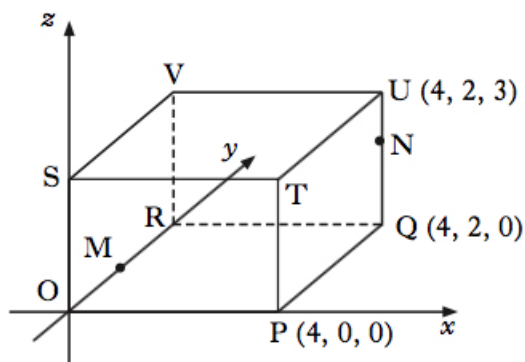


Dr Oliver Mathematics
Mathematics: Higher
2010 Paper 2: Calculator
1 hour 10 minutes

The total number of marks available is 60.

You must write down all the stages in your working.

1. The diagram shows a cuboid $OPQRSTUV$ relative to the coordinate axes.



P is the point $(4, 0, 0)$, Q is $(4, 2, 0)$, and U is $(4, 2, 3)$.

M is the midpoint of OR .

N is the point on UQ such that $UN = \frac{1}{3}UQ$.

- (a) State the coordinates of M and N . (2)
- (b) Express \overrightarrow{VM} and \overrightarrow{VN} in component form. (2)
- (c) Calculate the size of angle MVN . (5)

2.

$$12 \cos x^\circ - 5 \sin x^\circ$$

can be expressed in the form

$$k \cos(x + a)^\circ,$$

where $k > 0$ and $0 \leq a < 360$.

- (a) Calculate the values of k and a . (4)
- (b) (i) Hence state the maximum and minimum values of (3)

$$12 \cos x^\circ - 5 \sin x^\circ.$$

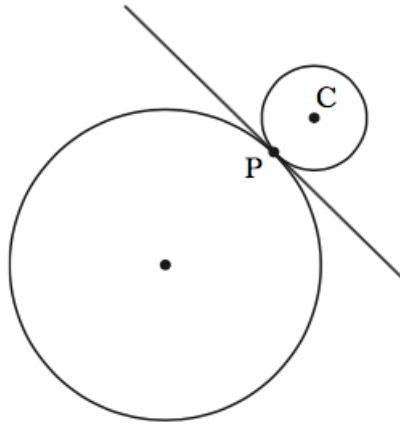
(ii) Determine the values of x , in the interval $0 \leq a < 360$, at which these maximum and minimum values occur.

3. (a) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation

$$x^2 + y^2 + 14x + 4y - 19 = 0.$$

(ii) Find the coordinates of the point of contact, P .

Relative to a suitable set of coordinate axes, the diagram below shows the circle from (a) and a second smaller circle with centre C .



The line $y = 3 - x$ is a common tangent at the point P .

The radius of the larger circle is three times the radius of the smaller circle.

- (b) Find the equation of the smaller circle. (6)

4. Solve (5)

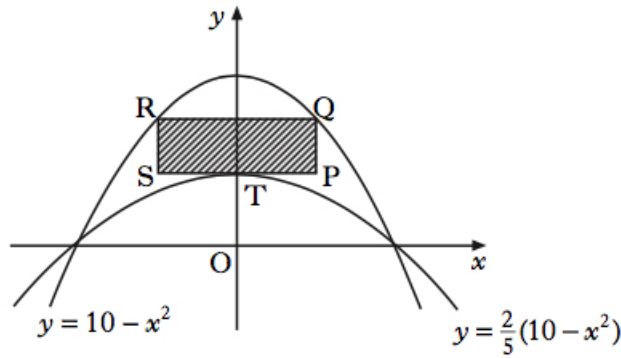
$$2 \cos 2x - 5 \cos x - 4 = 0$$

for $0 \leq x < 2\pi$.

5. The parabolas with equations

$$y = 10 - x^2 \text{ and } y = \frac{2}{5}(10 - x^2)$$

are shown in diagram below.



A rectangle $PQRS$ is placed between the two parabolas as shown, so that Q and R lie on the upper parabola, RQ and SP are parallel to the x -axis, and T , the turning point of the lower parabola, lies on SP .

(a) (i) If $TP = x$ units, find an expression for the length of PQ . (3)

(ii) Hence show that the area, A , of rectangle $PQRS$ is given by

$$A(x) = 12x - 2x^3.$$

(b) Find the maximum area of this rectangle. (6)

6. A curve has equation

$$y = (2x - 9)^{\frac{1}{2}}.$$

(a) Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$. (5)

Diagram 1 shows part of the curve and the tangent.

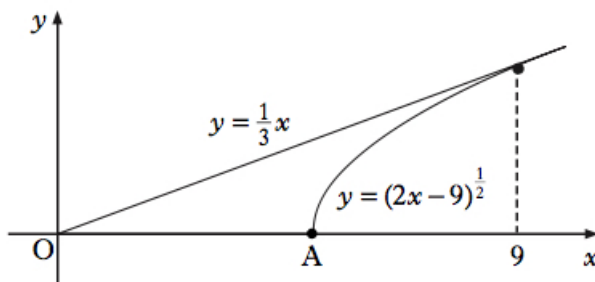


Diagram 1

The curve cuts the x -axis at the point A .

(b) Find the coordinates of point A . (1)

- (c) Calculate the shaded area shown in diagram 2. (7)

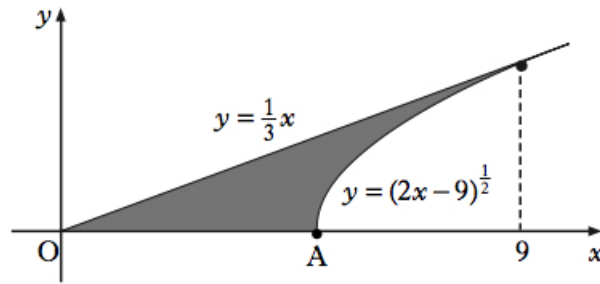


Diagram 2

7. (a) Given that $\log_4 x = P$, show that (3)

$$\log_{16} x = \frac{1}{2}P.$$

- (b) Solve (3)

$$\log_3 x + \log_9 x = 12.$$