Dr Oliver Mathematics
AQA GCSE Mathematics
2018 Jun Paper 3: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Circle the decimal that is closest in value to (1)

$$\frac{11}{20}.$$

$$0.56 \quad 0.6 \quad 0.525 \quad 0.5$$

Solution

Well,

$$\frac{11}{20} = 0.55$$

so

$$\underline{\underline{0.56}} \quad 0.6 \quad 0.525 \quad 0.5$$

2. Circle the list of all the integers that satisfy (1)

$$-2 < x \leqslant 4.$$

$$-2, -1, 0, 1, 2, 3 \quad -1, 0, 1, 2, 3 \quad -2, -1, 0, 1, 2, 3, 4 \quad -1, 0, 1, 2, 3, 4$$

Solution

$$-2, -1, 0, 1, 2, 3 \quad -1, 0, 1, 2, 3 \quad -2, -1, 0, 1, 2, 3, 4 \quad \underline{\underline{-1, 0, 1, 2, 3, 4}}$$

3. Circle the largest number. (1)

$$3.2\dot{7} \quad 3.27 \quad 3.277 \quad 3.20\dot{7}$$

Solution

$$\underline{\underline{3.2\dot{7}}} \quad 3.27 \quad 3.277 \quad 3.20\dot{7}$$

4. What is the size of an exterior angle of a regular decagon? (1)
Circle your answer.

$$18^\circ \quad 36^\circ \quad 144^\circ \quad 162^\circ$$

Solution

A regular decagon has 10 sides:

$$\frac{360}{10} = 36^\circ$$

so

$$18^\circ \quad \underline{\underline{36^\circ}} \quad 144^\circ \quad 162^\circ$$

5. a is a common factor of 72 and 120. (4)
b is a common multiple of 6 and 9.

Work out the highest possible value of

$$\frac{a}{b}.$$

Solution

$$\begin{array}{r|l} & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 2 & 9 \\ \hline 3 & 3 \\ \hline 3 & 1 \\ \hline \end{array}$$

and

$$\begin{array}{r|l} & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 2 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

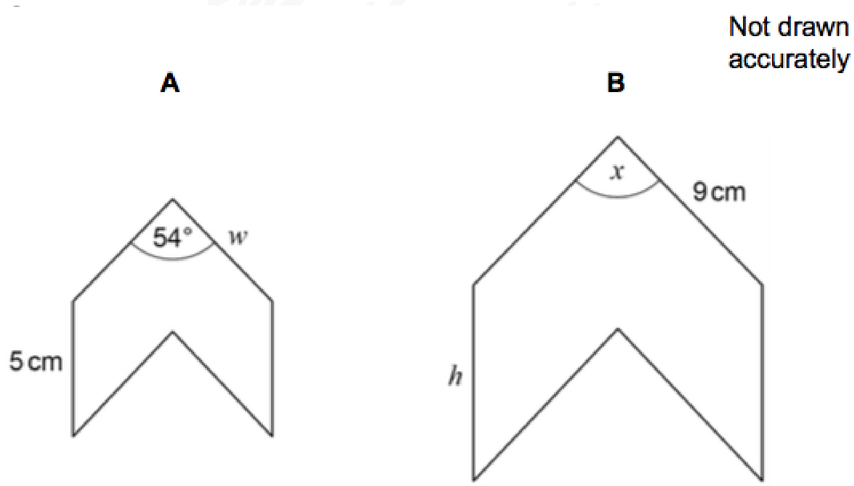
So
$$2^3 \times 3^2 \text{ and } 2^3 \times 3 \times 5.$$

Hence
$$a = 2^3 \times 3 = 24 \text{ as the maximum.}$$

Now, b is common multiple of 6 and 9:
$$18, 36, 54, \ldots$$

so we choose $b = 18$, as the minimum. Hence,
$$\frac{a}{b} = \frac{24}{18} = \underline{\underline{\tfrac{4}{3}}}.$$

6. **A** and **B** are similar shapes. (3)
B is an enlargement of **A** with scale factor 1.5.



Work out the values of x, h, and w.

Solution

Well,

$$\begin{aligned} x &= \underline{\underline{54^\circ}}; \\ h &= 1.5 \times 5 \\ &= \underline{\underline{7.5\text{ cm}}}; \\ w &= \frac{9}{1.5} \\ &= \underline{\underline{6\text{ cm}}}. \end{aligned}$$

7. (4)

Investment A : Save £150 per month for 2 years
2.5% interest is added to the total amount saved

Investment B : Invest £3 500
Compound interest is added at 3% per year

After 2 years, how much **more** is investment B worth than investment A?

Solution

Investment A:

$$\begin{aligned} (150 \times 24) \times 1.025 &= 3\,600 \times 1.025 \\ &= £3\,690. \end{aligned}$$

Investment B:

$$3\,500 \times (1.03)^2 = £3\,713.15.$$

So, the difference is

$$3\,713.15 - 3\,690 = \underline{\underline{23.15}}.$$

8. (a) Show that the lines (3)

$$y = 3x + 7 \text{ and } 2y - 6x = 8$$

are parallel.
Do **not** use a graphical method.

Solution

$$\begin{aligned} 2y - 6x = 8 &\Rightarrow 2y = 6x + 8 \\ &\Rightarrow y = 3x + 4. \end{aligned}$$

As the gradients are the same, the lines are parallel.

(b) Is the point $(-5, -6)$ above, below or on the line $y = 3x + 7$? (2)
Tick **one** box.

☐ Above ☐ Below ☐ On the line

You **must** show your working.
Do **not** use a graphical method.

Solution

Well,

$$\begin{aligned} x = -5 &\Rightarrow y = 3(-5) + 7 \\ &\Rightarrow y = -15 + 7 \\ &\Rightarrow y = -8 \end{aligned}$$

so the answer is left one: Above.

9. The cost of a ticket increases by 10% to £19.25. (3)

Work out the original cost.

Solution

$$\begin{aligned} \text{Original cost} &= \frac{19.25}{1.1} \\ &= \text{£}17.50. \end{aligned}$$

10. The nth term of a sequence is (3)

$$12n - 5.$$

Work out the numbers in the sequence that

have two digits and are **not** prime.

Solution

Well,

$$\begin{aligned}
n = 1 &\Rightarrow u_1 = 7 \text{ (only one digit)} \\
n = 2 &\Rightarrow u_2 = 19 \text{ (prime)} \\
n = 3 &\Rightarrow u_3 = 31 \text{ (prime)} \\
n = 4 &\Rightarrow u_4 = 43 \text{ (prime)} \\
n = 5 &\Rightarrow u_5 = 55\,(5 \times 11) \\
n = 6 &\Rightarrow u_6 = 67 \text{ (prime)} \\
n = 7 &\Rightarrow u_7 = 79 \text{ (prime)} \\
n = 8 &\Rightarrow u_8 = 91\,(7 \times 13) \\
n = 9 &\Rightarrow u_9 = 103 \text{ (has three digits)}.
\end{aligned}$$

Hence,

$$\underline{\underline{55 \text{ and } 91}}.$$

11.

$$\mathbf{a} = \begin{pmatrix} 6 \\ -10 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \text{ and } \mathbf{c} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}.$$

(a) Work out (2)

$$\mathbf{a} + \mathbf{b} + \mathbf{c}.$$

Solution

$$\begin{aligned}
\mathbf{a} + \mathbf{b} + \mathbf{c} &= \begin{pmatrix} 6 \\ -10 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ 7 \end{pmatrix} \\
&= \underline{\underline{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}}.
\end{aligned}$$

(b) Show that (2)

$$\mathbf{a} + 2\mathbf{c}$$

is parallel to is $\mathbf{b}$.

Solution

$$\begin{aligned}\mathbf{a} + 2\mathbf{c} &= \begin{pmatrix} 6 \\ -10 \end{pmatrix} + 2\begin{pmatrix} -4 \\ 7 \end{pmatrix} \\ &= \begin{pmatrix} -2 \\ 4 \end{pmatrix} \\ &= 2\begin{pmatrix} -1 \\ 2 \end{pmatrix} \\ &= 2\mathbf{b};\end{aligned}$$

hence, $\mathbf{a}+2\mathbf{c}$ is parallel to is $\mathbf{b}$.

12. A force of 40 Newtons is applied to an area of 3.2 square metres. (2)

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the pressure.
Give the units of your answer.

Solution

$$\begin{aligned}\text{Pressure} &= \frac{40}{3.2} \\ &= 12.5 \text{ Pa or N/m}^2.\end{aligned}$$

13. Tick **all** the statements that are true for any rhombus. (1)

☐ The diagonals are lines of symmetry

☐ The diagonals bisect each other

☐ The diagonals are perpendicular

☐ The diagonals are equal in length

Solution

Tick the top three boxes:

The diagonals are lines of symmetry,

The diagonals bisect each other,

and

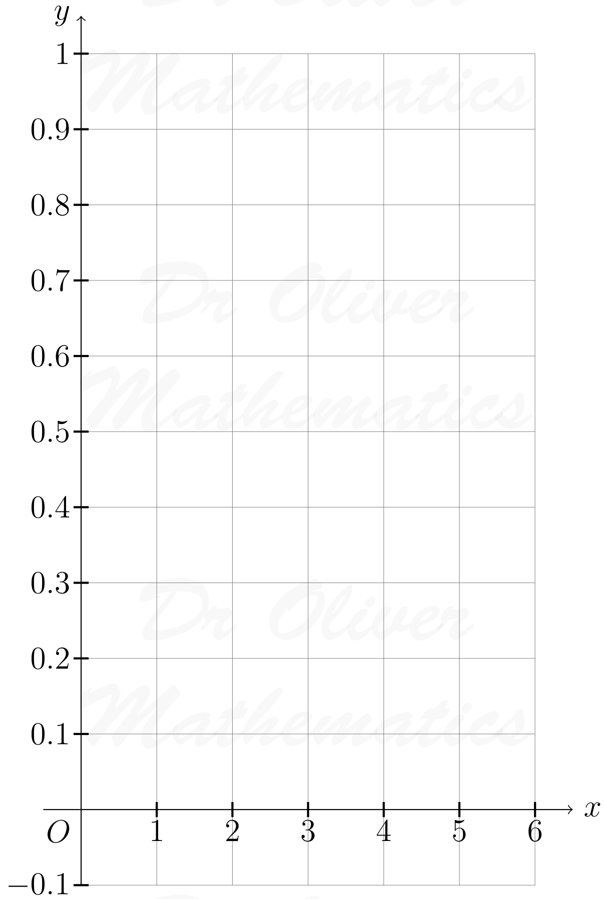
The diagonals are perpendicular.

14. Draw the graph of (3)

$$y = 0.8^x$$

for values of x from 0 to 6.

x	0	1	2	3	4	5	6
y							

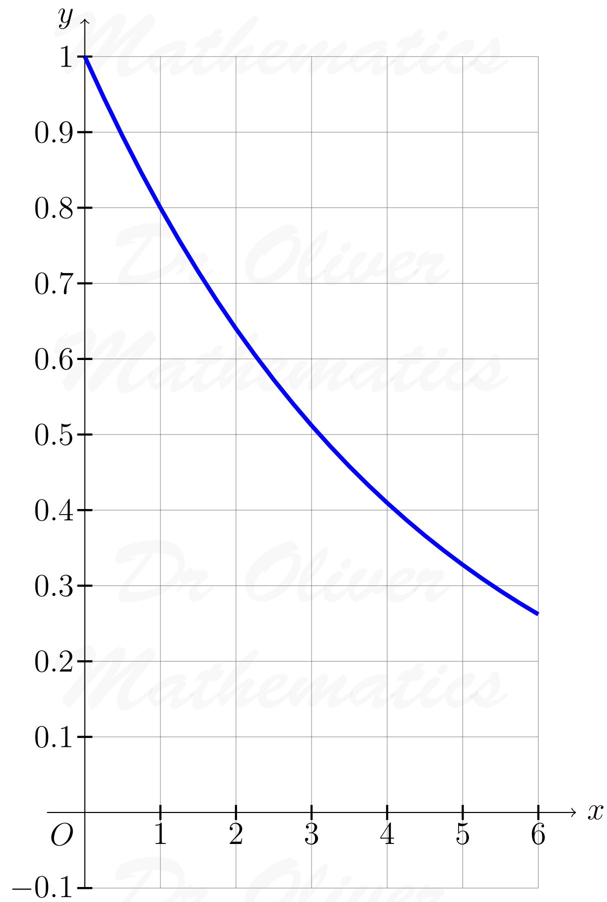


Solution

We will make up a table:

x	0	1	2	3	4	5	6
y	1	0.8	0.64	0.512	0.409 6	0.327 68	0.262 144

and we will draw the graph:



15. Amy has x beads. (1)

Billy has three more beads than Amy.

Carly has four times as many beads as Billy.

Circle the expression for the number of beads that Carly has.

$$4x + 3 \quad 3x + 4 \quad 4(x + 3) \quad x + 12$$

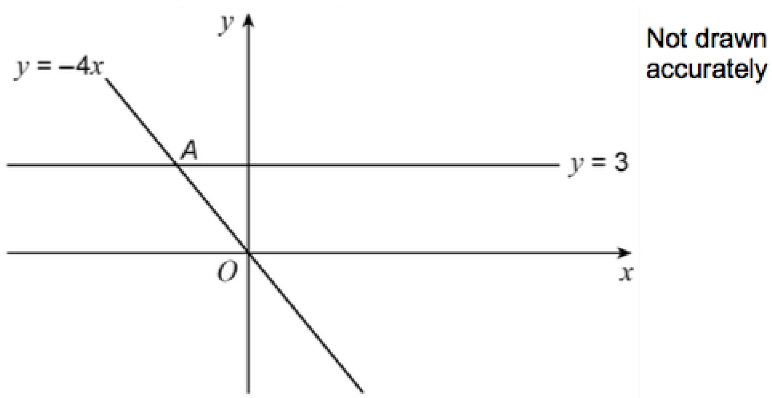
Solution

Amy : x
Billy : $x + 3$
Carly : $4(x + 3)$

so

$$4x + 3 \quad 3x + 4 \quad \underline{\underline{4(x + 3)}} \quad x + 12$$

16. Two straight lines intersect at point A. (1)



Circle the coordinates of A.

$$(-\tfrac{3}{4}, 3) \quad (-4, 3) \quad (-12, 3) \quad (-\tfrac{4}{3}, 3)$$

Solution

Well,

$$-4x = 3 \Rightarrow x = -\tfrac{3}{4}$$

so

$$\underline{\underline{(-\tfrac{3}{4}, 3)}} \quad (-4, 3) \quad (-12, 3) \quad (-\tfrac{4}{3}, 3)$$

17. Here are two methods to make a 4-digit code. (3)
Codes can have repeated digits.

Method A
For the first two digits use an odd number between 30 and 100
For the last two digits use a multiple of 11

Method B				
Use four digits in the order	even	odd	even	odd
Do **not** use the digit zero				

Which method gives the **greater** number of possible codes?
You **must** show your working.

Solution

Method A:
How many odd numbers are there between 30 and 100?

$$\frac{70}{2} = 35 \text{ ways.}$$

How many multiples of 11? 9 ways.
Hence, there are

$$35 \times 9 = 315 \text{ ways.}$$

Method B:
Even odd even odd:

$$4 \times 5 \times 4 \times 5 = 400 \text{ ways.}$$

Hence, there are more ways in Method B.

18. Show that, for $x \neq 0$, (3)

$$\frac{x+4}{3x} - \frac{5}{2x}$$

can be written in the form

$$\frac{ax+b}{cx},$$

where a, b, and c are integers.

Solution

LCM(3, 2) = 6:

$$\begin{aligned}\frac{x+4}{3x} - \frac{5}{2x} &= \frac{2(x+4)}{6x} - \frac{3(5)}{6x} \\ &= \frac{2x+8-15}{6x} \\ &= \underline{\underline{\frac{2x-7}{6x}}};\end{aligned}$$

so,

$$\underline{\underline{a = 2,\ b = -7,\ \text{and } c = 6}}.$$

19. The equation of a straight line is (1)

$$3x + 2y = 24.$$

Circle the point where the line crosses the x-axis.

$(0, 8)$ $(12, 0)$ $(0, 12)$ $(8, 0)$

Solution

Well,

$$\begin{aligned}3x + 2y = 24 &\Rightarrow 2y = -3x + 24 \\ &\Rightarrow y = -\tfrac{3}{2}x + 12\end{aligned}$$

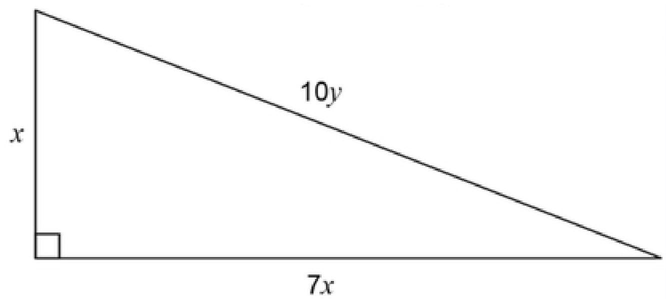
so

$$\begin{aligned}y = 0 &\Rightarrow 0 = -\tfrac{3}{2}x + 12 \\ &\Rightarrow \tfrac{3}{2}x = 12 \\ &\Rightarrow x = 8.\end{aligned}$$

Hence,

$(0, 8)$ $(12, 0)$ $(0, 12)$ $\underline{\underline{(8, 0)}}$

20. All dimensions are in centimetres. (3)



Use Pythagoras' theorem to work out the exact value of

$$\frac{x}{y}.$$

Solution

$$\begin{aligned} x^2 + (7x)^2 = (10y)^2 &\Rightarrow x^2 + 49x^2 = 100y^2 \\ &\Rightarrow 50x^2 = 100y^2 \\ &\Rightarrow x^2 = 2y^2 \\ &\Rightarrow x = \sqrt{2}y \text{ (as all dimensions are positive)} \\ &\Rightarrow \underline{\underline{\frac{x}{y} = \sqrt{2}.}} \end{aligned}$$

21. The mass of an ornament is m grams.
The height of the ornament is h centimetres.
m is directly proportional to the cube of h.
$m = 1\,600$ when $h = 8$.

(a) Work out an equation connecting m and h. (3)

Solution

Now,

$$m \propto h^3 \Rightarrow m = kh^3,$$

for some constant k. Next,

$$\begin{aligned} m = 1\,600, h = 8 &\Rightarrow 1\,600 = k \times 8^3 \\ &\Rightarrow 1\,600 = 512k \\ &\Rightarrow k = \tfrac{25}{8} \end{aligned}$$

and so

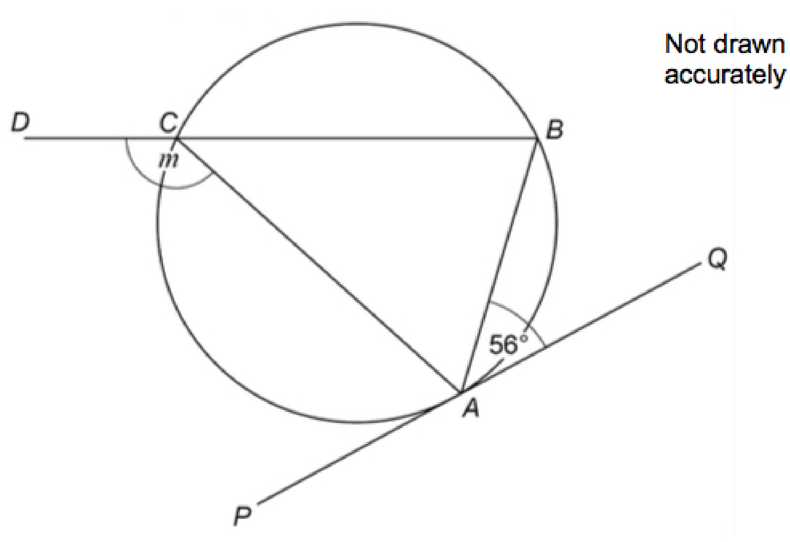
$$m = \tfrac{25}{8}h^3.$$

(b) Work out the mass of an ornament of height 12 centimetres. (2)

Solution

$$\begin{aligned} h = 12 &\Rightarrow m = \tfrac{25}{8} \times 12^3 \\ &\Rightarrow m = \tfrac{25}{8} \times 1728 \\ &\Rightarrow m = 5\,400 \text{ grams} \end{aligned}$$

22. A, B, and C are points on a circle. (1)
DCB is a straight line.
PAQ is a tangent to the circle.



Sam is trying to work out the size of angle m.
Here is his working.

angle $ACB = 56°$	angles in the same segment are equal
$m = 180° - 56°$	angles at a point on a straight line add up to 180°
$m = 124°$	

Make a criticism of his working.

Solution

He means alternate segment theorem in the first line.

23. A sequence of numbers is formed by the iterative process (2)

$$u_{n+1} = \frac{3}{u_n + 1}, \; u_1 = 4.$$

Work out the values of u_2 and u_3.

Solution

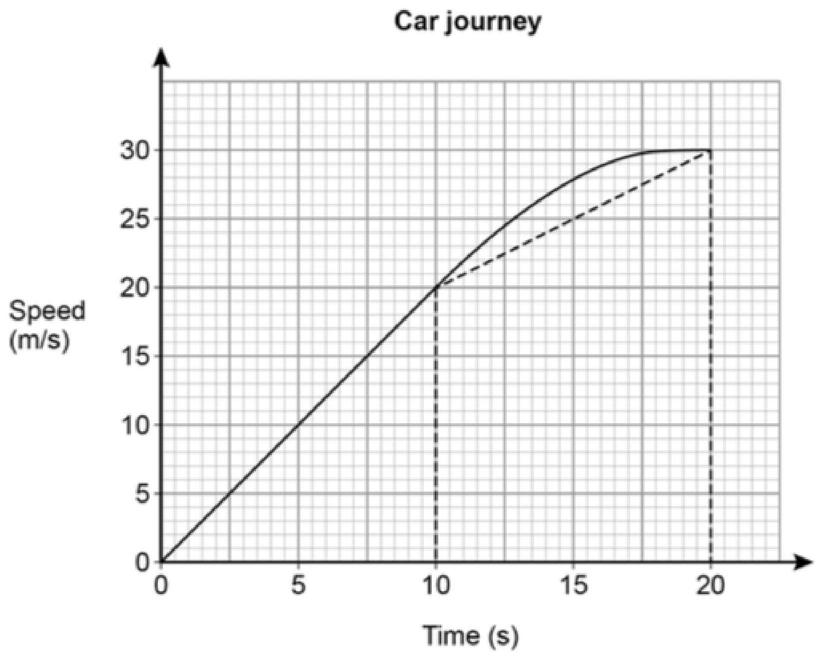
Well,

$$\begin{aligned} u_2 &= \frac{3}{u_1 + 1} \\ &= \frac{3}{4 + 1} \\ &= \underline{\underline{\tfrac{3}{5}}} \end{aligned}$$

and

$$\begin{aligned} u_3 &= \frac{3}{u_2 + 1} \\ &= \frac{3}{\frac{3}{5} + 1} \\ &= \underline{\underline{1\tfrac{7}{8}}}. \end{aligned}$$

24. The speed-time graph shows 20 seconds of a car journey.
Harry wants to estimate the distance the car travels in this time.



He uses a triangle and a trapezium, as shown, to estimate the area under the graph.

(a) Complete Harry's method to estimate the distance the car travels. (3)

Solution

$$\begin{aligned}\text{Distance} &= (\tfrac{1}{2} \times 10 \times 20) + [\tfrac{1}{2}(20+30)(10)] \\ &= 100 + 250 \\ &= \underline{\underline{350 \text{ m}}}.\end{aligned}$$

(b) For this journey, which of these is true for Harry's method? (1)
Tick **one** box.

☐ It works out an overestimate of the distance

☐ It works out an underestimate of the distance

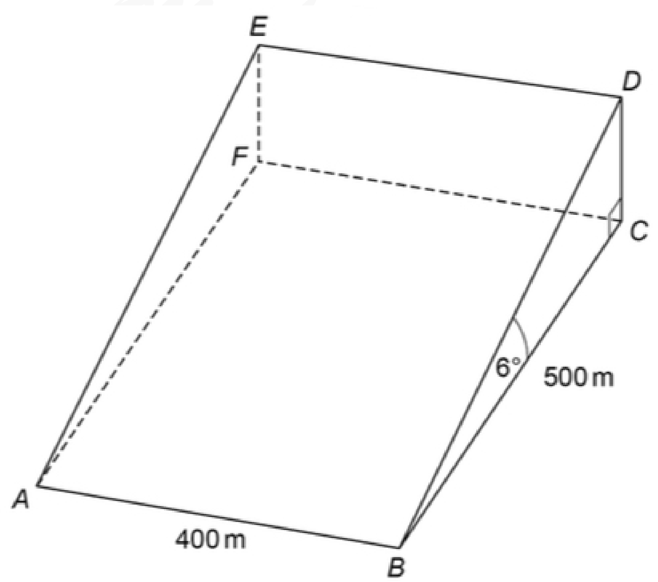
☐ It could work out an overestimate or an underestimate of the distance

Solution

Tick the second box: It works out an underestimate of the distance.

25. $ABCDEF$ is a triangular prism which represents part of a hill.

- $ABCF$ is the horizontal rectangular base.
- D is vertically above C.

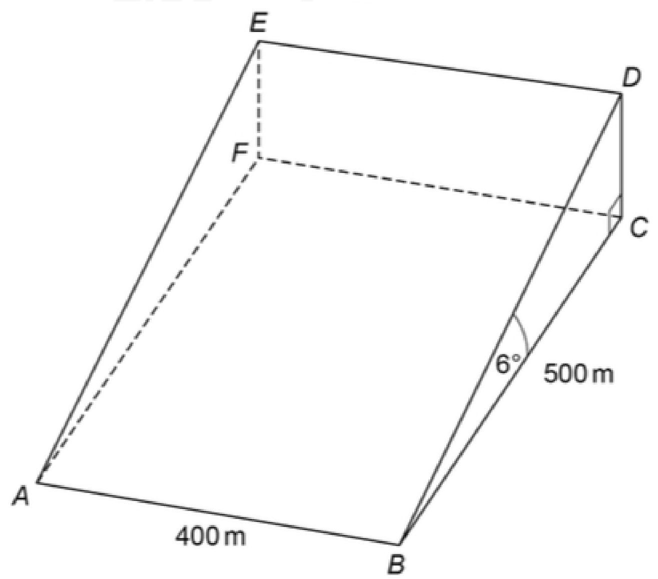


(a) Work out the height CD. (2)

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 6^\circ = \frac{CD}{500}$$
$$\Rightarrow CD = 500 \tan 6^\circ$$
$$\Rightarrow CD = 52.552\,117\,63 \text{ (FCD)}$$
$$\Rightarrow CD = 52.6 \text{ cm (3 sf)}.$$

Jamil walks in a straight line from A to D.



(b) Work out the size of angle DAC. (4)
You **must** show your working.

Solution

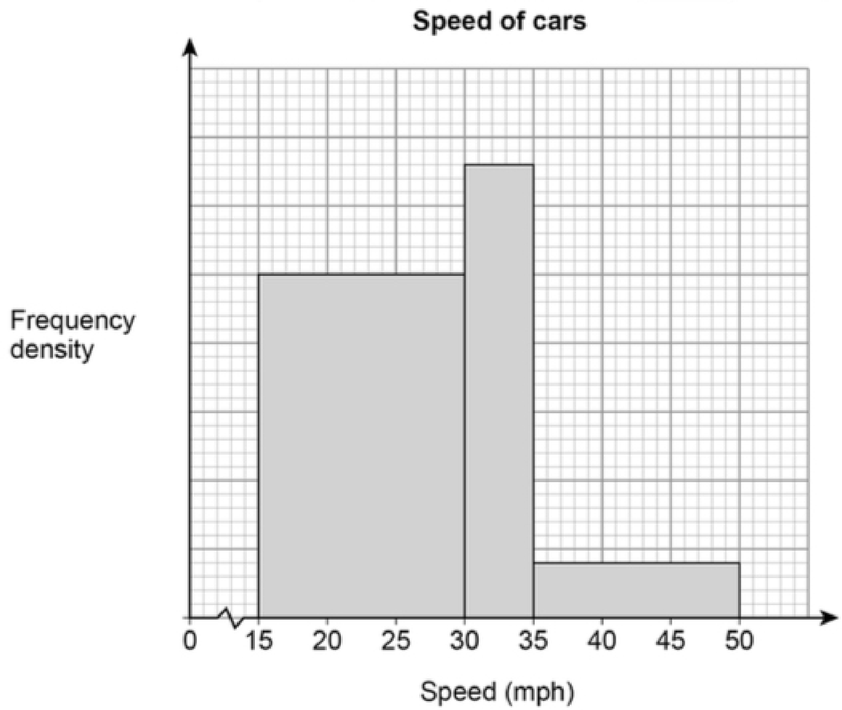
Pythagoras' theorem:

$$\begin{aligned} AC^2 = AB^2 + BC^2 &\Rightarrow AD^2 = 400^2 + 500^2 \\ &\Rightarrow AC^2 = 410\,000 \\ &\Rightarrow AC = \sqrt{410\,000} \end{aligned}$$

and

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan DAC = \frac{52.552\ldots}{\sqrt{410\,000}} \\ &\Rightarrow \angle DAC = 4.691\,898\,799 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\angle DAC = 4.69^\circ \text{ (3 sf)}}}. \end{aligned}$$

26. The histogram shows information about the speed of cars as they pass a checkpoint.
The scale on the frequency density axis is missing.



The histogram shows information about 480 cars.

(a) How many cars does the first bar represent? (4)

Solution

Let us count five squares on the frequency density axis as 1 unit.

Interval	Width	Frequency Density	Frequency
$15-30$	15	5	$15 \times 5 = 75$
$30-35$	5	6.6	$5 \times 6.6 = 33$
$35-50$	15	0.8	$15 \times 0.8 = 12$
Total			120

Now,

$$\frac{480}{120} = 4,$$

and we multiply by 4:

Interval	Width	Frequency Density	Frequency
$15-30$	15	5	$\underline{\underline{300}}$
$30-35$	5	6.6	132
$35-50$	15	0.8	48
Total			120

Cars with a speed greater than 40 mph are over the speed limit.

(b) Use the histogram to estimate the number of cars that are over the speed limit. (2)

Solution

$$\tfrac{2}{3} \times 48 = \underline{\underline{32 \text{ cars}}}.$$

27. A bag contains 30 discs. (3)
10 are red and 20 are blue.

- One disc is taken out at random and replaced by **two** of the other colour.
- Another disc is then taken out at random and replaced by **two** of the other colour.
- Another disc is then taken out at random.

Work out the probability that all three discs taken out are red.

Solution

Well,

$$\begin{aligned} &10 \text{ are red and } 20 \text{ are blue } (30 \text{ counters}) \\ \text{one red} \rightarrow\ &9 \text{ are red and } 22 \text{ are blue } (31 \text{ counters}) \\ \text{one red} \rightarrow\ &8 \text{ are red and } 24 \text{ are blue } (32 \text{ counters}) \end{aligned}$$

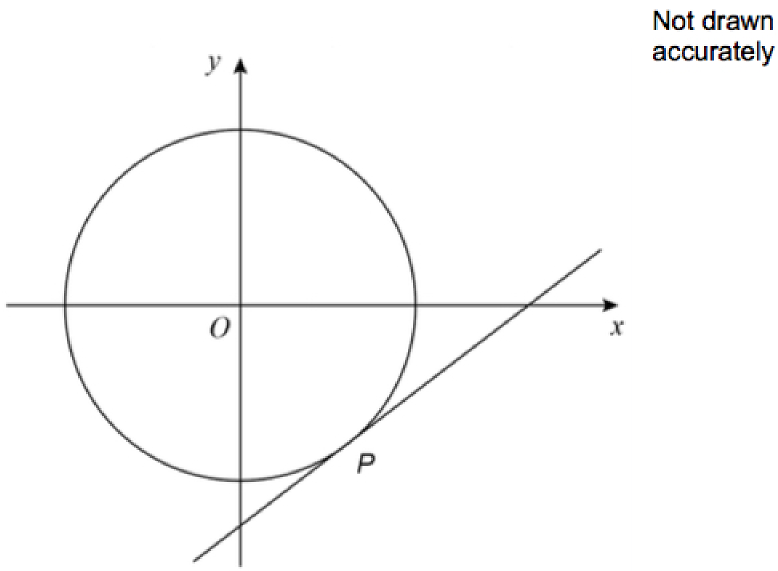
and

$$\begin{aligned} \mathrm{P}(R, R, R) &= \tfrac{10}{30} \times \tfrac{9}{31} \times \tfrac{8}{32} \\ &= \underline{\underline{\tfrac{3}{124}}}. \end{aligned}$$

28. P is a point on the circle with equation (5)

$$x^2 + y^2 = 80.$$

P has x-coordinate 4 and is below the x-axis.



Work out the equation of the tangent to the circle at P.

Solution

Well,

$$\begin{aligned} x = 4 \Rightarrow 4^2 + y^2 &= 80 \\ \Rightarrow 16 + y^2 &= 80 \\ \Rightarrow y^2 &= 64 \\ \Rightarrow y &= \pm 8; \end{aligned}$$

P is below the x-axis and that means $P(4, -8)$. Now,

$$\begin{aligned} \text{gradient}_{OP} &= \frac{-8-0}{4-0} \\ &= -2 \end{aligned}$$

which means

$$m_{normal} = -\frac{1}{-2} = -\tfrac{1}{2}.$$

Hence, the tangent to the circle at P is

$$\begin{aligned} y - (-8) = -\tfrac{1}{2}(x-4) &\Rightarrow y + 8 = \tfrac{1}{2}x - 2 \\ &\Rightarrow \underline{\underline{y = -\tfrac{1}{2}x - 10}}. \end{aligned}$$