Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2007 November Paper 2: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. The two variables x and y are related by the equation

$$yx^2 = 800.$$

   (a) Obtain an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x. (2)

   (b) Hence find the approximate change in y as x increases from 10 to $10 + p$, where p is small. (2)

2. Solve the equation (5)

$$3\sin(\tfrac{1}{2}x - 1) = 1,$$

   for $0 < x < 6\pi$ radians.

3. (a) Express (1)

$$9^{x+1}$$

   as a power of 3.

   (b) Express (1)

$$\sqrt[3]{27^{2x}}$$

   as a power of 3.

   (c) Express (3)

$$\frac{54 \times \sqrt[3]{27^{2x}}}{9^{x+1} + 216(3^{2x-1} - 1)}$$

   as a fraction in its simplest form.

4. A cycle shop sells three models of racing cycles, A, B, and C.

   The table below shows the numbers of each model sold over a four-week period and the cost of each model in £.

	A	B	C
1	8	12	4
2	7	10	2
3	10	12	0
4	6	8	4
Cost, £	300	500	800

In the first two weeks the shop banked 30% of all money received, but in the last two weeks the shop only banked 20% of all money received.

(a) Write down three matrices such that matrix multiplication will give the total amount of money banked over the four-week period. (2)

(b) Hence evaluate this total amount. (4)

5. (a) Expand (1)

$$(1+x)^5.$$

(b) Hence express (3)

$$(1+\sqrt{2})^5$$

in the form

$$a+b\sqrt{2},$$

where a and b are integers.

(c) Obtain the corresponding result for (2)

$$(1-\sqrt{2})^5$$

and **hence** evaluate

$$(1+\sqrt{2})^5+(1-\sqrt{2})^5.$$

6. Two circular flower beds have a combined area of $\frac{29}{2}\pi$ m^2. (6)

The sum of the circumferences of the two flower beds is 10π m.

Determine the radius of each flower bed.

7. The position vectors of points A and B, relative to an origin O, are $2\mathbf{i}+4\mathbf{j}$ and $6\mathbf{i}+10\mathbf{j}$ respectively.

The position vector of C, relative to O, is $k\mathbf{i}+25\mathbf{j}$, where k is a positive constant.

(a) Find the value of k for which the length of BC is 25 units. (3)

(b) Find the value of k for which ABC is a straight line. (3)

8. Given that $x \in \mathbb{R}$ and that (7)

- $\mathscr{E} = \{x : 2 < x < 10\}$,
- $A = \{x : 3x + 2 < 20\}$, and
- $B = \{x : x^2 < 11x - 28\}$,

find the set of values of x which define

(a) $A \cap B$,

(b) $(A \cup B)'$.

9. A particle travels in a straight line so that, t s after passing through a fixed point O, its speed, v ms^{-1}, is given by

$$v = 8\cos(\tfrac{1}{2}t).$$

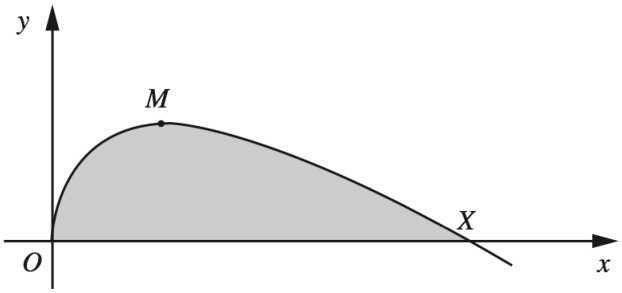
(a) Find the acceleration of the particle when $t = 1$. (3)

The particle first comes to instantaneous rest at the point P.

(b) Find the distance OP. (4)

10. The diagram shows part of the curve

$$y = 4\sqrt{x} - x.$$



The origin O lies on the curve and the curve intersects the positive x-axis at X.

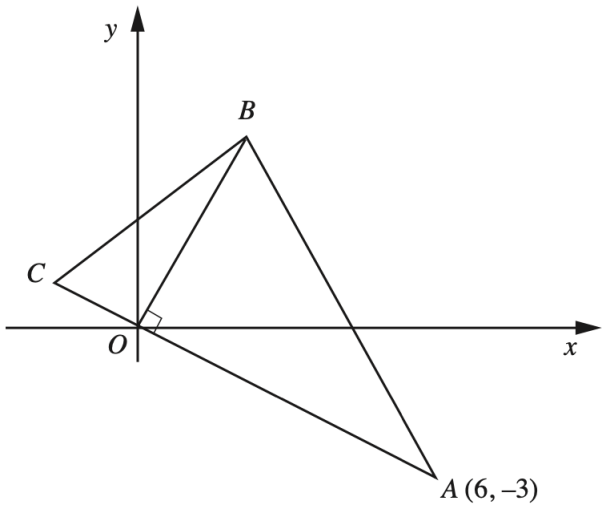
The maximum point of the curve is at M.

Find

(a) the coordinates of X and of M, (5)

(b) the area of the shaded region. (4)

11. **Solutions to this question by accurate drawing will not be accepted.**
The diagram shows a triangle ABC in which A is the point $(6, -3)$.



The line AC passes through the origin O.

The line OB is perpendicular to AC.

(a) Find the equation of OB. (2)

The area of triangle AOB is 15 units^2.

(b) Find the coordinates of B. (3)

The length of AO is 3 times the length of OC.

(c) Find the coordinates of C. (2)

The point D is such that the quadrilateral $ABCD$ is a kite.

(d) Find the area of $ABCD$. (2)

EITHER

12. The function f is defined, for $x > 0$, by

$$\mathrm{f} : x \mapsto \ln x.$$

(a) State the range of f. (1)

(b) State the range of f^{-1}. (1)

(c) On the same diagram, sketch and label the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. (2)

The function g is defined, for $x > 0$, by

$$\mathrm{g} : x \mapsto 3x + 2.$$

(d) Solve the equation (2)

$$\mathrm{f}\,\mathrm{g}(x) = 3.$$

(e) Solve the equation (4)

$$\mathrm{f}^{-1}\,\mathrm{g}^{-1}(x) = 7.$$

OR

13. (a) Find the values of k for which (4)

$$y = kx + 2$$

is a tangent to the curve

$$y = 4x^2 + 2x + 3.$$

(b) Express (3)

$$4x^2 + 2x + 3$$

in the form

$$a(x + b)^2 + c,$$

where a, b, and c are constants.

(c) Determine, with explanation, whether or not the curve (2)

$$y = 4x^2 + 2x + 3$$

meets the x-axis.

The function f is defined by

$$\mathrm{f} : x \mapsto 4x^2 + 2x + 3,$$

where $x \geqslant p$.

(d) Determine the smallest value of p for which f has an inverse. (1)