

Dr Oliver Mathematics

Worked Examples

Capture-Recapture 1

From: Corbett Maths

1. Rhys has a large tub of yellow counters.
Alex has a large tub of blue counters.

40 yellow counters are taken from Rhys's tub and placed into Alex's tub.
40 blue counters are taken from Alex's tub and placed into Rhys's tub.

Rhys randomly selects 100 counters from his tub.
8 of the 100 counters are blue.

Alex randomly selects 50 counters from his tub.
48 of the 50 counters are blue.

All the counters are then placed into one tub.

Work out an estimate for the ratio of yellow to blue counters in the tub.

Solution

- Let y be the number of counters that Rhys initially.
- Let b be the number of counters that Alex initially.

Since the transfer is neutral in terms of the number of counters that each has, Rhys has y counters and Alex has b counters:

- Rhys has $(y - 40)$ yellow counters and 40 blue counters.
- Alex has $(b - 40)$ blue counters and 40 yellow counters.

Rhys:

Now, from Rhys, 8 of the 100 counters are blue and we cross-multiply:

$$\frac{8}{100} = \frac{40}{y} \Rightarrow 8y = 4000$$
$$\Rightarrow y = 500.$$

Alex:

Next, from Alex, 48 of the 50 counters are blue and we cross-multiply:

$$\begin{aligned}\frac{48}{50} &= \frac{(b - 40)}{b} \Rightarrow 48b = 50(b - 40) \\ &\Rightarrow 48b = 50b - 2000 \\ &\Rightarrow 2b = 2000 \\ &\Rightarrow b = 1000.\end{aligned}$$

Finally,

$$\begin{aligned}\text{yellow counters} : \text{blue counters} &= 500 : 1000 \\ &= \underline{\underline{1 : 2}}.\end{aligned}$$