = 2$, $b = 3$, and $c = -7$:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-7)}}{2 \times 2} \\ &= \frac{-3 \pm \sqrt{65}}{4} \\ &= -2.765\ 564\ 437, 1.265\ 564\ 437 \text{ (FCD)} \\ &= \underline{\underline{-2.8, 1.3 \text{ (2 sf)}}}. \end{aligned}$$

4. A car is valued at £3780.

This is 16% less than last year's value.

What was the value of the car last year?

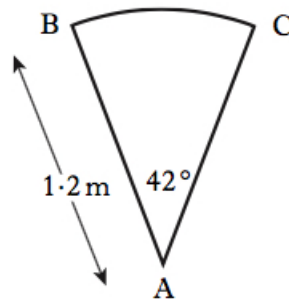
(3)

Solution

$$\begin{aligned} \text{Last year} &= \frac{3780}{1 - 0.16} \\ &= \frac{3780}{0.84} \\ &= \underline{\underline{\pounds 4500}}. \end{aligned}$$

5. A spiral staircase is being designed.

(4)



Each step is made from a sector of a circle as shown.

The radius is 1.2 metres.

Angle BAC is 42° .

For the staircase to pass safety regulations, the arc BC must be at least 0.9 metres.

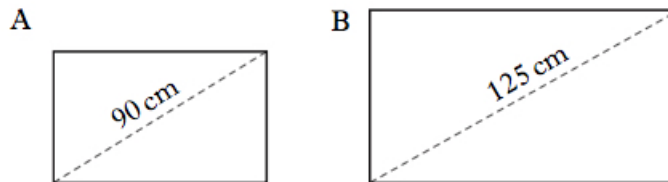
Will the staircase pass safety regulations?

Solution

$$\begin{aligned}\text{Arc} &= \frac{1}{2}r^2\theta \\ &= 2 \times \pi \times 1.2 \times \frac{42}{360} \\ &= 0.879\,645\,943 \text{ (FCD)};\end{aligned}$$

as this is less than 0.9 metres, the staircase does not pass the safety regulations.

6. Two rectangular solar panels, A and B , are mathematically similar. (4)
Panel A has a diagonal of 90 centimetres and an area of 4 020 square centimetres.



A salesman claims that panel B , with a diagonal of 125 centimetres, will be double the area of panel A .

Is this claim justified?

Show all your working.

Solution

The length scale factor (LSF) is

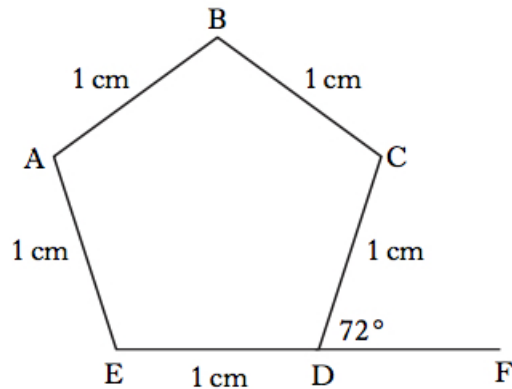
$$\frac{125}{90} = \frac{25}{18}$$

and the area scale factor (ASF) is

$$\left(\frac{25}{18}\right)^2 = 1\frac{301}{324}$$

So, the salesman is not justified.

7. $ABCDE$ is a regular pentagon with each side 1 centimetre.



Angle CDF is 72° .

EDF is a straight line.

(a) Write down the size of angle ABC .

(1)

Solution

$$\angle ABC = 180 - 72 = \underline{108^\circ}.$$

(b) Calculate the length of AC .

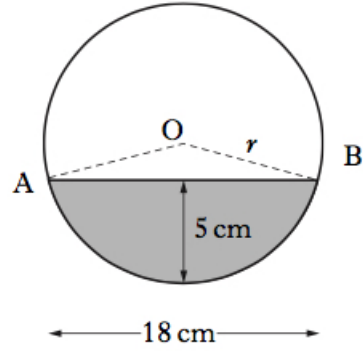
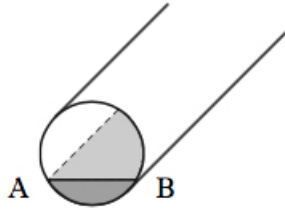
(3)

Solution

$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos ABC} \\ &= \sqrt{1^2 + 1^2 - 2 \cdot 1 \cdot 1 \cdot \cos 108^\circ} \\ &= 1.618\,033\,989 \text{ (FCD)} \\ &= \underline{1.62 \text{ cm (3 sf)}}. \end{aligned}$$

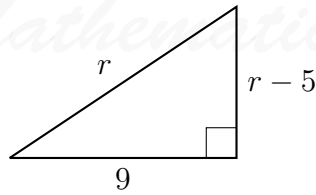
8. A pipe has water in it as shown.

(3)



The depth of the water is 5 centimetres.
 The width of the water surface, AB , is 18 centimetres.
 Calculate r , the radius of the pipe.

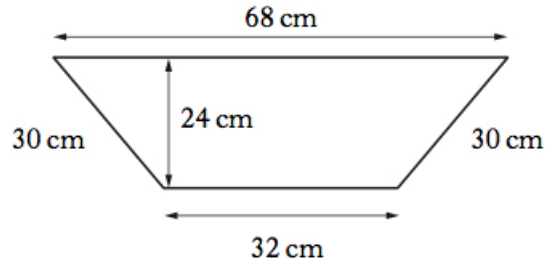
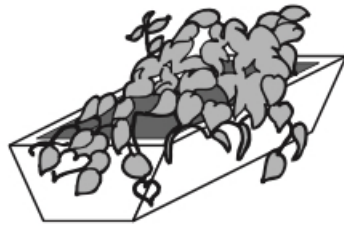
Solution



$$\begin{array}{r|l} \times & r \quad -5 \\ \hline r & r^2 \quad -5r \\ -5 & -5r \quad +25 \\ \hline \end{array}$$

$$\begin{aligned} r^2 &= 9^2 + (r - 5)^2 \Rightarrow r^2 = 81 + (r^2 - 10r + 25) \\ &\Rightarrow 10r = 106 \\ &\Rightarrow \underline{\underline{r = 10.6 \text{ cm.}}} \end{aligned}$$

9. A flower planter is in the shape of a prism.
 The cross-section is a trapezium with dimensions as shown.



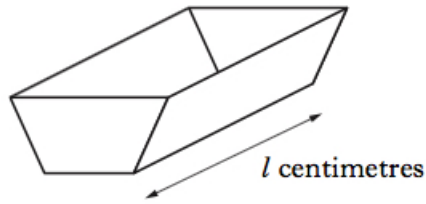
- (a) Calculate the area of the cross-section of the planter.

(2)

Solution

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times (68 + 32) \times 24 \\ &= \underline{1\,200 \text{ cm}^2}.\end{aligned}$$

The volume of the planter is 156 litres.



- (b) Calculate the length, l centimetres, of the planter.

(3)

Solution

Well, 156 litres equals $156\,000 \text{ cm}^3$. Finally,

$$\begin{aligned}\text{length} &= \frac{156\,000}{1\,200} \\ &= \underline{130 \text{ cm}}.\end{aligned}$$

10. Tom and Samia are paid the same hourly rate.

(3)

Harry is paid $\frac{1}{3}$ more per hour than Tom.

Tom worked 15 hours, Samia worked 8 hours, and Harry worked 12 hours.

They were paid a total of £429.

How much was Tom paid?

Solution

Let T , S , and H be the time that Tom, Samia, and Harry worked.
The time they worked is

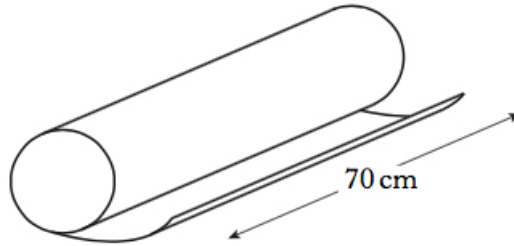
$$15 + 8 + 12 = 35 \text{ hours.}$$

Finally,

$$\begin{aligned} 15T + 8S + 12H &= 429 \Rightarrow 15T + 8T + 12\left(\frac{4}{3}T\right) = 429 \\ &\Rightarrow 39T = 429 \\ &\Rightarrow T = 11 \\ &\Rightarrow 15T = \underline{\underline{\pounds 165}}. \end{aligned}$$

11. Paper is wrapped round a cardboard cylinder **exactly** 3 times.
The cylinder is 70 centimetres long.

(4)



The area of the paper is 3 000 square centimetres.
Calculate the diameter of the cylinder.

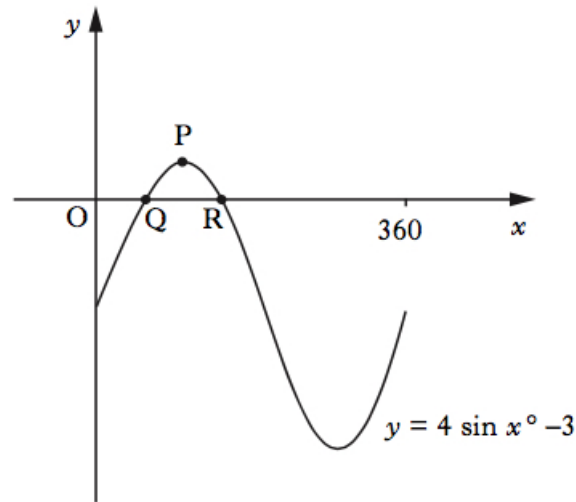
Solution

$$\begin{aligned} 3 \times 2 \times \pi \times r \times 70 &= 3\,000 \Rightarrow r = \frac{3\,000}{420\pi} \\ &\Rightarrow 2r = \frac{3\,000}{210\pi} \\ &\Rightarrow \text{diameter} = 4.547\,284\,088 \text{ (FCD)} \\ &\Rightarrow \text{diameter} = \underline{\underline{4.55 \text{ cm (3 sf)}}} \end{aligned}$$

12. Part of the graph of

$$y = 4 \sin x^\circ - 3$$

is shown below.



The graph cuts the x -axis at Q and R .
 P is the maximum turning point.

(a) Write down the coordinates of P .

(1)

Solution

$P(90, 1)$.

(b) Calculate the x -coordinates of Q and R .

(4)

Solution

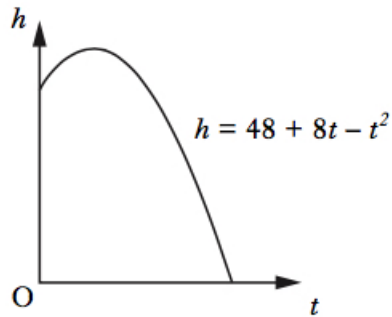
$$\begin{aligned}
 4 \sin x^\circ - 3 &\Rightarrow 4 \sin x^\circ = 3 \\
 &\Rightarrow \sin x^\circ = \frac{3}{4} \\
 &\Rightarrow x = 48.590\,377\,89, 131.409622\,1 \text{ (FCD)} \\
 &\Rightarrow \underline{\underline{x = 48.6, 131.4 \text{ (1 dp)}}}.
 \end{aligned}$$

13. The diagram shows the path of a flare after it is fired.
 The height, h metres above sea level, of the flare is given by

(4)

$$h = 48 + 8t - t^2,$$

where t is the number of seconds after firing.



Calculate, **algebraically**, the time taken for the flare to enter the sea.

Solution

$$\begin{aligned}48 + 8t - t^2 = 0 &\Rightarrow t^2 - 8t = 48 \\&\Rightarrow t^2 - 8t + 16 = 48 + 16 \\&\Rightarrow (t - 4)^2 = 64 \\&\Rightarrow t - 4 = \pm 8 \\&\Rightarrow t = -4 \text{ or } t = 12;\end{aligned}$$

since $t \neq -4$, $t = 12$.

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