

Dr Oliver Mathematics
AQA GCSE Mathematics
2020 June Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

1. Which of these is a correct identity?

(1)

Circle your answer.

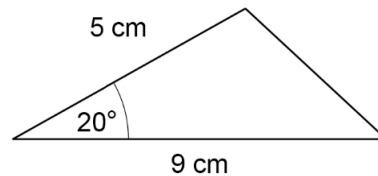
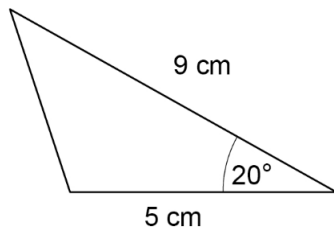
$$x + 4x \equiv 5x \quad 6x \equiv 18 \quad 2x + 1 \equiv 7 \quad 7x + 9 \equiv x$$

Solution

$$\underline{x + 4x \equiv 5x} \quad 6x \equiv 18 \quad 2x + 1 \equiv 7 \quad 7x + 9 \equiv x$$

2. Circle the reason why these triangles are congruent.

(1)



Not drawn
accurately

RHS ASA SSS SAS

Solution

RHS ASA SSS SAS

3. Circle the number that is written in standard form. (1)

$$0.9 \times 10^{-3} \quad 6 \times 10^{0.5} \quad 5.2 \times 10^{-4} \quad 12 \times 10^7$$

Solution

$$0.9 \times 10^{-3} \quad 6 \times 10^{0.5} \quad \underline{\underline{5.2 \times 10^{-4}}} \quad 12 \times 10^7$$

4. Circle the expression that has the **largest** value when $a < -1$. (1)

$$\frac{1}{2}a \quad a \quad a^2 \quad a^3$$

Solution

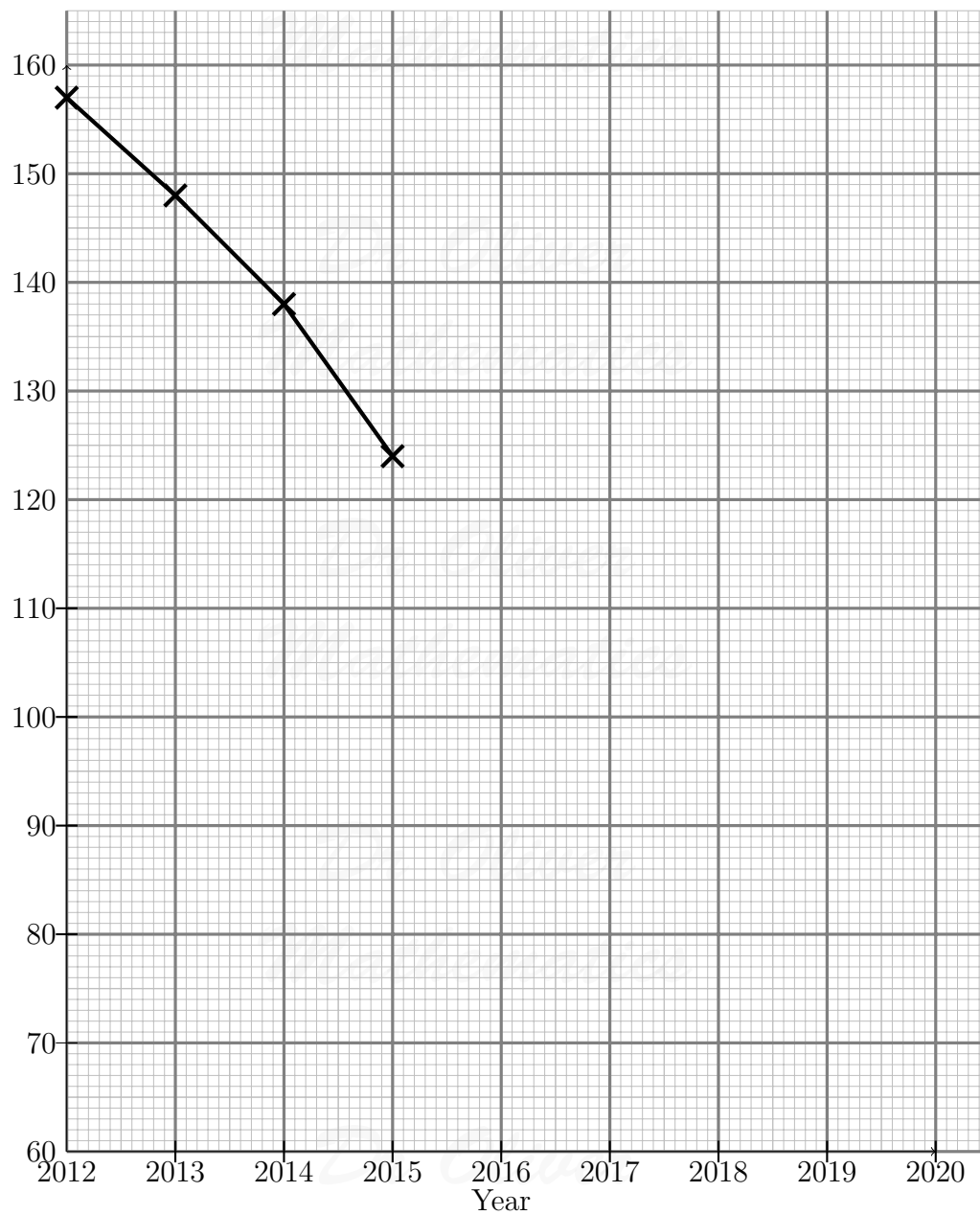
$$\frac{1}{2}a \quad a \quad \underline{\underline{a^2}} \quad a^3$$

5. The time students spent watching TV was recorded.
The table shows the average daily time per student each year from 2012 to 2019.

Year	2012	2013	2014	2015	2016	2017	2018	2019
Time (minutes)	157	145	138	124	113	100	90	82

A time series graph is drawn to represent the data.
The first four points have been plotted.

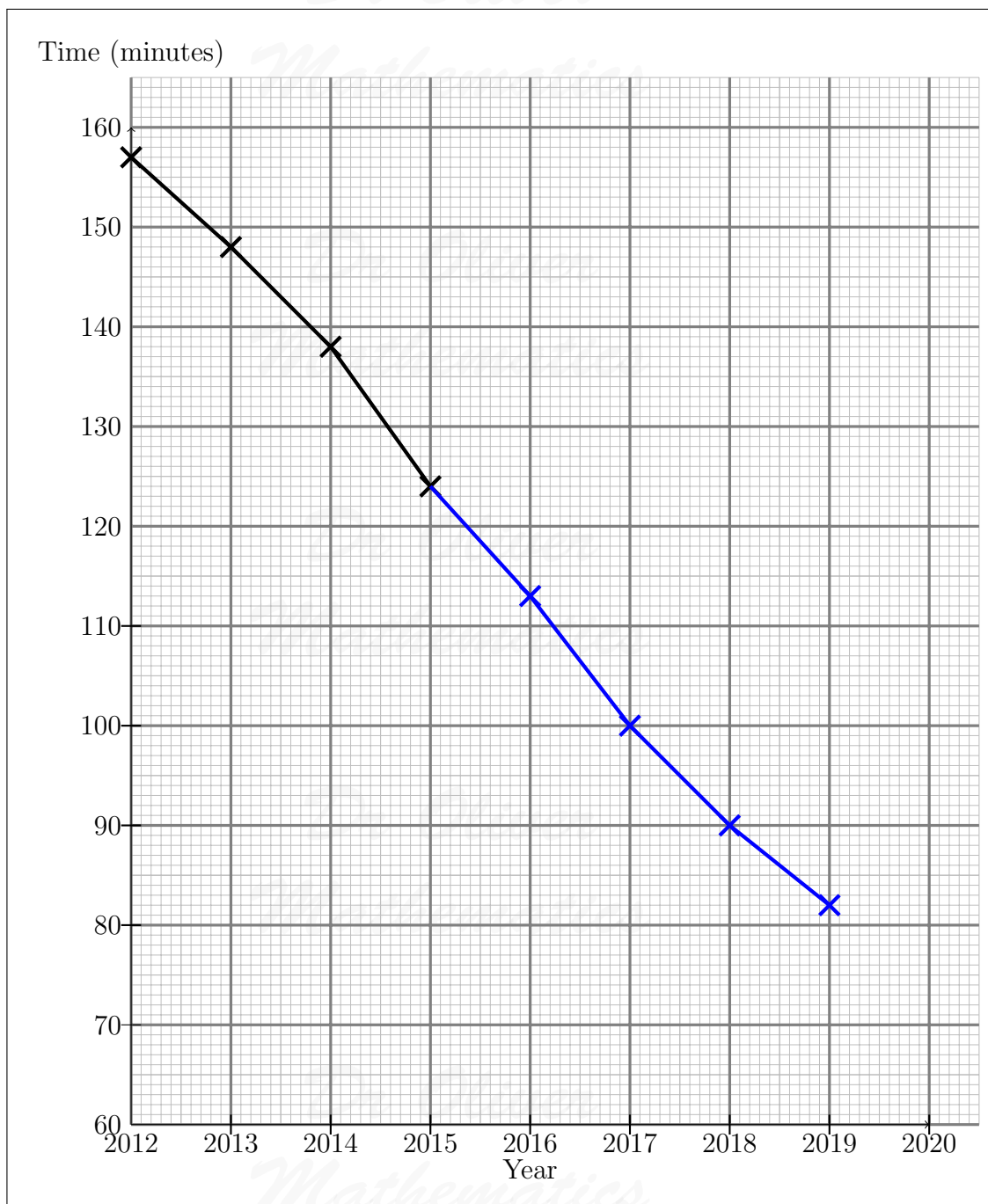
Time (minutes)



(a) Complete the graph.

(2)

Solution

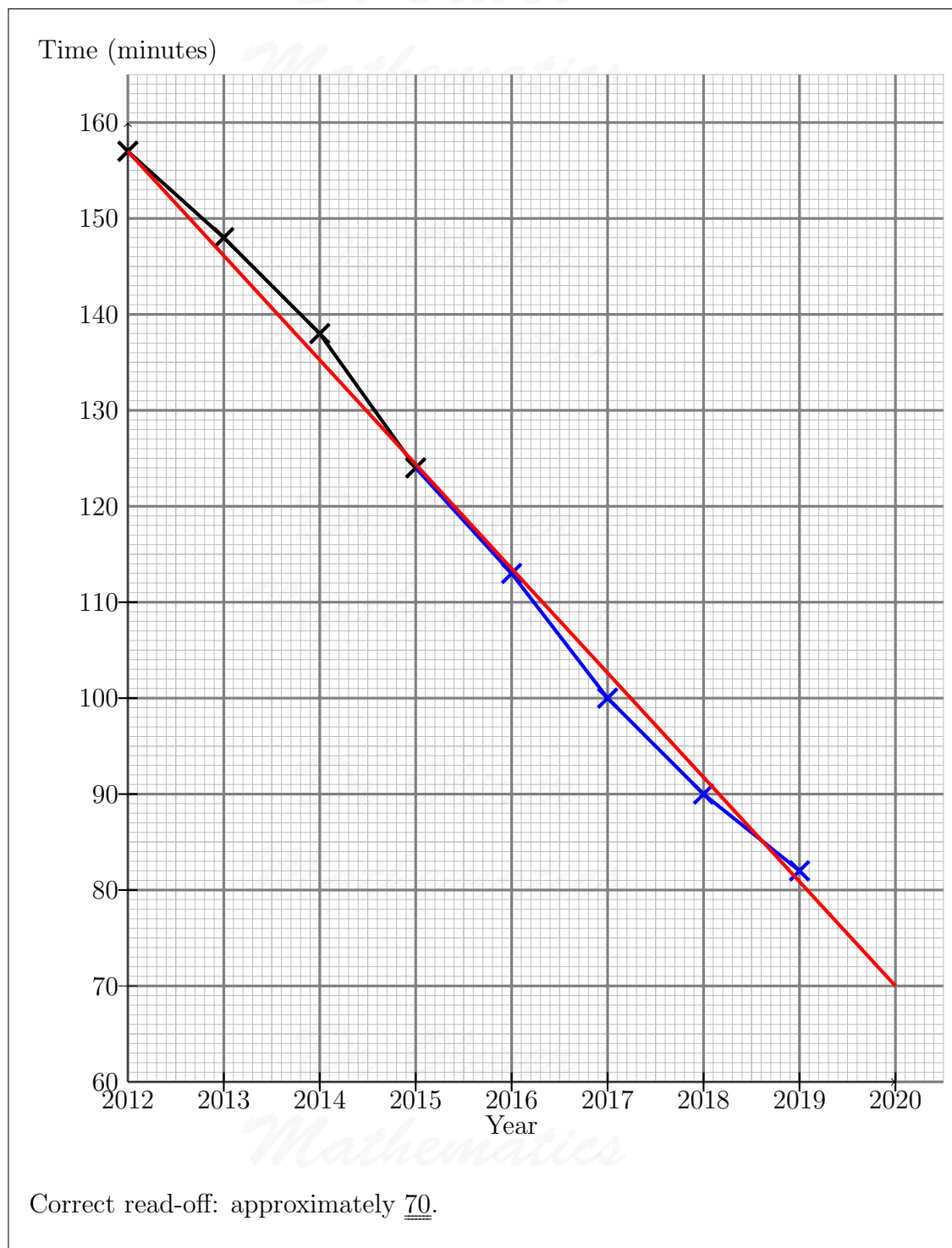


(b) Use the graph to estimate the average daily time per student in 2020.

(1)

Solution

Draw a line of best fit:



6. Work out the highest common factor (HCF) of 75 and 105.

(2)

Solution

Well,

$$\begin{array}{r|l} & 75 \\ 3 & 25 \\ 5 & 5 \\ 5 & 1 \end{array}$$

and

$$\begin{array}{r|l} & 105 \\ 3 & 35 \\ 5 & 7 \\ 7 & 1 \end{array}$$

so

$$75 = 3 \times 5^2 \text{ and } 105 = 3 \times 5 \times 7.$$

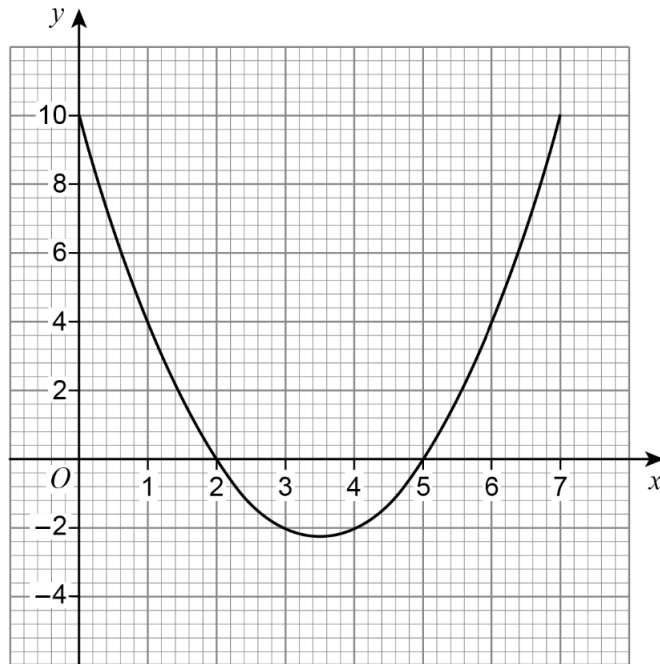
Hence, highest common factor is

$$3 \times 5 = \underline{\underline{15}}.$$

7. Here is the graph of

$$y = x^2 - 7x + 10$$

for values of x from 0 to 7.



- (a) Write down the roots of

$$x^2 - 7x + 10 = 0.$$

(2)

Solution

$$\underline{x = 2 \text{ or } x = 5.}$$

- (b) Write down the x -coordinate of the turning point of the curve.

(1)

Solution

3.5.

8. At a party there are 90 people.
48 are women and 42 are men.

(2)

Some women leave.
Some men arrive.

The ratio of women to men is now 10 : 11.

Are there now more than 90 people at the party?

Tick **one** box.

☐

Yes

☐

No

☐

Cannot tell

Show working to support your answer.

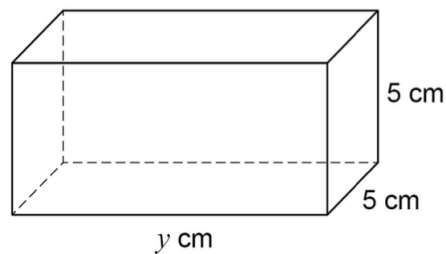
Solution

Well,

$$\begin{aligned}\text{women : men} &= 10 : 11 \\ &= 4 \times 10 : 4 \times 11 \\ &= 40 : 44;\end{aligned}$$

hence, we can be certain of the fact that are less than 90 people at the party: the answer is No.

9. Here is a cuboid.



(a) Assume that the total surface area of the cuboid is 200 cm^2 .

(3)

Work out the volume of the cuboid.

Solution

Well,

$$\begin{aligned}\text{total surface area} &= 200 \Rightarrow (4 \times 5y) + (2 \times 5^2) = 200 \\ &\Rightarrow 20y + 50 = 200 \\ &\Rightarrow 20y = 150 \\ &\Rightarrow y = 7.5\end{aligned}$$

and, finally,

$$\begin{aligned}\text{volume} &= 7.5 \times 5^2 \\ &= \underline{\underline{187.5 \text{ cm}^3}}.\end{aligned}$$

- (b) In fact, the total surface area of the cuboid is smaller than 200 cm^2 . (1)

What does this mean about the volume of the cuboid?

Tick **one** box.

☐

It is smaller than the answer to part (a)

☐

It is bigger than the answer to part (a)

☐

It is the same as the answer to part (a)

☐

It could be any of the above

Solution

It is smaller than the answer to part (a).

10. Alex and Bev sat six tests, each with 50 marks. (4)
The table shows their mean percentages after five tests.

Alex	60%
Bev	52%

After all six tests, their mean percentages were equal.
In the sixth test, Alex scored 24 out of 50.

Work out Bev's score, out of 50, in the sixth test.

Solution

On the first five tests, Alex scored

$$5 \times 50 \times 0.6 = 150$$

whereas Bev scored

$$5 \times 50 \times 0.52 = 130.$$

In total, Alex has

$$150 + 24 = 174$$

and, hence, Bev needs

$$174 - 130 = \underline{\underline{44 \text{ marks}}}.$$

11. A solid piece of silver

(2)

- has mass 2.625 kilograms and
- volume 250 cm^3 .

Work out the density of the piece of silver.

Give your answer in grams per cubic centimetre.

Solution

Well,

$$\begin{aligned} \text{density} &= \frac{\text{mass}}{\text{volume}} \\ &= \frac{2.625 \text{ kg}}{250 \text{ cm}^3} \\ &= \frac{2\,625 \text{ g}}{250 \text{ cm}^3} \\ &= \underline{\underline{10.5 \text{ grams per cubic centimetre}}}. \end{aligned}$$

12. Work out the gradient of the straight line through $(-2, 3)$ and $(1, 9)$.

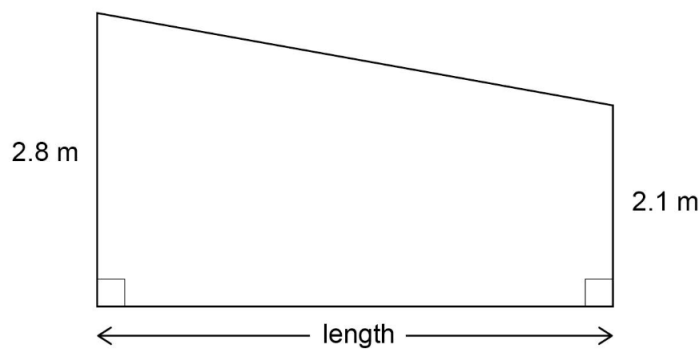
(2)

Solution

$$\begin{aligned}\text{Gradient} &= \frac{9 - 3}{1 - (-2)} \\ &= \frac{6}{3} \\ &= \underline{\underline{2}}.\end{aligned}$$

13. The diagram shows a wall.

(3)



Not drawn
accurately

The area of the wall is 39.2 m^2 .

Work out the length of the wall.

Solution

Well,

$$\begin{aligned}\frac{1}{2} \times \text{length} \times (2.8 + 2.1) &= 39.2 \Rightarrow \frac{1}{2} \times \text{length} \times 4.9 = 39.2 \\ \Rightarrow \text{length} &= \frac{2 \times 39.2}{4.9} \\ \Rightarrow \underline{\underline{\text{length} = 16 \text{ m}}}.\end{aligned}$$

14. A marathon takes place each year.

(3)

In 2020 there were 6 500 runners.

Prediction : For each of the next 3 years the number of runners will increase by 5%.

Does this predict that in 2023 there will be more than 7 500 runners?
You **must** show your working.

Solution

Well, the prediction is

$$6\,500 \times 1.05^3 = 7\,524\frac{9}{16};$$

so the prediction could be true.

15. Rearrange

(3)

$$a = \frac{b}{c} + 5$$

to make c the subject.

Solution

$$a = \frac{b}{c} + 5 \Rightarrow \frac{b}{c} = a - 5$$

invert:

$$\begin{aligned} \Rightarrow \frac{c}{b} &= \frac{1}{a - 5} \\ \Rightarrow c &= \frac{b}{a - 5}. \end{aligned}$$

16. On a restaurant menu there are

(3)

- 22 main dishes, of which $\frac{4}{11}$ are gluten-free,
- 7 rice dishes, which are all gluten-free, and
- 5 naan breads, of which 40% are gluten-free.

This Meal Deal is on the menu.

Choose one main dish, one rice dish, and one naan bread.

How many of the possible Meal Deals are totally gluten-free?

Solution

Well,

$$22 \times \frac{4}{11} = 8$$

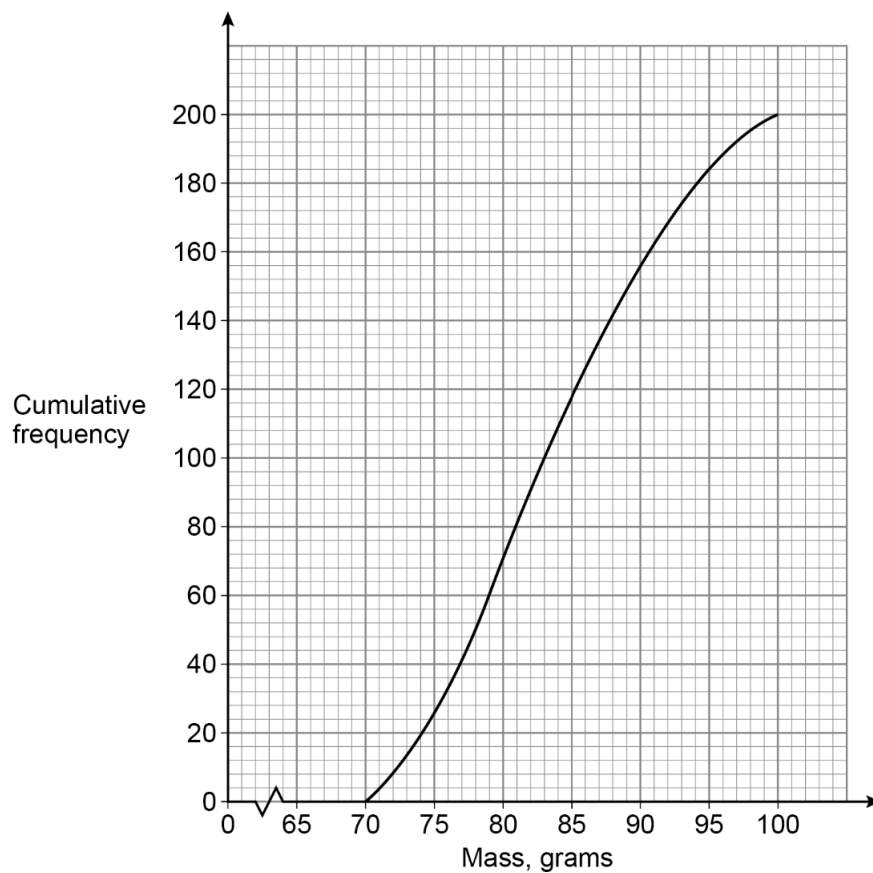
and

$$5 \times 0.4 = 2.$$

Hence, there are

$$8 \times 7 \times 2 = \underline{\underline{112 \text{ ways}}}.$$

17. The cumulative frequency graph shows information about the masses of 200 apples.



- (a) Estimate the median mass.

(1)

Solution

Draw a horizontal line from 100 and read-off: 83.

- (b) Apples with mass 90 grams or less cost 32p each. (3)
Apples with mass more than 90 grams cost 39p each.

Estimate the **total** cost of the 200 apples.

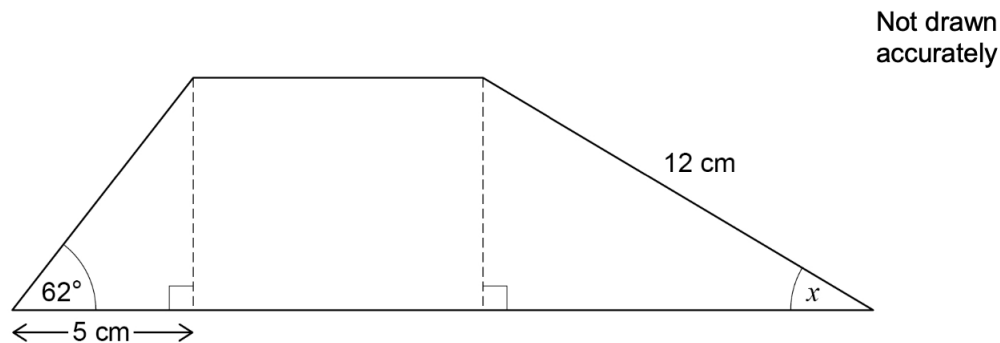
Solution

Draw a vertical line from 90 and read-off: 156.

Now,

$$\begin{aligned}\text{total cost} &= (32 \times 156) + [39 \times (200 - 156)] \\ &= (32 \times 156) + (39 \times 44) \\ &= 4\,992 + 1\,716 \\ &= \underline{\underline{6\,708 \text{ p or } \pounds 67.08.}}\end{aligned}$$

18. This shape is made from two right-angled triangles and a rectangle. (4)



Work out the size of angle x .

Solution

Let the vertical side be x cm. Now,

$$\begin{aligned}\tan &= \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 62^\circ = \frac{x}{5} \\ &\Rightarrow x = 5 \tan 62^\circ\end{aligned}$$

and

$$\begin{aligned}\sin x &= \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin x = \frac{5 \tan 62^\circ}{12} \\ \Rightarrow x &= 51.594\,702\,85 \text{ (FCD)} \\ \Rightarrow x &= \underline{\underline{51.6^\circ \text{ (3 sf)}}}.\end{aligned}$$

19. a and b are positive values.

(3)

Show that

$$\frac{7a + 2b - 3a}{8a + 6b + 2a - b}$$

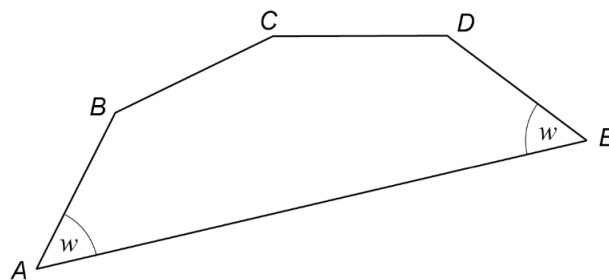
always simplifies to the same value.

Solution

$$\begin{aligned}\frac{7a + 2b - 3a}{8a + 6b + 2a - b} &= \frac{4a + 2b}{10a + 5b} \\ &= \frac{2(2a + b)}{5(2a + b)} \\ &= \underline{\underline{\frac{2}{5}}}.\end{aligned}$$

20. AB , BC , CD , and DE are four of the sides of a regular decagon.

(3)



Not drawn
accurately

Work out the size of angle w .

Solution

Well, a decagon has 10 sides and

$$180 - \frac{360}{10} = 144^\circ.$$

The total angle is 540° . Hence,

$$\begin{aligned} w &= \frac{1}{2}(540 - 3 \times 144) \\ &= \frac{1}{2}(540 - 432) \\ &= \underline{\underline{54}}^\circ. \end{aligned}$$

21. (a) Circle the point that is on the graph of (1)

$$y = \frac{1}{x}.$$

$(-1, 1)$ $(0.3, 3)$ $(0.8, 0.2)$ $(2.5, 0.4)$

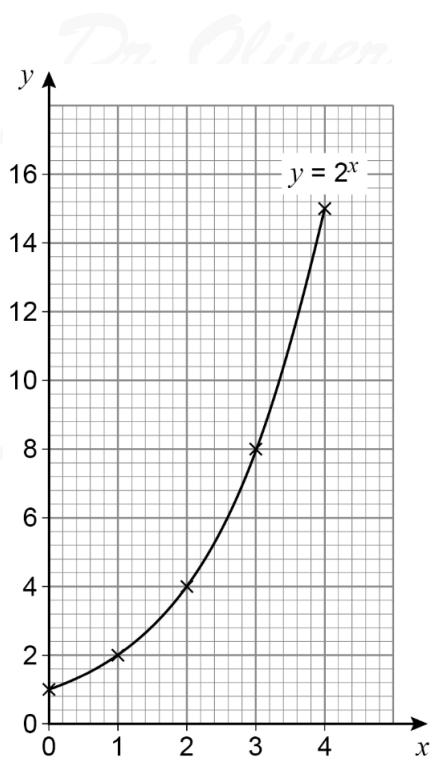
Solution

$(-1, 1)$ $(0.3, 3)$ $(0.8, 0.2)$ $(2.5, 0.4)$

- (b) Leo wants to draw the graph of (1)

$$y = 2^x,$$

for values of x from 0 to 4. Here is his graph.



Make one criticism of his graph.

Solution

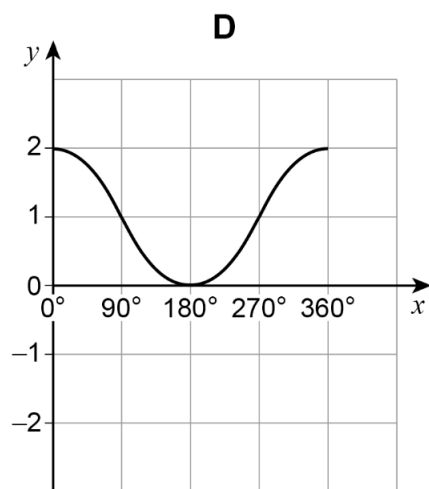
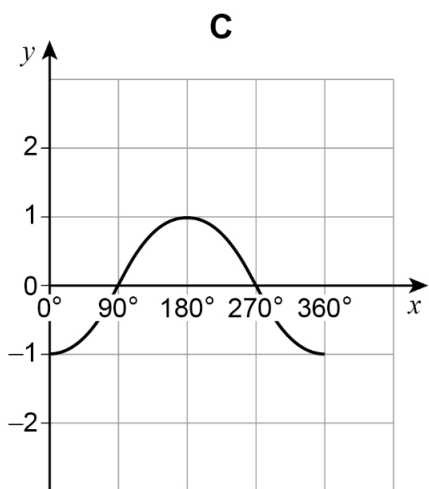
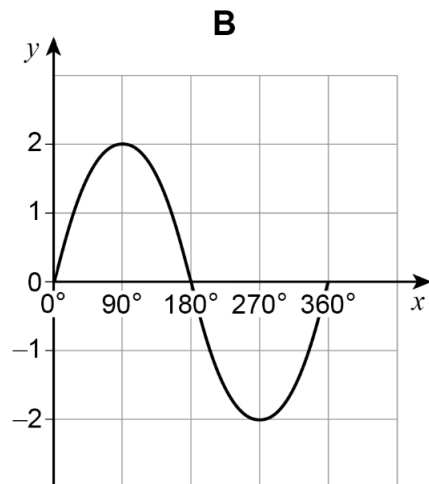
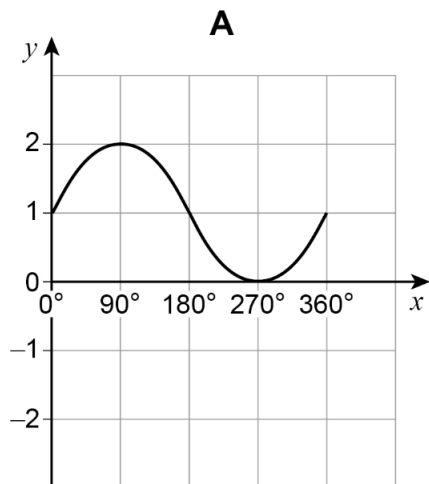
He plotted the point (4, 15) but it should be (4, 16).

22. One of these is the graph of

$$y = 1 + \sin x$$

(1)

for $0^\circ \leq x \leq 360^\circ$.



Circle the letter above the correct graph.

Solution

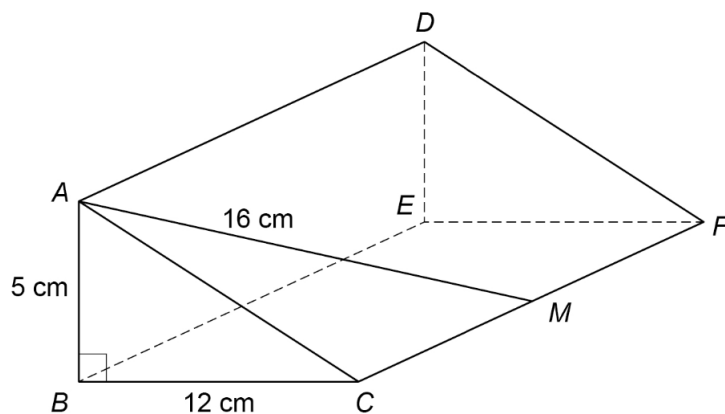
A.

23. Right-angled triangle ABC is the cross section of a prism.

- $AB = 5$ cm.
- $BC = 12$ cm.

(4)

- M is the midpoint of CF .
- $AM = 16$ cm.



Work out the volume of the prism.

Solution

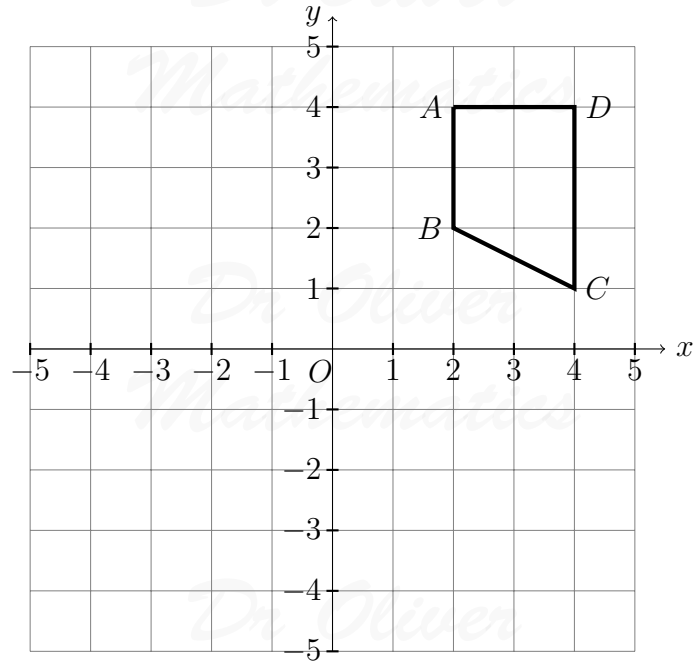
3-dimensional Pythagoras:

$$\begin{aligned}
 AB^2 + BC^2 + CM^2 &= AM^2 \Rightarrow 5^2 + 12^2 + CM^2 = 16^2 \\
 &\Rightarrow 25 + 144 + CM^2 = 256 \\
 &\Rightarrow CM^2 = 87 \\
 &\Rightarrow CM = \sqrt{87} \\
 &\Rightarrow CF = 2\sqrt{87}.
 \end{aligned}$$

Hence,

$$\begin{aligned}
 \text{volume} &= \left(\frac{1}{2} \times 5 \times 12\right) \times 2\sqrt{87} \\
 &= \underline{\underline{60\sqrt{87} \text{ cm}^3}}.
 \end{aligned}$$

24. Quadrilateral $ABCD$ is shown.

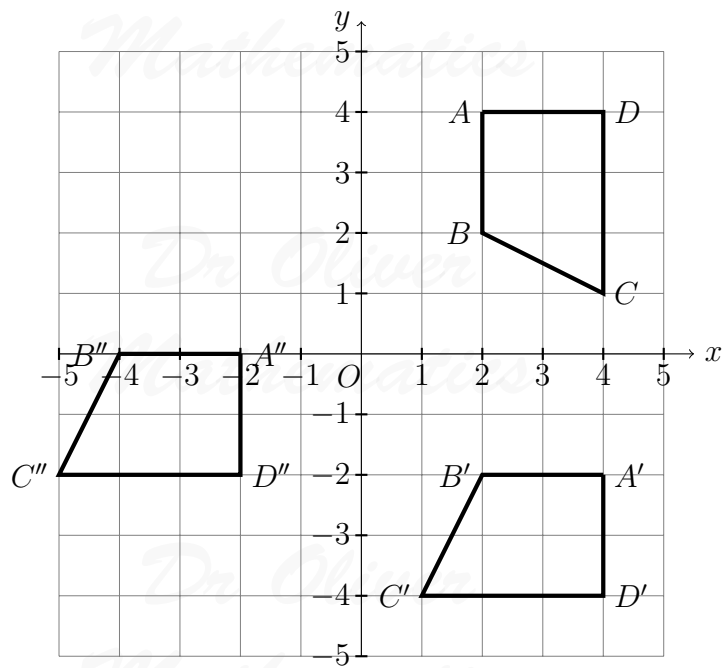


(a) Work out the coordinates of C when $ABCD$ (2)

- is rotated 90° clockwise about O then
- translated by

$$\begin{pmatrix} -6 \\ 2 \end{pmatrix}.$$

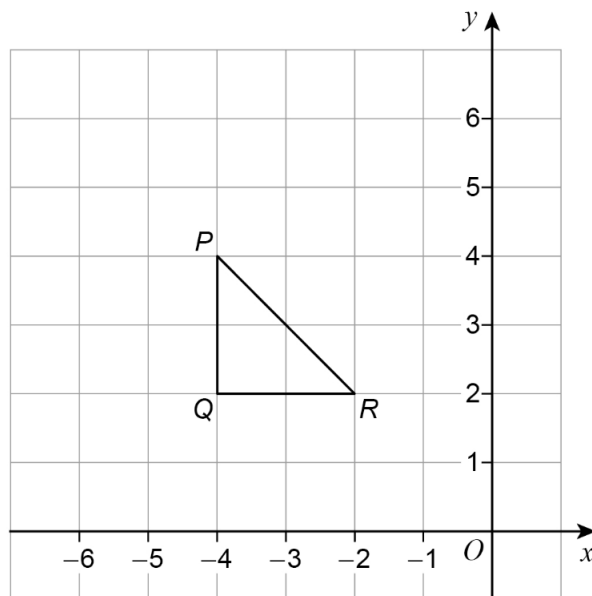
Solution



Hence, $C''(-5, -2)$.

(b) Triangle PQR is shown.

(1)



When PQR is reflected in a line, P and R are invariant points.

Circle the equation of the line.

$$y = x + 6 \quad y = -x \quad y = 2 \quad x = -4$$

Solution

$$y = x + 6 \quad \underline{y = -x} \quad y = 2 \quad x = -4$$

25. Factorise

$$3x^2 + 11x - 20.$$

(2)

Solution

$$\left. \begin{array}{l} \text{add to:} \quad +11 \\ \text{multiply to: } (+3) \times (-20) = -60 \end{array} \right\} +15, -4$$

E.g.,

$$\begin{aligned} 3x^2 + 11x - 20 &= 3x^2 + 15x - 4x - 20 \\ &= 3x(x + 5) - 4(x + 5) \\ &= \underline{(3x - 4)(x + 5)}. \end{aligned}$$

26. Edith's van can safely carry a maximum load of 920 kilograms.

(4)

She wants to use her van to carry

- 30 sacks of potatoes, each of mass 25 kilograms to the nearest kilogram and
- 20 sacks of carrots, each of mass 7.5 kilograms to 1 decimal place.

Can she definitely use her van safely in one journey?

You **must** show your working.

Solution

Well,

$$\begin{aligned} 24.5 &\leq \text{one sack of potatoes} < 25.5 \\ 7.45 &\leq \text{one sack of carrots} < 7.55. \end{aligned}$$

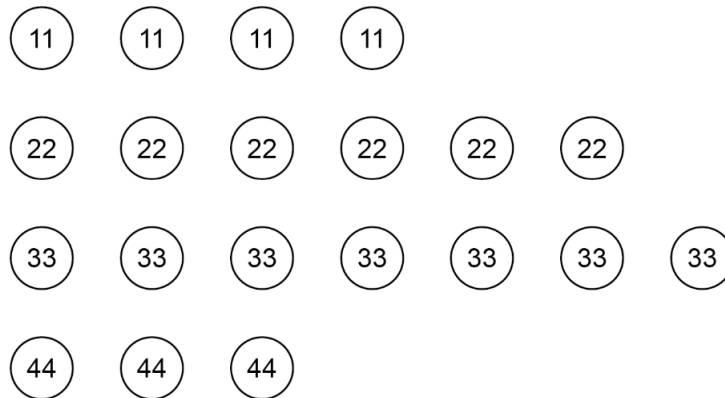
Now,

$$\begin{aligned}\text{total mass} &< (30 \times 25.5) + (20 \times 7.55) \\ &= 765 + 151 \\ &= 916;\end{aligned}$$

hence, she can her van safely in one journey.

27. These 20 discs are in a bag.

(4)



Two of the discs are taken at random from the bag.

Work out the probability that the first disc has a smaller number than the second disc.

Solution

Well, there are 4 number 11s, 6 number 22s, 7 number 33s, and 4 number 44s.

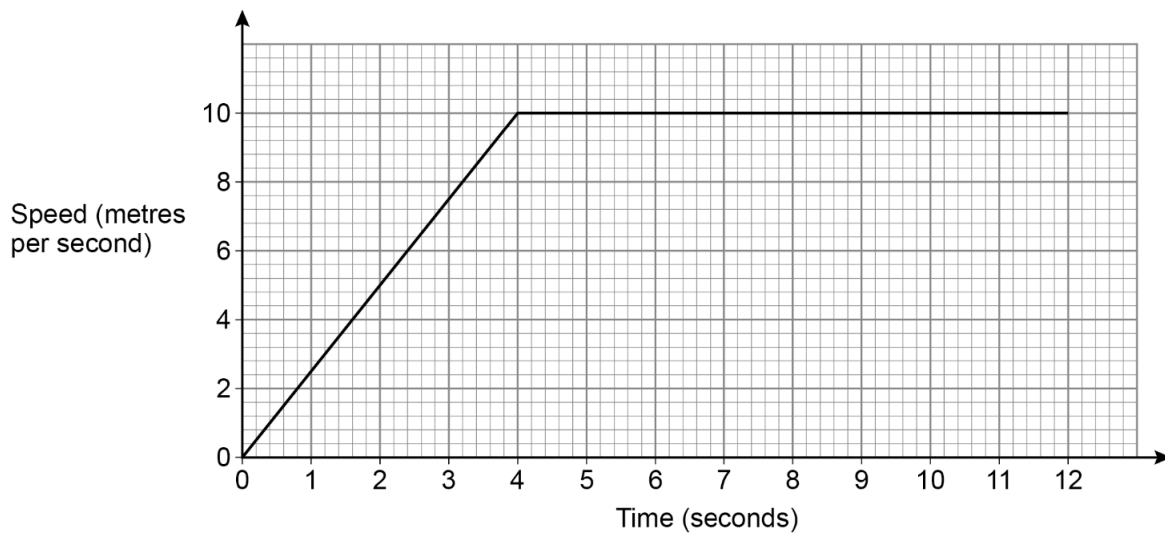
Now,

$$\begin{aligned}&P(\text{smaller number than the second disc}) \\ &= P(11, \geq 22) + P(22, \geq 33) + P(33, 44) \\ &= \left(\frac{4}{20} \times \frac{16}{19}\right) + \left(\frac{6}{20} \times \frac{10}{19}\right) + \left(\frac{7}{20} \times \frac{3}{19}\right) \\ &= \frac{64}{380} + \frac{60}{380} + \frac{21}{380} \\ &= \frac{145}{380} \\ &= \frac{29}{76}.\end{aligned}$$

28. A horse runs in a field.

(3)

The speed-time graph represents the first 12 seconds of the run.



After how many seconds had the horse run a distance of 75 metres?

Solution

Now,

$$\begin{aligned}\text{distance run after 4 s} &= \frac{1}{2} \times 4 \times 10 \\ &= 20\end{aligned}$$

and so we have

$$75 - 20 = 55 \text{ m left to run.}$$

Next,

$$\frac{55}{10} = 5.5$$

and we add that to 4:

$$4 + 5.5 = \underline{\underline{9.5 \text{ s}}}.$$

29. Solve

(5)

$$\frac{5}{4x+1} = \frac{2x}{x^2+3}.$$

Give your solutions to 3 significant figures.

You **must** show your working.

Solution

Cross-multiply:

$$\begin{aligned}\frac{5}{4x+1} &= \frac{2x}{x^2+3} \Rightarrow 5(x^2+3) = 2x(4x+1) \\ &\Rightarrow 5x^2 + 15 = 8x^2 + 2x \\ &\Rightarrow 3x^2 + 2x - 15 = 0\end{aligned}$$

Quadratic formula: $a = 2$, $b = 2$, and $c = -15$:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times (-15)}}{2 \times 3} \\ &= \frac{-2 \pm \sqrt{184}}{6} \\ &= -2.594\,109\,94, \, 1.927\,443\,328 \text{ (FCD)} \\ &= \underline{\underline{-2.59, \, 1.93 \text{ (3 sf)}}}.\end{aligned}$$