

Dr Oliver Mathematics
GCSE Mathematics
2011 November Paper 3H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Theo earns £20 one weekend.
He gives £4 to his brother.

- (a) Express £4 as a fraction of £20.
Give your answer in its simplest form.

(2)

Solution

$$\frac{4}{20} = \underline{\underline{\frac{1}{5}}}$$

Theo gives £6 to his mother.

- (b) Express £6 as a percentage of £20.

(2)

Solution

$$\frac{6}{20} \times 100\% = \underline{\underline{30\%}}$$

Theo spent the remaining £10 on bus fares and food.
He spent £1.50 more on bus fares than on food.

- (c) How much did he spend on bus fares?

(2)

Solution

Let x and y be the bus fares and food respectively. Then

$$x + y = 10$$

$$x - y = 1.5.$$

Add:

$$2x = 11.5 \Rightarrow \underline{\underline{x = £5.75}}$$

2. Here is a number pattern.

Line Number			
1	$1^2 + 3^2$	$2 \times 2^2 + 2$	10
2	$2^2 + 4^2$	$2 \times 3^2 + 2$	20
3	$3^2 + 5^2$	$2 \times 4^2 + 2$	34
4	52
...
10

(a) Complete Line Number 4 of the pattern.

(1)

Solution

Line Number			
1	$1^2 + 3^2$	$2 \times 2^2 + 2$	10
2	$2^2 + 4^2$	$2 \times 3^2 + 2$	20
3	$3^2 + 5^2$	$2 \times 4^2 + 2$	34
4	<u>$4^2 + 6^2$</u>	<u>$2 \times 5^2 + 2$</u>	52
...
10

(b) Complete Line Number 10 of the pattern.

(2)

Solution

Line Number			
1	$1^2 + 3^2$	$2 \times 2^2 + 2$	10
2	$2^2 + 4^2$	$2 \times 3^2 + 2$	20
3	$3^2 + 5^2$	$2 \times 4^2 + 2$	34
4	<u>$4^2 + 6^2$</u>	<u>$2 \times 5^2 + 2$</u>	52
...
10	<u>$10^2 + 12^2$</u>	<u>$2 \times 11^2 + 2$</u>	<u>244</u>

(c) Use the number pattern to find the answer to $999^2 + 1001^2$.

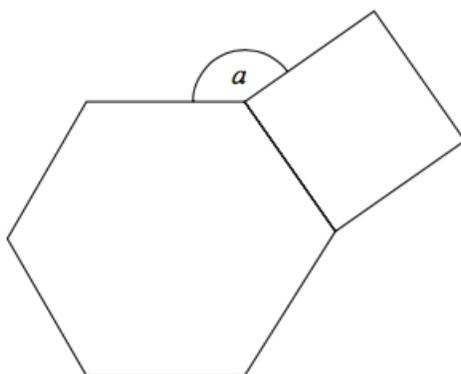
(2)

Solution

$$\begin{aligned}
 999^2 + 1\,001^2 &= 2 \times 1\,000^2 + 2 \\
 &= 2 \times 1\,000\,000 + 2 \\
 &= 2\,000\,000 + 2 \\
 &= \underline{\underline{2\,000\,002}}.
 \end{aligned}$$

3. The diagram shows a regular hexagon and a square.

(4)



Calculate the size of the angle a .

Solution

$$\begin{aligned}
 360 - (120 + 90) &= 360 - 210 \\
 &= \underline{\underline{150^\circ}}.
 \end{aligned}$$

4. Jim did a survey on the lengths of caterpillars he found on a field trip. Information about the lengths is given in the stem and leaf diagram.

(2)

1	3	5	7	7				
2	0	6	8	8	8	9		
3	1	5	5	5	5	6	8	9
4	1	5						
5	2							

Key: 5|2 means 5.2 cm.

Work out the median.

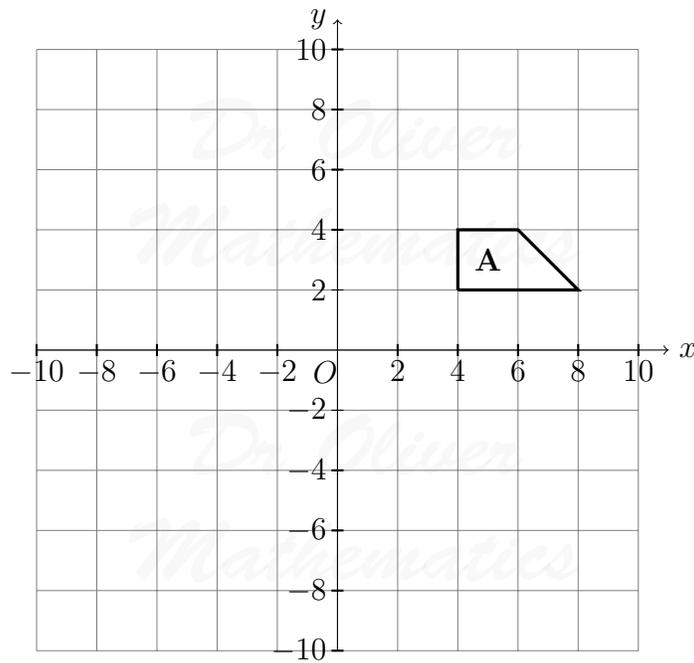
Solution

There are 21 caterpillars and so the median is the

$$\frac{21 + 1}{2} = 11\text{th}$$

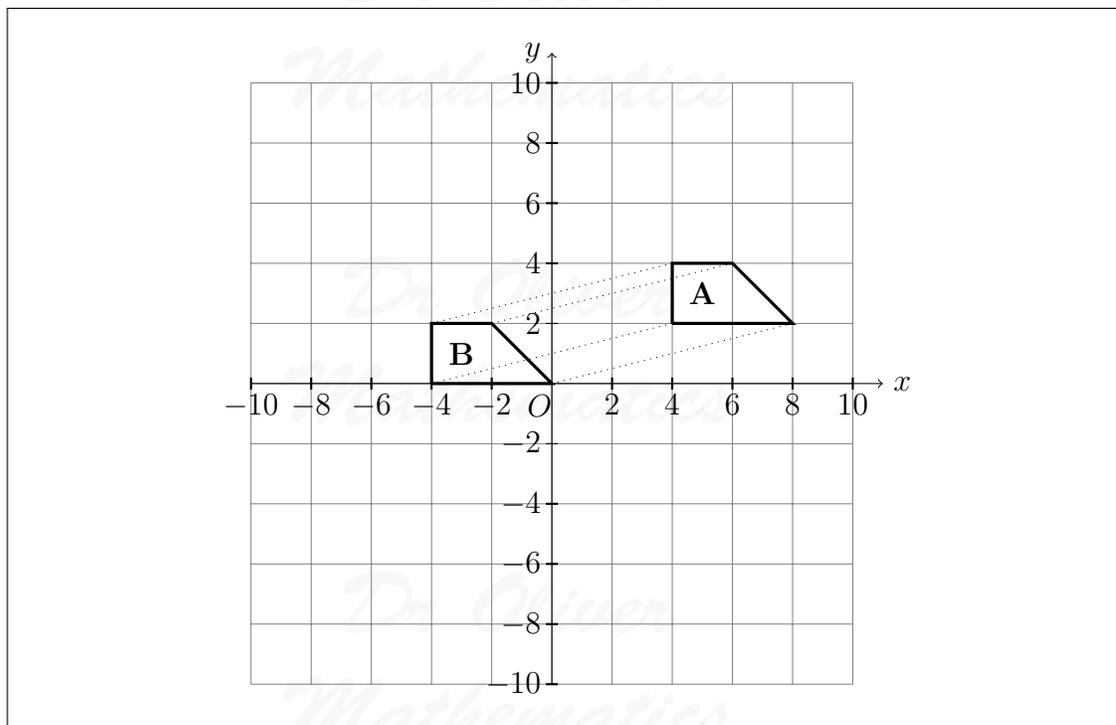
number in our list; the median is 3.1 cm.

5. (a) Translate shape **A** by $\begin{pmatrix} -8 \\ -2 \end{pmatrix}$. (2)



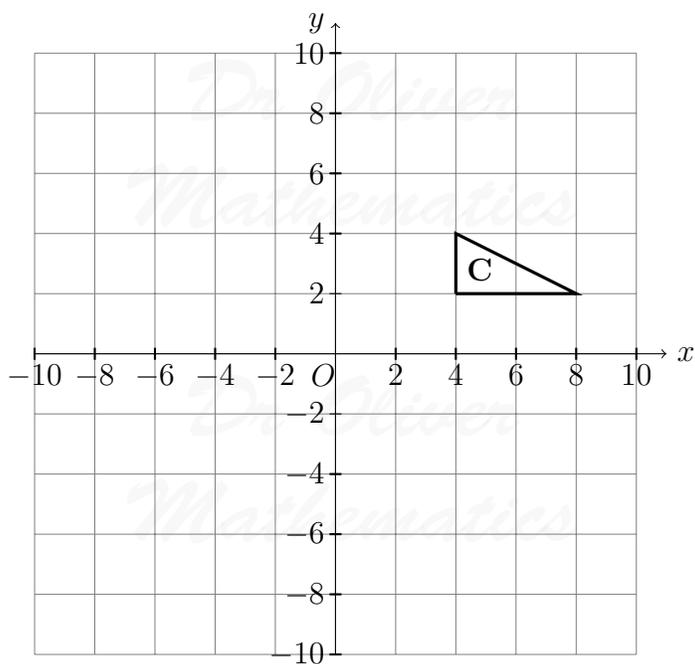
Label the new shape **B**.

Solution

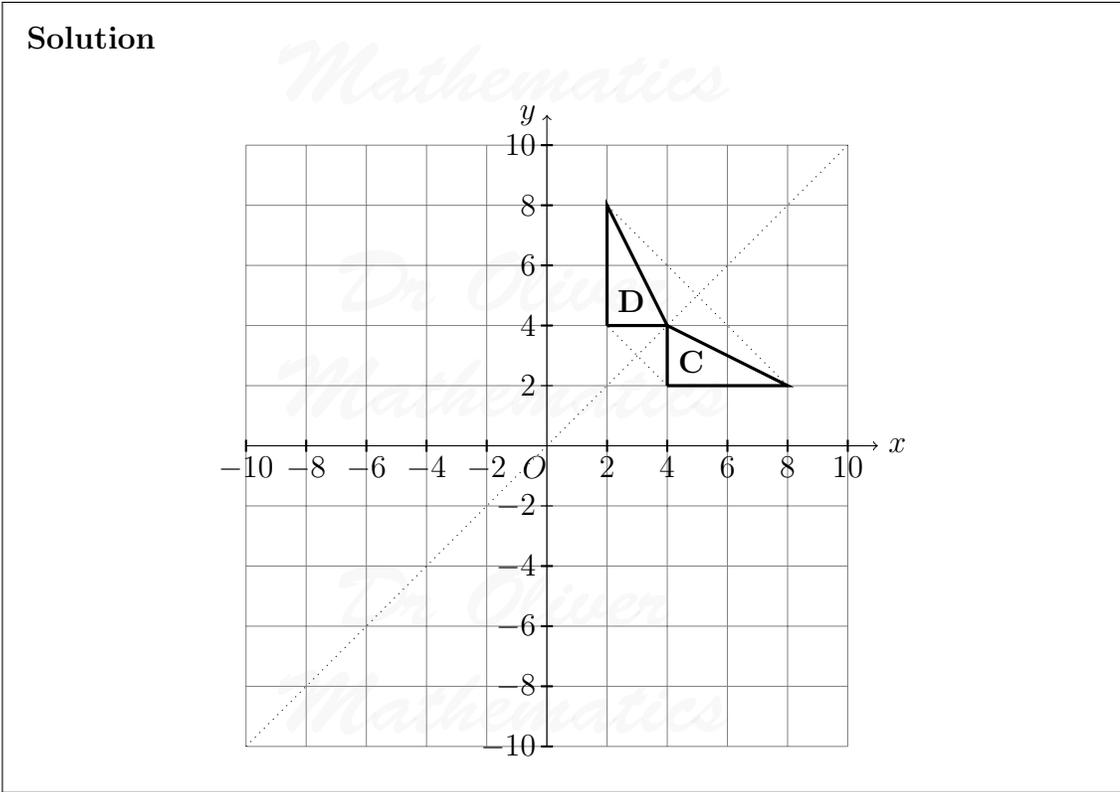


(b) Reflect shape **C** in the line $y = x$.

(2)



Label the new shape **D**.



6. The table gives distances in miles by road between some towns.

(4)

Reading				
22	Slough			
28	40	Guildford		
30	22	47	Oxford	
45	28	66	25	Buckingham

Izzy lives in Oxford.

She has to drive to a meeting in Buckingham and then from Buckingham to Reading to pick up a friend.

After she picks up her friend she will drive back to Oxford.

She plans to drive at a speed of 50 miles per hour.

The meeting will last 3 hours, including lunch.

She leaves Oxford at 9 am.

Work out the time at which she should get back to Oxford.

Solution

Distance	Miles
Oxford to Buckingham	25
Buckingham to Reading	45
Reading to Oxford	30
Total	100

So, the time taken is

$$\frac{100}{50} = 2 \text{ hours}$$

and, if she leaves at 9 am, she will get back at

$$9 + 2 + 3 = \underline{\underline{1400}} \text{ or } \underline{\underline{2 \text{ pm.}}}$$

7. (a) Solve

$$3(2t - 4) = 2t + 12.$$

(3)

Solution

$$\begin{aligned} 3(2t - 4) &= 2t + 12 \Rightarrow 6t - 12 = 2t + 12 \\ &\Rightarrow 4t = 24 \\ &\Rightarrow \underline{\underline{t = 6.}} \end{aligned}$$

(b) Expand and simplify

$$2(x - y) - 3(x - 2y)$$

(2)

Solution

$$\begin{aligned} 2(x - y) - 3(x - 2y) &= 2x - 2y - 3x + 6y \\ &= \underline{\underline{-x + 4y.}} \end{aligned}$$

(c) Expand and simplify

$$(x - 5)(x + 7).$$

(2)

Solution

×		x	-5
x		x^2	$-5x$
$+7$		$+7x$	-35

Hence,

$$(x - 5)(x + 7) = \underline{\underline{x^2 + 2x - 35}}.$$

8. Work out an estimate for the value of

$$(0.49 \times 0.61)^2.$$

(2)

Solution

Round to 1 significant figure:

$$\begin{aligned}(0.49 \times 0.61)^2 &\approx (0.5 \times 0.6)^2 \\ &= 0.3^2 \\ &= \underline{\underline{0.09}}.\end{aligned}$$

9. Two shops both sell the same type of suit.

In both shops the price of the suit was £180.

One shop increases the price of the suit by $17\frac{1}{2}\%$.

The other shop increases the price of the suit by $22\frac{1}{2}\%$.

Calculate the difference between the new prices of the suits in the two shops.

(3)

Solution

The difference in price is

$$22\frac{1}{2}\% - 17\frac{1}{2}\% = 5\%$$

and the difference between the new prices of the suits in the two shops is

$$\begin{aligned}\frac{5}{100} \times 180 &= \frac{5}{10} \times 18 \\ &= \underline{\underline{\pounds 9}}.\end{aligned}$$

10. $ABCD$ is a rhombus.

(3)

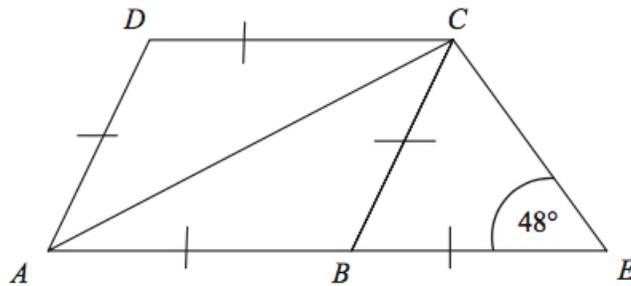


Diagram NOT
accurately drawn

BCE is an isosceles triangle.

ABE is a straight line.

Work out the size of angle DCA .

Solution

$\angle BCE = 48^\circ$ (base angles in an isosceles triangle).

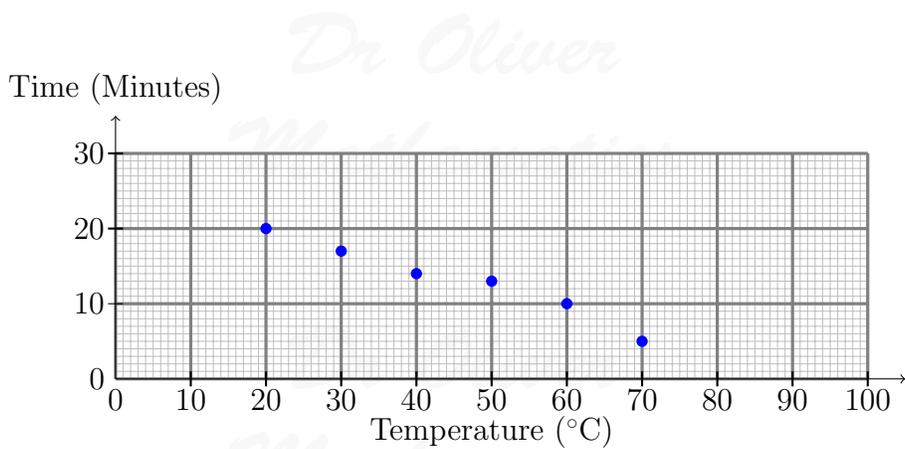
$\angle CBE = 180 - 48 - 48 = 84^\circ$ (angles in a triangle).

$\angle DCB = 180 - 84 = 96^\circ$ (supplementary angles).

$\angle DCB = 180 - 96 = 84^\circ$ (interior angles).

$\angle DCA = \angle DCB \div 2 = \underline{\underline{42^\circ}}$ (half that).

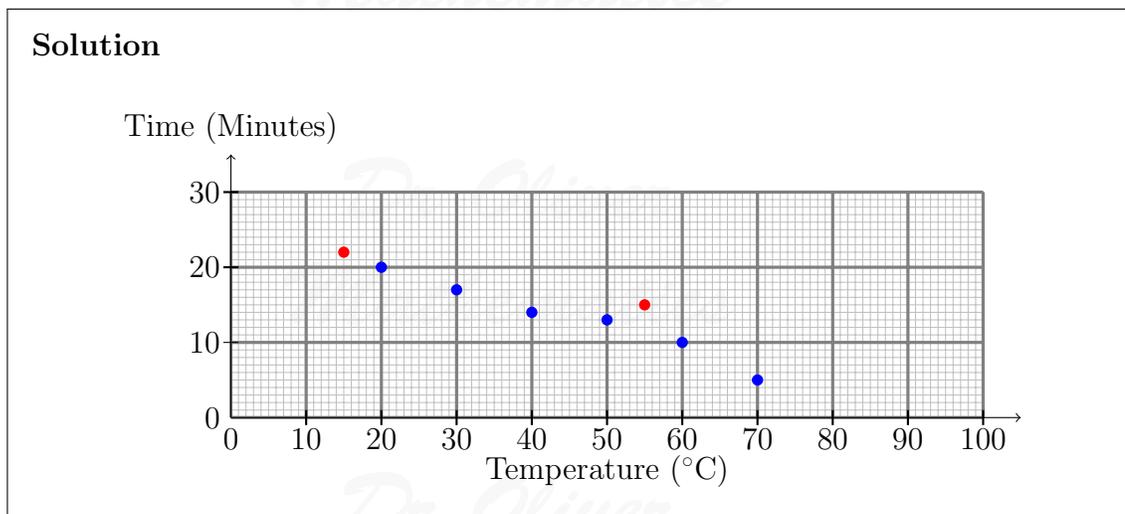
11. Suzy did an experiment to study the times, in minutes, it took 1 cm ice cubes to melt at different temperatures.
Some information about her results is given in the scatter graph.



The table shows information from two more experiments.

Temperature (°C)	15	55
Time (Minutes)	22	15

- (a) On the scatter graph, plot the information from the table. (1)



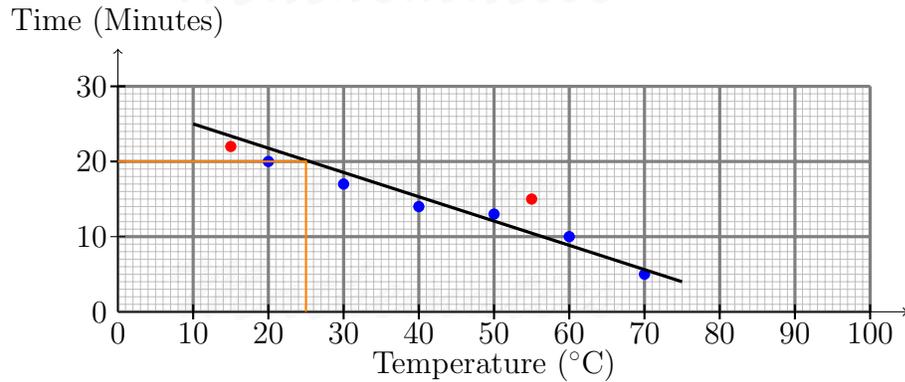
- (b) Describe the relationship between the temperature and the time it takes a 1 cm ice cube to melt. (1)

Solution

E.g., Negative correlation, if the temperature increases, the time taken decreases.

- (c) Find an estimate for the time it takes a 1 cm ice cube to melt when the temperature is 25°C. (2)

Solution



Correct read-off: approximately 20 minutes.

Suzy's data cannot be used to predict how long it will take a 1 cm ice cube to melt when the temperature is 100°C.

(d) Explain why.

(1)

Solution

E.g., because it is steam, the line of best fit would give negative values, you should not use your line of best fit beyond your data.

12. Solve the simultaneous equations

$$3x + 4y = 200$$

$$2x + 3y = 144.$$

(4)

Solution

E.g.,

$$3x + 4y = 200 \quad (1)$$

$$2x + 3y = 144 \quad (2)$$

$$2 \times (1) : 6x + 8y = 400 \quad (3)$$

$$3 \times (2) : 6x + 9y = 432 \quad (4)$$

(4) – (3):

$$\underline{y = 32} \Rightarrow 3x + 128 = 200$$

$$\Rightarrow 3x = 72$$

$$\Rightarrow \underline{x = 24}.$$

13. (a) Work out the value of

$$(6 \times 10^8) \times (4 \times 10^7).$$

(2)

Give your answer in standard form.

Solution

$$\begin{aligned}(6 \times 10^8) \times (4 \times 10^7) &= (6 \times 4) \times (10^8 \times 10^7) \\ &= 24 \times 10^{15} \\ &= \underline{2.4 \times 10^{16}}.\end{aligned}$$

(b) Work out the value of

$$(6 \times 10^8) + (4 \times 10^7).$$

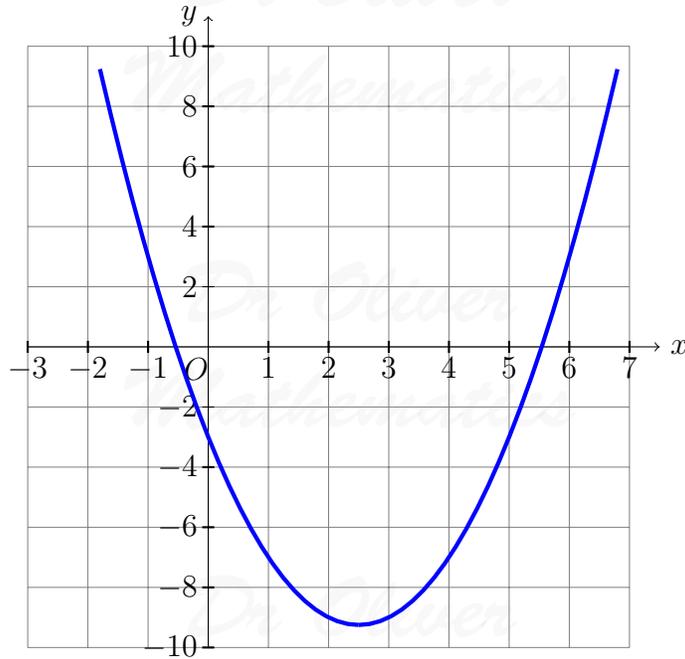
(2)

Give your answer in standard form.

Solution

$$\begin{aligned}(6 \times 10^8) + (4 \times 10^7) &= (6 \times 10^8) \times (0.4 \times 10^8) \\ &= \underline{6.4 \times 10^8}.\end{aligned}$$

14. The diagram shows the graph of $y = x^2 - 5x - 3$.



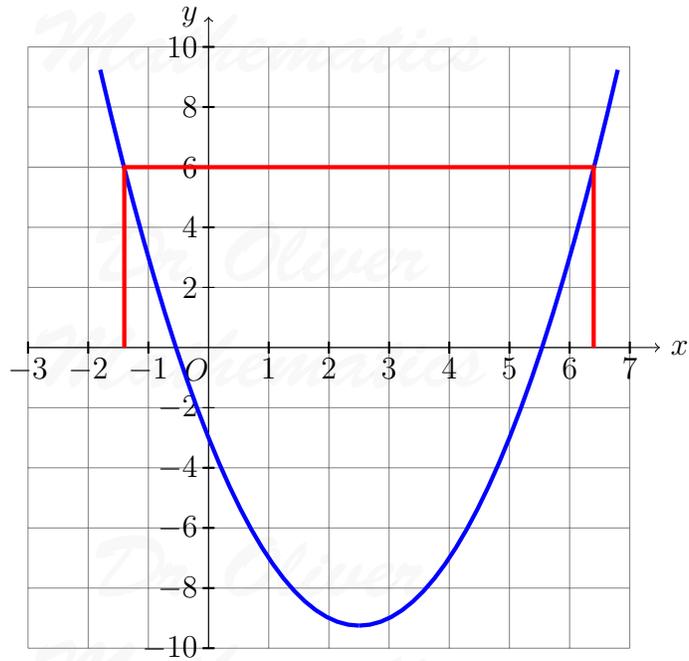
- (a) Use the graph to find estimates for the solutions of (3)
 (i) $x^2 - 5x - 3 = 0$,

Solution

$x = -0.5$ or $x = 5.5$.

- (ii) $x^2 - 5x - 3 = 6$.

Solution



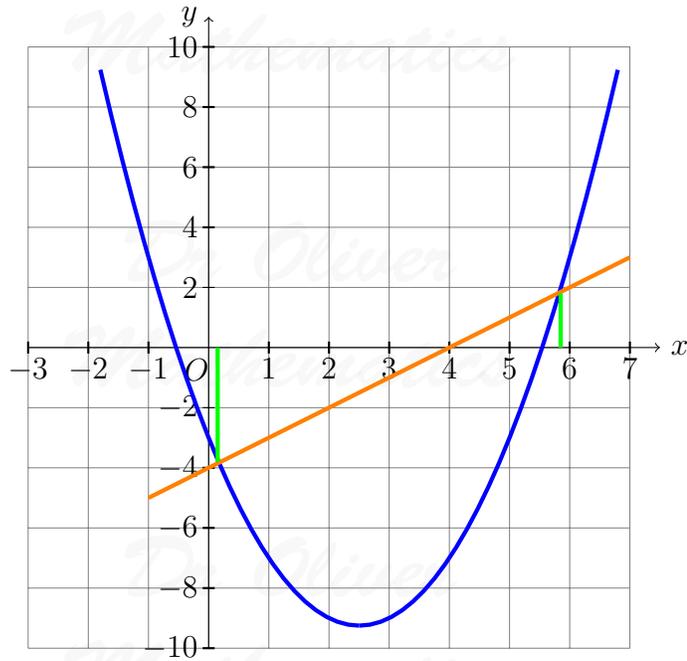
Correct read-off: approximately $x = -1.4$ or $x = 6.4$.

- (b) Use the graph to find estimates for the solutions of the simultaneous equations (3)

$$y = x^2 - 5x - 3$$

$$y = x - 4.$$

Solution



Correct read-off: approximately $x = 0.15, y = -3.85$ or $x = 5.85, y = 1.85$.

15. A garage keeps records of the costs of repairs to customers' cars. The table gives information about these costs for one month.

Cost $\pounds C$	Frequency
$0 < C \leq 200$	7
$200 < C \leq 400$	11
$400 < C \leq 600$	9
$600 < C \leq 800$	10
$800 < C \leq 1\,000$	8
$1\,000 < C \leq 1\,200$	5

- (a) Write down the modal class interval. (1)

Solution

$200 < C \leq 400$.

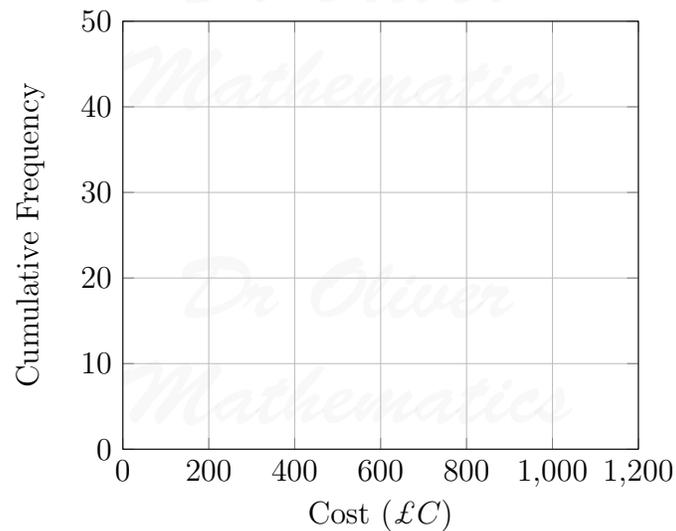
- (b) Complete the cumulative frequency table. (1)

Cost (£C)	Cumulative Frequency
$0 < C \leq 200$	
$0 < C \leq 400$	
$0 < C \leq 600$	
$0 < C \leq 800$	
$0 < C \leq 1000$	
$0 < C \leq 1200$	

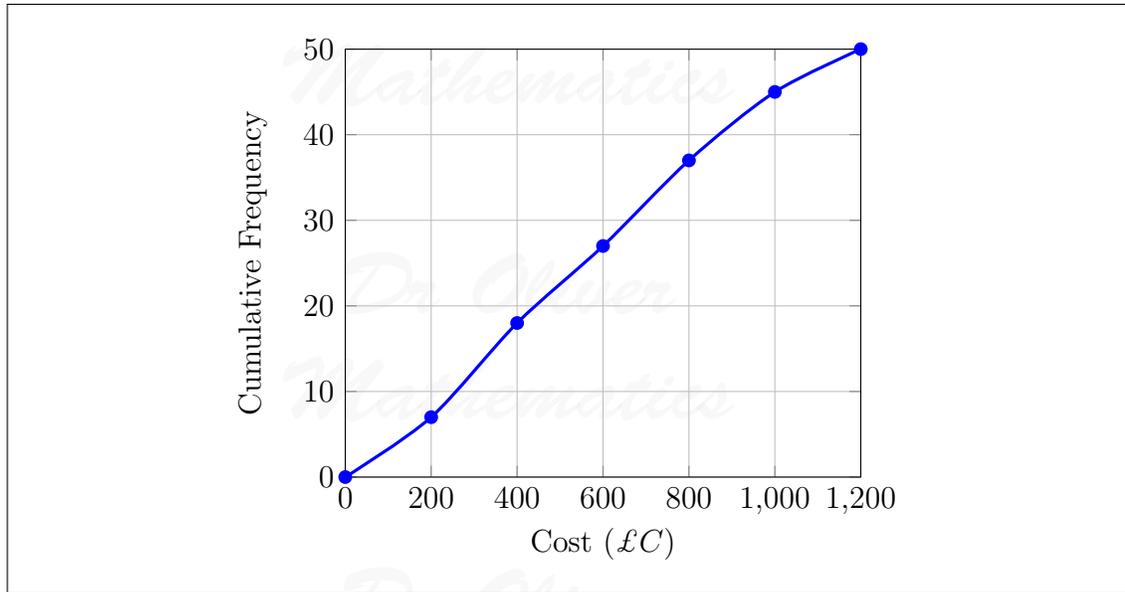
Solution

Cost (£C)	Cumulative Frequency
$0 < C \leq 200$	<u>7</u>
$0 < C \leq 400$	$7 + 11 = \underline{18}$
$0 < C \leq 600$	$18 + 9 = \underline{27}$
$0 < C \leq 800$	$27 + 10 = \underline{37}$
$0 < C \leq 1000$	$37 + 8 = \underline{45}$
$0 < C \leq 1200$	$45 + 5 = \underline{50}$

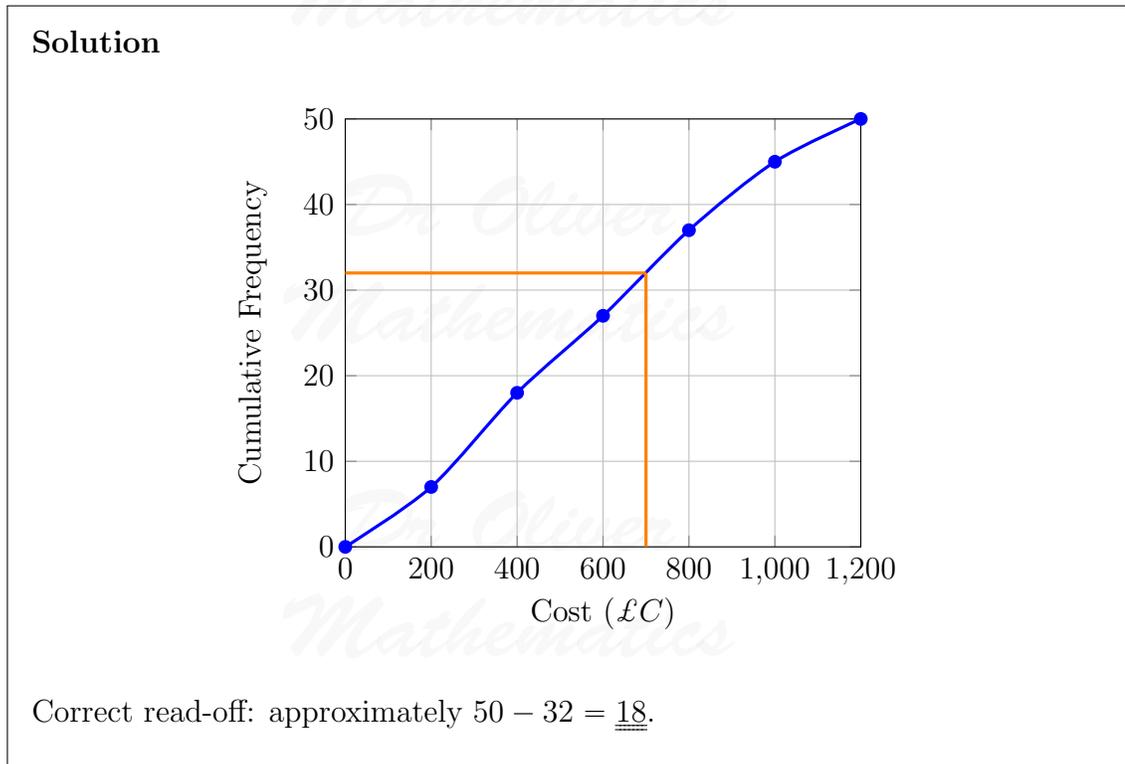
(c) On the grid, draw a cumulative frequency diagram for your table. (2)



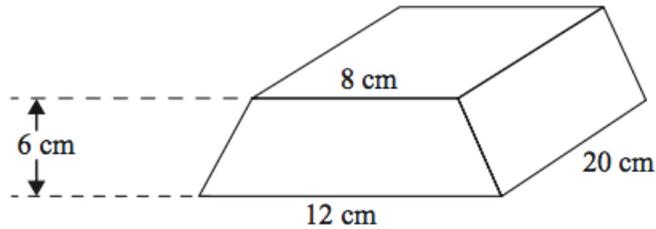
Solution



- (d) Use the graph to find an estimate for the number of repairs which cost more than £700. (2)



16. The diagram shows a solid prism made from metal. (5)



The cross-section of the prism is a trapezium.
 The parallel sides of the trapezium are 8 cm and 12 cm.
 The height of the trapezium is 6 cm.
 The length of the prism is 20 cm.
 The density of the metal is 5 g/cm^3 .
 Calculate the mass of the prism.
 Give your answer in kilograms.

Solution

$$\begin{aligned} \text{Area of the cross-section} &= \frac{1}{2} \times (8 + 12) \times 6 \\ &= \frac{1}{2} \times 20 \times 6 \\ &= 10 \times 6 \\ &= 60 \text{ cm}^2 \end{aligned}$$

and

$$\begin{aligned} \text{volume} &= 60 \times 20 \\ &= 1\,200 \text{ cm}^3. \end{aligned}$$

Now, the mass of metal is

$$\begin{aligned} 5 \times 1\,200 &= 6\,000 \text{ g} \\ &= \underline{\underline{6 \text{ kg}}}. \end{aligned}$$

17. $y = p - 2qx^2$.
 $p = -10$.
 $q = 3$.
 $x = -5$.

(a) Work out the value of y .

(2)

Solution

$$\begin{aligned}y &= -10 - 2 \times 3 \times (-5)^2 \\ &= -10 - 6 \times 25 \\ &= -10 - 150 \\ &= \underline{\underline{-160}}.\end{aligned}$$

(b) Rearrange

$$y = p - 2qx^2$$

(3)

to make x the subject of the formula.

Solution

$$\begin{aligned}y = p - 2qx^2 &\Rightarrow 2qx^2 = p - y \\ \Rightarrow x^2 &= \frac{p - y}{2q} \\ \Rightarrow x &= \underline{\underline{\pm \sqrt{\frac{p - y}{2q}}}}.\end{aligned}$$

18. (a) Write down the value of 2^0 .

(1)

Solution

$$2^0 = \underline{\underline{1}}.$$

$$2^y = \frac{1}{4}.$$

(b) Write down the value of y .

(1)

Solution

$$2^{-2} = \frac{1}{4}$$

so $\underline{\underline{y = -2}}$.

(c) Work out the value of $9^{-\frac{3}{2}}$.

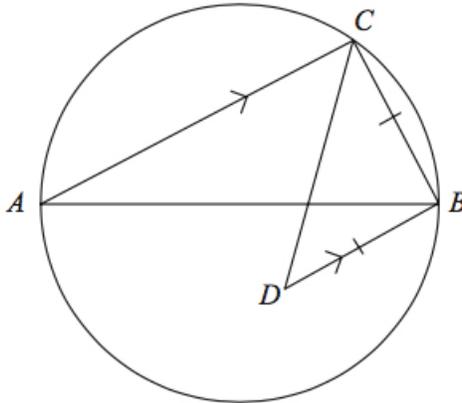
(2)

Solution

$$\begin{aligned}9^{-\frac{3}{2}} &= \frac{1}{9^{\frac{3}{2}}} \\ &= \frac{1}{(9^{\frac{1}{2}})^3} \\ &= \frac{1}{3^3} \\ &= \frac{1}{27}.\end{aligned}$$

19. AB is a diameter of a circle.

(4)



C is a point on the circle.

D is the point inside the circle such that $BD = BC$ and BD is parallel to CA .

Find the size of angle CDB .

You must give reasons for your answer.

Solution

Let $\angle ACD = y^\circ$.

Then $\angle CDB = y^\circ$ (alternate angles) and $\angle BCD = 90 - y^\circ$ (because $\angle ACB$ is a right-angle).

Finally, base angles is an isosceles triangle:

$$\begin{aligned}y &= 90 - y \Rightarrow 2y = 90 \\ &\Rightarrow \underline{y = 45}.\end{aligned}$$

20. (a) Factorise

$$2x^2 - 9x + 4.$$

(2)

Solution

$$\left. \begin{array}{l} \text{add to:} \quad \quad \quad -9 \\ \text{multiply to: } (+2) \times (+4) = +8 \end{array} \right\} -8, -1$$

E.g.,

$$\begin{aligned} 2x^2 - 9x + 4 &= 2x^2 - 8x - x + 4 \\ &= 2x(x - 4) - (x - 4) \\ &= \underline{\underline{(2x - 1)(x - 4)}}. \end{aligned}$$

Hence, or otherwise,

(b) solve

$$2x^2 - 9x + 4 = (2x - 1)^2.$$

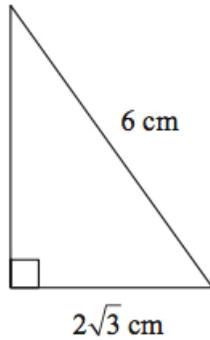
(4)

Solution

$$\begin{aligned} 2x^2 - 9x + 4 = (2x - 1)^2 &\Rightarrow (2x - 1)(x - 4) = (2x - 1)^2 \\ &\Rightarrow (2x - 1)^2 - (2x - 1)(x - 4) = 0 \\ &\Rightarrow (2x - 1)[(2x - 1) - (x - 4)] = 0 \\ &\Rightarrow (2x - 1)(x + 3) = 0 \\ &\Rightarrow 2x - 1 = 0 \text{ or } x + 3 = 0 \\ &\Rightarrow \underline{\underline{x = \frac{1}{2} \text{ or } x = -3.}} \end{aligned}$$

21. The diagram shows a right-angled triangle.

(5)



The length of the base of the triangle is $2\sqrt{3}$ cm.

The length of the hypotenuse of the triangle is 6 cm.

The area of the triangle is A cm².

Show that $A = k\sqrt{2}$, giving the value of k .

Solution

The third side of our triangle is

$$\begin{aligned} \sqrt{6^2 - (2\sqrt{3})^2} &= \sqrt{36 - 12} \\ &= \sqrt{24} \\ &= \sqrt{4 \times 6} \\ &= 2\sqrt{6}. \end{aligned}$$

Now,

$$\begin{aligned} A &= \frac{1}{2} \times 2\sqrt{3} \times 2\sqrt{6} \\ &= 2\sqrt{18} \\ &= 2\sqrt{9 \times 2} \\ &= 6\sqrt{2}, \end{aligned}$$

and hence $k = 6$.

22. Jan has two boxes.

There are 6 black and 4 white counters in box A.

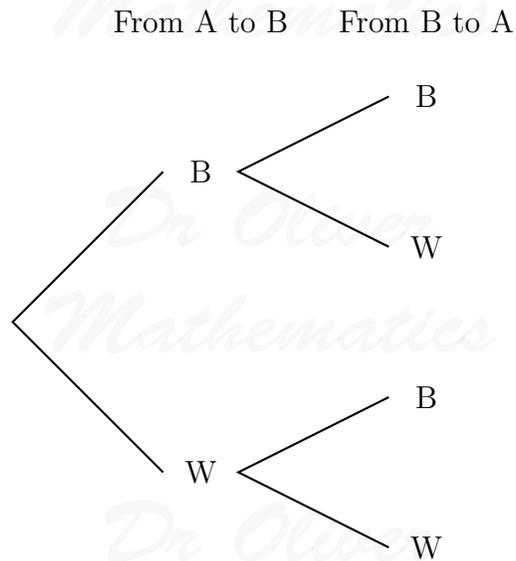
There are 7 black and 3 white counters in box B.

Jan takes at random a counter from box A and puts it in box B.

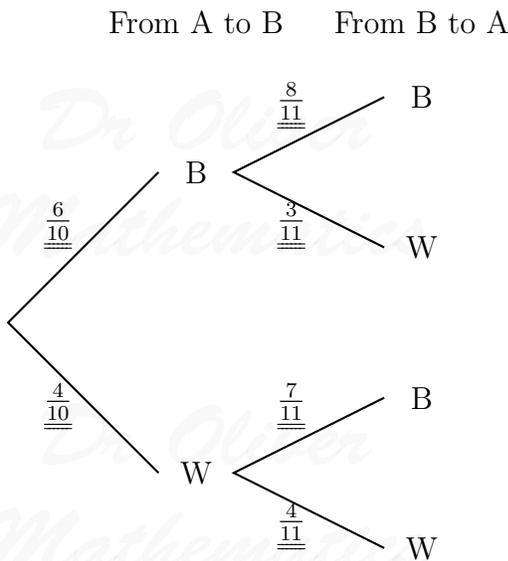
She then takes at random a counter from box B and puts it in box A.

(a) Complete the probability tree diagram.

(2)



Solution



(b) Find the probability that after Jan has put the counter from box B into box A there will still be 6 black counters and 4 white counters in box A.

(4)

Solution

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$$\begin{aligned} P(6 \text{ B}, 4 \text{ W}) &= P(\text{BB}) + P(\text{WW}) \\ &= \left(\frac{6}{10} \times \frac{8}{11}\right) + \left(\frac{4}{10} \times \frac{4}{11}\right) \\ &= \frac{48}{110} + \frac{16}{110} \\ &= \frac{64}{110} \\ &= \frac{32}{55}. \end{aligned}$$

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