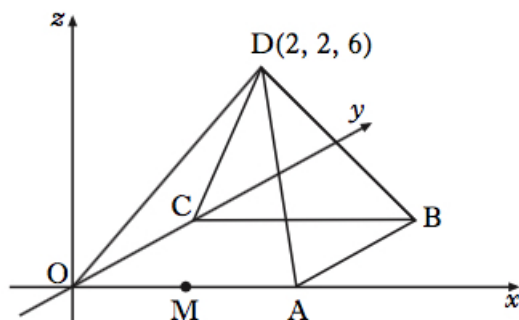


**Dr Oliver Mathematics**  
**Mathematics: Higher**  
**2011 Paper 2: Calculator**  
**1 hour 10 minutes**

The total number of marks available is 60.

You must write down all the stages in your working.

1.  $OABCD$  is a square based pyramid as shown in the diagram below.



$O$  is the origin,  $D$  is the point  $(2, 2, 6)$ , and  $OA = 4$  units.

$M$  is the mid-point of  $OA$ .

- (a) State the coordinates of  $B$ . (1)
- (b) Express  $\overrightarrow{DB}$  and  $\overrightarrow{DM}$  in component form. (3)
- (c) Find the size of angle  $BDM$ . (5)
2. Functions  $f$ ,  $g$ , and  $h$  are defined on the set of real numbers by

$$f(x) = x^3 - 1$$

$$g(x) = 3x + 1$$

$$h(x) = 4x - 5.$$

- (a) Find  $g(f(x))$ . (2)
- (b) Show that (1)

$$g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2.$$

- (c) (i) Show that  $(x - 1)$  is a factor of (5)

$$3x^3 + 4x^2 - 5x - 2.$$

(ii) Factorise

$$3x^3 + 4x^2 - 5x - 2$$

fully.

(d) Hence solve

$$g(f(x)) + x h(x) = 0.$$

(1)

3. A sequence is defined by

$$u_{n+1} = -\frac{1}{2}u_n \text{ with } u_0 = -16.$$

(a) Write down the values of  $u_1$  and  $u_2$ .

(1)

A second sequence is given by 4, 5, 7, 11, ...

It is generated by the recurrence relation

$$v_{n+1} = pv_n + q \text{ with } v_1 = 4.$$

(b) Find the values of  $p$  and  $q$ .

(3)

Either the sequence in (a) or the sequence in (b) has a limit.

(c) (i) Calculate this limit.

(3)

(ii) Why does the other sequence not have a limit?

4. The diagram shows the curve with equation

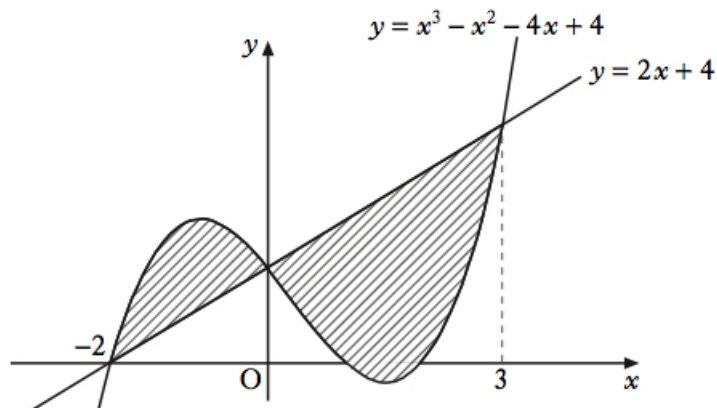
(10)

$$y = x^3 - x^2 - 4x + 4$$

and the line with equation

$$y = 2x + 4.$$

The curve and the line intersect at the points  $(-2, 0)$ ,  $(0, 4)$ , and  $(3, 10)$ .

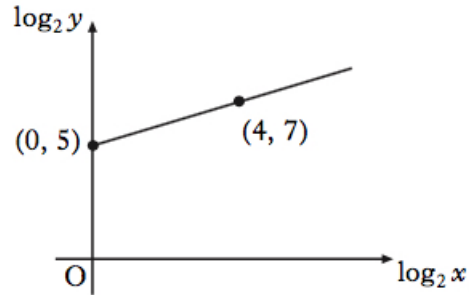


Calculate the total shaded area.

5. Variables  $x$  and  $y$  are related by the equation (5)

$$y = kx^n.$$

The graph of  $\log_2 y$  against  $\log_2 x$  is a straight line through the points  $(0, 5)$  and  $(4, 7)$ , as shown in the diagram.



Find the values of  $k$  and  $n$ .

6. (a) The expression (4)

$$3 \sin x - 5 \cos x$$

can be written in the form

$$R \sin(x + a)$$

where  $R > 0$  and  $0 \leq a < 2\pi$ .

Calculate the values of  $R$  and  $a$ .

- (b) Hence find the value of  $t$ , where  $0 \leq t \leq 2$ , for which (7)

$$\int_0^t (3 \cos x + 5 \sin x) dx = 3.$$

7. Circle  $C_1$  has equation (9)

$$(x + 1)^2 + (y - 1)^2 = 121.$$

A circle  $C_2$  with equation

$$x^2 + y^2 - 4x + 6y + p = 0$$

is drawn inside  $C_1$ .

The circles have no points of contact.

What is the range of values of  $p$ ?