Dr Oliver Mathematics
Mathematics
Sequences
Past Examination Questions

This booklet consists of 19 questions across a variety of examination topics. The total number of marks available is 117.

1. The sequence of positive numbers u_1, u_2, u_3, ... is given by

$$u_{n+1} = (u_n - 3)^2,\ u_1 = 1.$$

(a) Find u_2, u_3, and u_4. (3)

(b) Write down the value of u_{20}. (1)

2. A sequence a_1, a_2, a_3, ... is defined by

$$a_1 = 3,$$
$$a_{n+1} = 3a_n - 5,\ n \geqslant 1.$$

(a) Find the value of a_2 and the value of a_3. (2)

(b) Calculate the value of $\displaystyle\sum_{r=1}^{5} a_r$. (3)

3. A sequence a_1, a_2, a_3, ... is defined by

$$a_1 = k,$$
$$a_{n+1} = 3a_n + 5,\ n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k. (1)

(b) Show that $a_3 = 9k + 20$. (2)

(c) (i) Find $\displaystyle\sum_{r=1}^{4} a_r$ in terms of k. (4)

(ii) Show that $\displaystyle\sum_{r=1}^{4} a_r$ is divisible by 10.

4. A sequence is given by:

$$x_1 = 1,$$
$$x_{n+1} = x_n(p + x_n),$$

where p is a constant ($p \neq 0$).

(a) Find x_2 in terms of p. (1)

(b) Show that $x_3 = 1 + 3p + 2p^2$. (2)

Given that $x_3 = 1$,

(c) find the value of p, (3)

(d) write down the value of x_{2008}. (2)

5. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$
$$x_{n+1} = ax_n - 3, \, n \geqslant 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a. (1)

(b) Show that $x_3 = a^2 - 3a - 3$. (2)

Given that $x_3 = 7$,

(c) find the possible values of a. (3)

6. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$
$$a_{n+1} = 2a_n - 7, \, n \geqslant 1,$$

where k is a constant.

(a) Find an expression for a_2 in terms of a. (1)

(b) Show that $a_3 = 4k - 21$. (2)

Given that $\sum_{r=1}^{4} a_r = 43$,

(c) find the value of k. (4)

7. A sequence of positive numbers is defined by

$$a_1 = 2,$$
$$a_{n+1} = \sqrt{a_n^2 + 3}, \, n \geqslant 1.$$

(a) Find a_2 and a_3, leaving your answers in surd form. (2)

(b) Show that $a_5 = 4$. (2)

8. A sequence a_1, a_2, a_3, … is defined by

$$a_1 = 2,$$
$$a_{n+1} = 3a_n - c,\ n \geqslant 1,$$

where c is a constant.

(a) Find an expression for a_2 in terms of c. (1)

Given that $\sum_{i=1}^{3} a_i = 0$,

(b) find the value of c. (4)

9. A sequence a_1, a_2, a_3, … is defined by

$$a_1 = k,$$
$$a_{n+1} = 5a_n + 3,\ n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k. (1)

(b) Show that $a_3 = 25k + 18$. (2)

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form. (4)

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 6.

10. A sequence x_1, x_2, x_3, … is defined by

$$x_1 = 1,$$
$$x_{n+1} = ax_n + 5,\ n \geqslant 1,$$

where a is a constant.

(a) Write down an expression for x_2 in terms of a. (1)

(b) Show that $x_3 = a^2 + 5a + 5$. (2)

Given that $x_3 = 41$,

(c) find the possible values of a. (3)

11. A sequence a_1, a_2, a_3, … is defined by

$$a_1 = 3,$$
$$a_{n+1} = 2a_n - c,\ n \geqslant 1,$$

where c is a constant.

(a) Write down an expression, in terms of c, for a_2. (1)

(b) Show that $a_3 = 12 - 3c$. (2)

Given that $\sum_{r=1}^{4} a_r \geqslant 23$,

(c) find the range of values of c. (4)

12. A sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+1} = 2u_n - 1, \; n \geqslant 1.$$

Given that $u_2 = 9$,

(a) find the value of u_3 and the value of u_4, (2)

(b) evaluate $\sum_{r=1}^{4} u_r$. (3)

13. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$
$$a_{n+1} = k(a_n + 2), \; n \geqslant 1,$$

where k is a constant.

(a) Find an expression for a_2 in terms of k. (1)

Given that $\sum_{i=1}^{3} a_i = 2$,

(b) find the two possible values of k. (6)

14. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$
$$x_{n+1} = (x_n)^2 - kx_n, \; n \geqslant 1,$$

where k is a constant, $k \neq 0$.

(a) Find an expression for x_2 in terms of k. (1)

(b) Show that $x_3 = 1 - 3k + 2k^2$. (2)

Given also that $x_3 = 1$,

(c) calculate the value of k. (3)

(d) Hence find the value of $\sum_{n=1}^{100} x_n$. (3)

15. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 5a_n - 3,\ n \geqslant 1.$$

Given that $a_2 = 7$,

(a) find the value of a_1, (2)

(b) Find the value of $\sum_{r=1}^{4} a_r$ (3)

16. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$
$$a_{n+1} = 4a_n - 3,\ n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k. (1)

Given that $\sum_{r=1}^{3} a_r = 66$,

(b) find the value of k. (4)

17. (a) A sequence $U_1, U_2, U_3, \ldots$ is defined by

$$U_1 = 4,$$
$$U_2 = 4,$$
$$U_{n+2} = 2U_{n+1} - U_n,\ n \geqslant 1.$$

Find the value of

(i) U_3, (1)

(ii) $\sum_{n=1}^{20} U_n$. (2)

(b) A sequence $V_1, V_2, V_3, \ldots$ is defined by

$$V_1 = k,$$
$$V_2 = 2k,$$
$$V_{n+2} = 2V_{n+1} - V_n,\ n \geqslant 1,$$

where k is a constant.

(i) Find V_3 and V_4 in terms of k. (2)

Given that $\sum_{n=1}^{5} V_n = 165$,

(ii) find the value of k. (3)

18. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$
$$a_{n+1} = 5 - ka_n,\ n \geqslant 1,$$

where k is a constant.

(a) Write down expressions for a_2 and a_3 in terms of k. (2)

Find

(b) $\sum_{r=1}^{3}(1 + a_r)$ in terms of k, giving your answer in its simplest form, (3)

(c) $\sum_{r=1}^{100}(a_{r+1} + ka_r)$. (1)

19. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 1,$$
$$a_{n+1} = \frac{k(a_n + 1)}{a_n},\ n \geqslant 1,$$

where k is a positive constant.

(a) Write down expressions for a_2 and a_3 in terms of k, giving your answers in their simplest form. (3)

Given that $\sum_{r=1}^{3} a_r = 10$,

(b) find an exact value for k. (3)