Dr Oliver Mathematics
Applied Mathematics: Mechanics or Statistics
Section B
2008 Paper
1 hour

The total number of marks available is 32.
You must write down all the stages in your working.

1. Given that $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are square matrices where:

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 & 6 \\ 0 & -3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x & 2 \\ 0 & y \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 & 7 \\ 12 & -1 \end{pmatrix}.$$

   (a) Find $\mathbf{AB}$. (1)

   (b) Express $4\mathbf{C} + \mathbf{D}$ as a single matrix. (2)

   (c) Given that (2)

$$\mathbf{AB} = 4\mathbf{C} + \mathbf{D},$$

   find the values of x and y.

2. Given that (3)

$$y = \mathrm{e}^{2x} \cos x,$$

   find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

3. Express (4)

$$y = \frac{4x - 3}{x(x^2 + 3)}, \ x \neq 0,$$

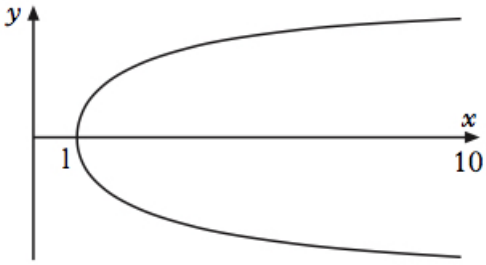
   in partial fractions.

4. (a) Use integration by parts to show that (2)

$$\int \ln x \, \mathrm{d}x = x \ln x - x + c.$$

A goblet consists of a bowl and a short stem.

The diagram below shows the bowl section of the goblet (on its side).



The equation of the upper half of the curve is

$$y = 2\sqrt{\ln x}$$

for $1 \leqslant x \leqslant 10$.

(b) Given that the stem has length 1 and the overall height is 10, what is the capacity of the bowl? (4)

5. (a) Use the standard formulas for (3)

$$\sum_{r=1}^{n} r \text{ and } \sum_{r=1}^{n} r^2$$

to show that

$$\sum_{r=1}^{n}(6r^2 - r) = \tfrac{1}{2}n(n+1)(4n+1).$$

(b) Hence evaluate (2)

$$\sum_{r=5}^{10}(6r^2 - r).$$

6. Newton's law of cooling states that a body loses heat at a rate which is proportional to the difference in temperature between itself and its surroundings. So, in a room with constant temperature 22°C, the temperature T°C of a body after a time t minutes satisfies

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(T - 22),$$

where k is a negative constant.

(a) Hence show that T can be expressed in the form (4)

$$T = A\mathrm{e}^{kt} + 22$$

for some arbitrary constant A.

In a restaurant, where the temperature remains constant at 22°C, a freshly baked roll, with temperature 82°C, is placed on a cooling tray. After 5 minutes, the temperature of the roll has fallen by 20°C.

(b) (i) Calculate the values of A and k. (2)

(ii) Write down an expression for the temperature of the roll after t minutes. (2)

(iii) Supposing the roll remains uneaten after a further 5 minutes, what will its temperature be? (1)