

Dr Oliver Mathematics
Mathematics: National Qualifications N5
2017 Paper 1: Non-Calculator
1 hour

The total number of marks available is 40.

You must write down all the stages in your working.

1. Given that

$$f(x) = x^2 + 3x,$$

evaluate $f(-5)$.

(2)

Solution

$$\begin{aligned} f(-5) &= (-5)^2 + 3(-5) \\ &= 25 - 15 \\ &= \underline{10}. \end{aligned}$$

2. The number of calls received by the police was recorded over 10 days.
The results are shown below.

198 216 218 230 232 247 248 250 265 267

Find the semi-interquartile range of this data.

(2)

Solution

The lower quartile (LQ) is

$$\frac{10 + 1}{4} = 2\frac{3}{4}\text{th value}$$

and so LQ is 218. The upper quartile (UQ) is

$$\frac{3(10 + 1)}{4} = 8\frac{1}{4}\text{th value}$$

and so UQ is 250. Hence,

$$\begin{aligned} \text{semi-interquartile range} &= \frac{1}{2}(250 - 218) \\ &= \frac{1}{2}(32) \\ &= \underline{16}. \end{aligned}$$

3. Evaluate

(2)

$$1\frac{5}{6} \div \frac{3}{4}.$$

Give your answer in its simplest form.

Solution

$$\begin{aligned} 1\frac{5}{6} \div \frac{3}{4} &= \frac{11}{6} \times \frac{4}{3} \\ &= \frac{11}{3} \times \frac{2}{3} \\ &= \frac{22}{9} \\ &= \underline{\underline{2\frac{4}{9}}}. \end{aligned}$$

4. Expand and simplify

(3)

$$(2x + 3)(x^2 - 4x + 1).$$

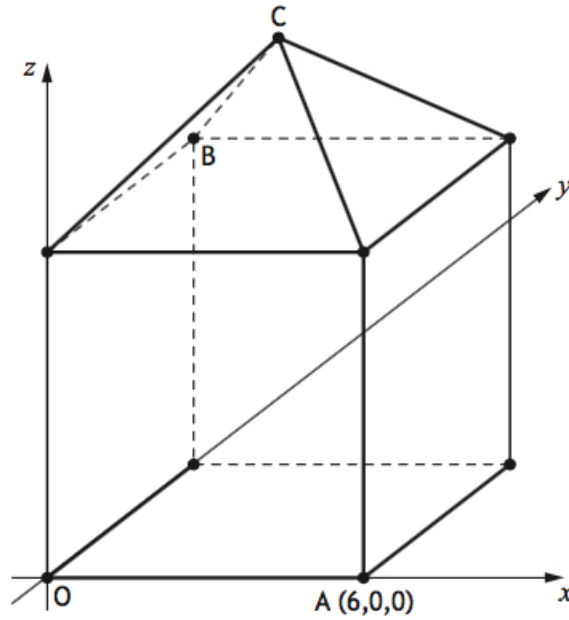
Solution

| | | | |
|----------|---------|---------|-------|
| \times | x^2 | $-4x$ | $+1$ |
| $2x$ | $2x^3$ | $-8x^2$ | $+2x$ |
| $+3$ | $+3x^2$ | $-12x$ | $+3$ |

$$(2x + 3)(x^2 - 4x + 1) = \underline{\underline{2x^3 - 5x^2 - 10x + 3}}.$$

5. The diagram shows a square-based pyramid placed on top of a cube, relative to the coordinate axes.

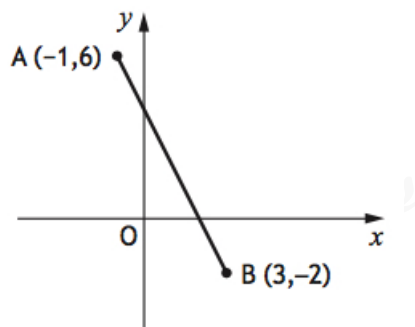
(2)



The height of the pyramid is half of the height of the cube.
 A is the point $(6, 0, 0)$.
 The point C is directly above the centre of the base.
 Write down the coordinates of B and C .

Solution
 $B(0, 6, 6)$ and $C(3, 3, 9)$.

6. The diagram below shows the straight line joining points A and B . (3)



Find the equation of the line AB .
 Give the equation in its simplest form.

Solution

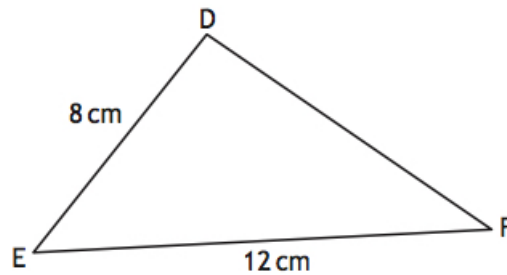
$$\begin{aligned}\text{Gradient} &= \frac{6 - (-2)}{-1 - 3} \\ &= \frac{8}{-4} \\ &= -2\end{aligned}$$

and the equation is

$$\begin{aligned}y - 6 &= -2(x + 1) \Rightarrow y - 6 = -2x - 2 \\ &\Rightarrow \underline{\underline{y = -2x + 4.}}\end{aligned}$$

7. In triangle DEF :

- $DE = 8$ centimetres,
- $EF = 12$ centimetres, and
- $\sin DEF = \frac{2}{3}$.



Calculate the area of triangle DEF .

Solution

$$\begin{aligned}\text{Area} &= \frac{1}{2} \cdot DE \cdot EF \cdot \sin DEF \\ &= \frac{1}{2} \times 8 \times 12 \times \frac{2}{3} \\ &= 4 \times 8 \\ &= \underline{\underline{32 \text{ cm}^2}}.\end{aligned}$$

8. Solve, algebraically, the inequality

(3)

$$19 + x > 15 + 3(x - 2).$$

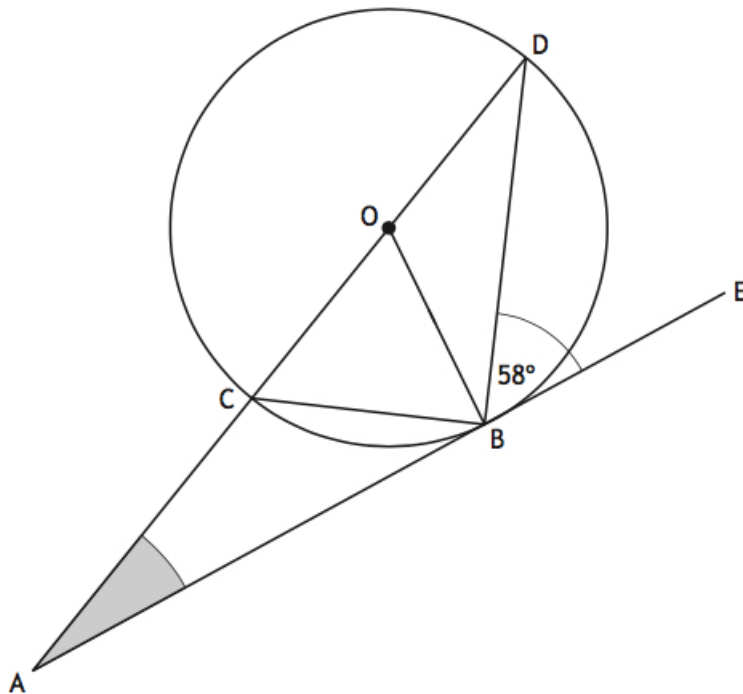
Solution

$$\begin{aligned} 19 + x > 15 + 3(x - 2) &\Rightarrow 4 + x > 3x - 6 \\ &\Rightarrow 10 > 2x \\ &\Rightarrow \underline{x < 5}. \end{aligned}$$

9. In the diagram shown below:

(3)

- ABE is a tangent to the circle, centre O and
- Angle DBE is 58° .



Calculate the size of angle CAB .

Solution

$$\angle BCD = 58^\circ \text{ (alternating segment theorem)}$$

$$\angle BOC = 180 - 58 - 58 = 180 - 116 = 64^\circ \text{ (completing the triangle)}$$

$$\angle CAB = 180 - 90 - 64 = \underline{\underline{26^\circ}} \text{ (completing the } OAB)$$

10. Change the subject of the formula

(3)

$$F = \frac{t^2 + 4b}{c}$$

to b .

Solution

$$\begin{aligned} F &= \frac{t^2 + 4b}{c} \Rightarrow cF = t^2 + 4b \\ &\Rightarrow 4b = cF - t^2 \\ &\Rightarrow \underline{\underline{b = \frac{1}{4}(cF - t^2)}}. \end{aligned}$$

11. Express

(2)

$$\frac{3}{a^2} - \frac{2}{a}, a \neq 0,$$

as a single fraction in its simplest form.

Solution

$$\begin{aligned} \frac{3}{a^2} - \frac{2}{a} &= \frac{3}{a^2} - \frac{2a}{a^2} \\ &= \underline{\underline{\frac{3 - 2a}{a^2}}}. \end{aligned}$$

12. Gym members are asked to fill out a questionnaire to rate the quality of service provided. They are asked to give a rating on a scale of 1 to 6. The ratings given by five members were as follows:

(4)

1 4 6 3 6

In its simplest form, the standard deviation of these ratings can be written as

$$\frac{a\sqrt{b}}{2}.$$

Find the values of a and b .

Solution

| x | x^2 |
|-----|-------|
| 1 | 1 |
| 4 | 16 |
| 6 | 36 |
| 3 | 9 |
| 6 | 36 |
| 20 | 98 |

$$\begin{aligned}\text{Mean} &= \frac{\sum x}{n} \\ &= \frac{20}{5} \\ &= 4\end{aligned}$$

and

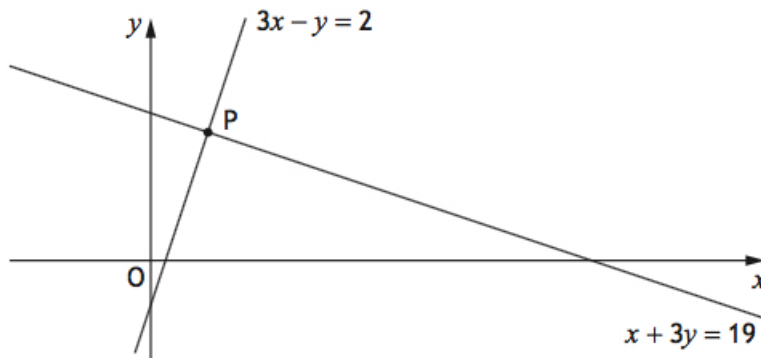
$$\begin{aligned}\text{standard deviation} &= \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}} \\ &= \sqrt{\frac{98 - (20)^2/5}{4}} \\ &= \sqrt{\frac{98 - 400/5}{4}} \\ &= \sqrt{\frac{98 - 80}{4}} \\ &= \sqrt{\frac{18}{4}} \\ &= \frac{\sqrt{18}}{2} \\ &= \frac{\sqrt{9 \times 2}}{2} \\ &= \frac{\sqrt{9} \times \sqrt{2}}{2} \\ &= \frac{3\sqrt{2}}{2};\end{aligned}$$

hence, $a = 3$ and $b = 2$.

13. The graph below shows two straight lines with the equations:

(3)

- $3x - y = 2$
- $x + 3y = 19$



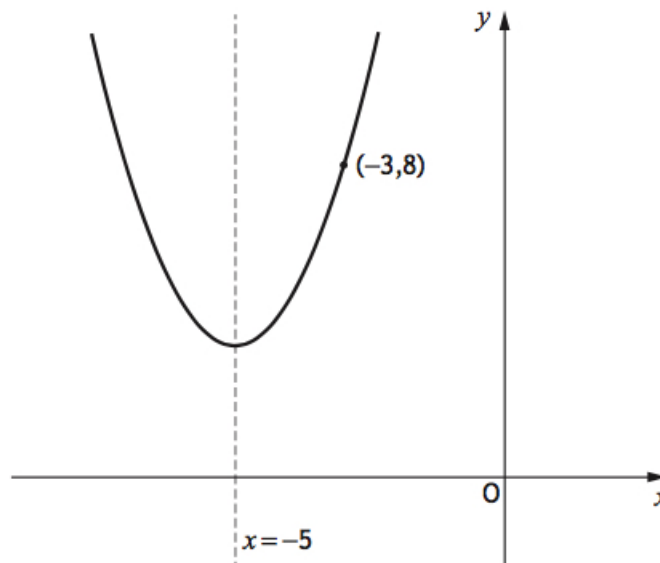
The lines intersect at the point P .
Find, **algebraically**, the coordinates of P .

Solution

$$\begin{aligned}3x - y = 2 &\Rightarrow y = 3x - 2 \\ \Rightarrow x + 3(3x - 2) &= 19 \\ \Rightarrow x + 9x - 6 &= 19 \\ \Rightarrow 10x &= 25 \\ \Rightarrow x &= \underline{\underline{2\frac{1}{2}}} \\ \Rightarrow y &= \underline{\underline{5\frac{1}{2}}}.\end{aligned}$$

14. The graph below shows a parabola with equation of the form

$$y = (x + a)^2 + b.$$



The equation of the axis of symmetry of the parabola is $x = -5$.

(a) State the value of a .

(1)

Solution

$a = 5$.

The point $(-3, 8)$ lies on the parabola.

(b) Calculate the value of b .

(2)

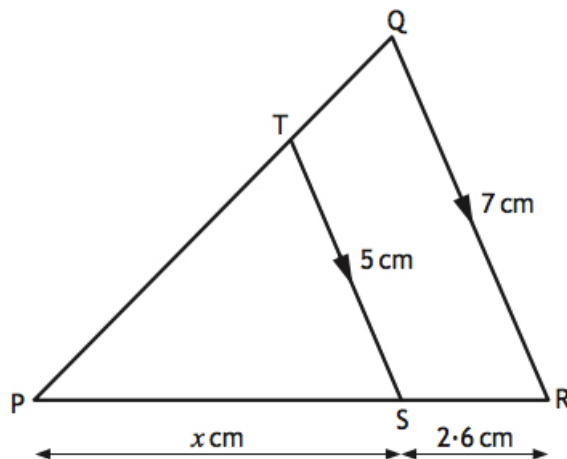
Solution

$$\begin{aligned}(-3 + 5)^2 + b &= 8 \Rightarrow 2^2 + b = 8 \\ &\Rightarrow 4 + b = 8 \\ &\Rightarrow \underline{b = 4}.\end{aligned}$$

15. In the diagram below:

(3)

- TS is parallel to QR ,
- $TS = 5$ centimetres,
- $QR = 7$ centimetres, and
- $SR = 2.6$ centimetres,



The length of PS is x centimetres.

Calculate the value of x .

Solution

$$\begin{aligned}\frac{PS}{ST} = \frac{PR}{QR} &\Rightarrow \frac{x}{5} = \frac{x + 2.6}{7} \\ &\Rightarrow 7x = 5(x + 2.6) \\ &\Rightarrow 7x = 5x + 13 \\ &\Rightarrow 2x = 13 \\ &\Rightarrow \underline{\underline{x = 6\frac{1}{2}}}.\end{aligned}$$