Dr Oliver Mathematics
GCSE Mathematics
2024 November Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Use your calculator to work out the value of (2)

$$\sqrt{\frac{208.3 - 15.7}{5.694 + 1.8^2}}.$$

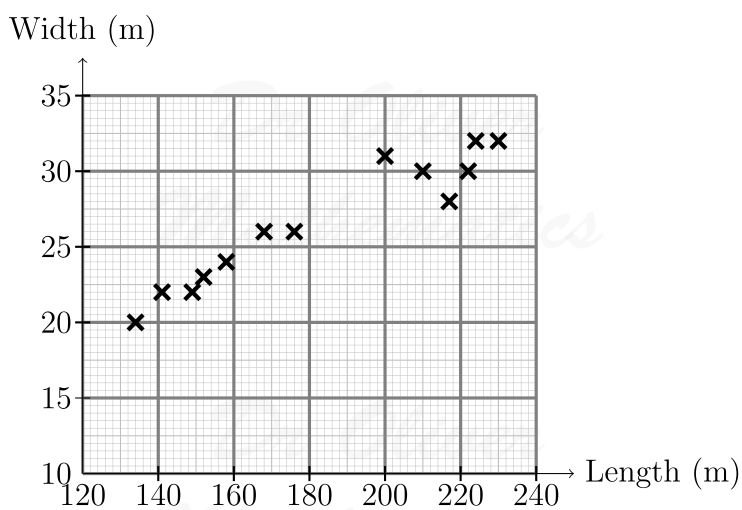
Write down all the digits on your calculator display.

Solution

Well,

$$\begin{aligned}\sqrt{\frac{208.3 - 15.7}{5.694 + 1.8^2}} &= \sqrt{\frac{192.6}{5.694 + 3.24}} \\ &= \sqrt{\frac{192.6}{8.934}} \\ &= \underline{\underline{4.643\,069\,317 \text{ (FCD)}}}.\end{aligned}$$

2. The scatter graph shows information about some ships.
It shows the length and the width of each ship.



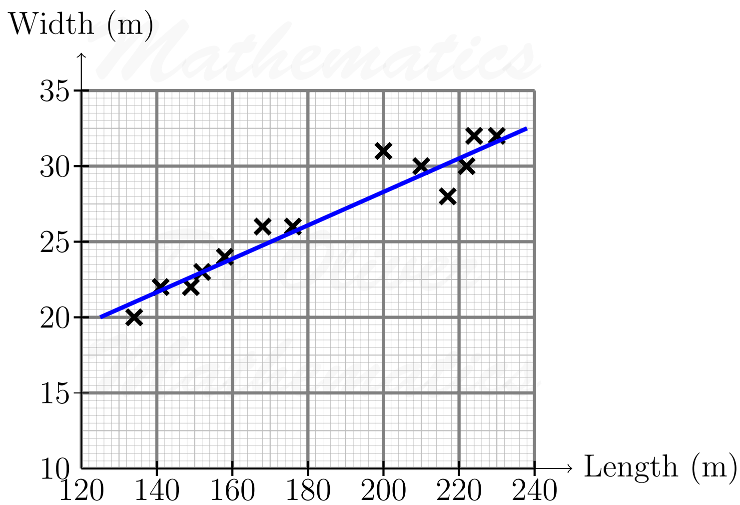
(a) What type of correlation does this scatter graph show? (1)

Solution

Positive correlation.

(b) Draw a line of best fit on the scatter graph. (1)

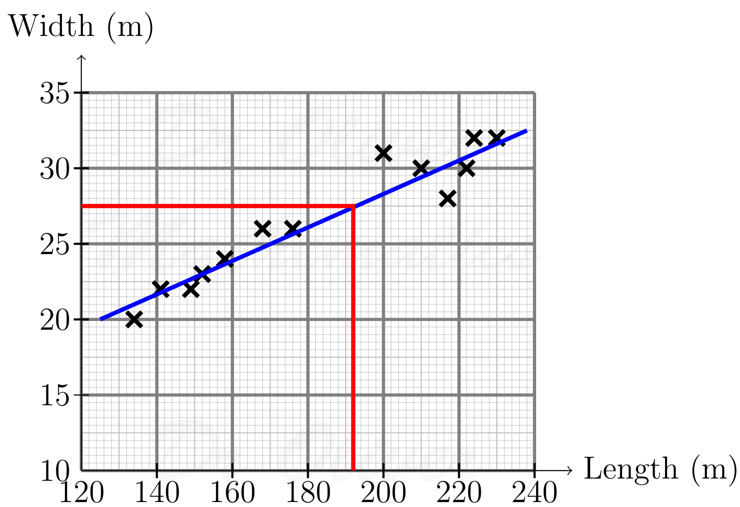
Solution



A different ship has a length of 194 metres.

(c) Use your line of best fit to find an estimate for the width of this ship. (1)

Solution



Correct read-off: approximately 25 m.

3. In London, a 200 g Choci bar costs £3.50. (3)
In Zurich, a 360 g Choci bar costs 7.20 Swiss francs.

London	Zurich
Choci bar	**Choci bar**
200 g	360 g
£3.50	7.20 Swiss francs

The exchange rate is £1 = 1.25 Swiss francs.

In which city is the Choci bar the better value for money, in London or in Zurich?
You must show how you get your answer.

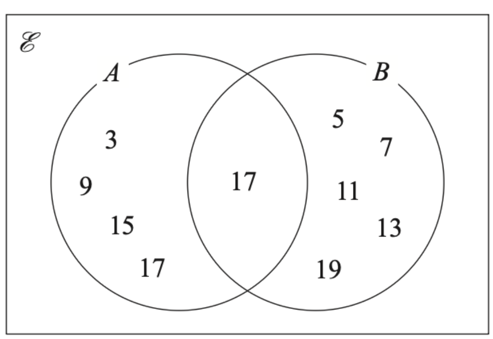
Solution

In Zurich, the cost of the 200 g Choci, in pounds, is

$$\begin{aligned}\frac{200}{360} \times 7.20 \text{ SF} &= \frac{200}{360} \times \frac{1}{1.25} \times £7.20 \\ &= £3.20;\end{aligned}$$

hence, the bar is better value for money in Zurich.

4. $\mathscr{E}$ = {odd numbers between 0 and 20}. (2)
$\mathscr{A}$ = $\{3, 915, 17\}$.
$\mathscr{B}$ = $\{5, 7, 11, 13, 17, 19\}$.



Write down two things Tom should do to make his answer fully correct.

Solution

He needs to remove the 17 that is sitting in the only A component.
He need to have 1 in the section outside of A and B.

5. (a) Complete the table of values for (2)

$$y = x^2 - 2x - 3.$$

x	-2	-1	0	1	2	3	4
y		0			-3		

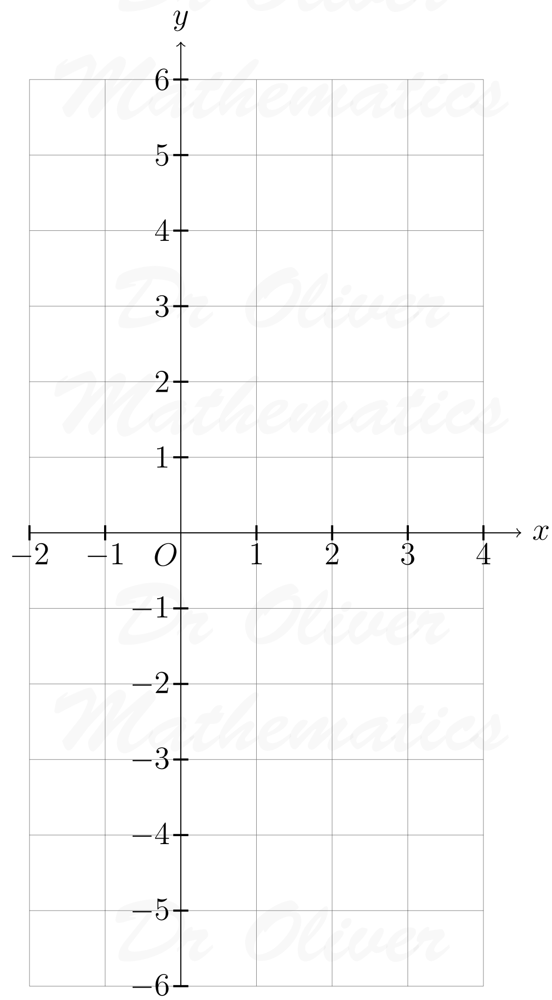
Solution

x	-2	-1	0	1	2	3	4
y	$\underline{\underline{5}}$	$\underline{\underline{0}}$	-3	$\underline{\underline{-5}}$	-3	$\underline{\underline{0}}$	$\underline{\underline{5}}$

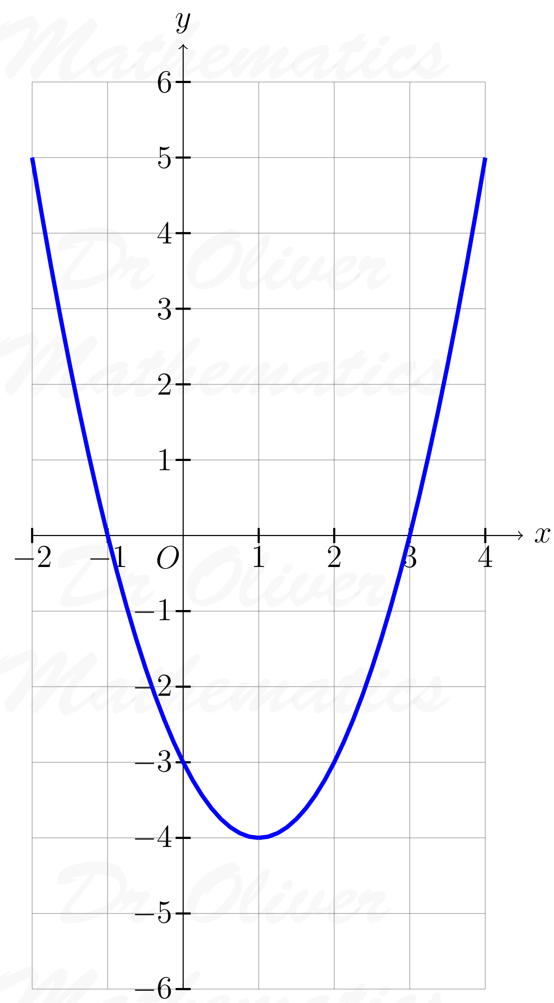
(b) On the grid, draw the graph of (2)

$$y = x^2 - 2x - 3,$$

for values of x from -2 to 4.



Solution



6. The cost of a first class stamp increased from 76p to 85p. (4)
   The cost of a second class stamp increased from 65p to 66p.

   Filip says, "The percentage increase in the cost of a first class stamp is more than 7 times the percentage increase in the cost of a second class stamp."

   Is Filip correct?
   You must show all your working.

Solution

First class stamp:

$$\left(\frac{85-76}{76}\right)\times 100\% = \frac{9}{76}\times 100\%$$
$$= 11\tfrac{16}{19}\%.$$

Second class stamp:

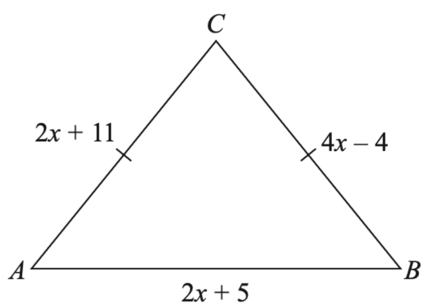
$$\left(\frac{66-65}{65}\right)\times 100\% = \frac{1}{65}\times 100\%$$
$$= 1\tfrac{7}{13}\%.$$

Now,

$$\frac{11\frac{16}{19}}{1\frac{7}{13}} = 7\tfrac{53}{76};$$

hence, Filip is correct.

7. The diagram shows triangle ABC. (5)



In the diagram, all measurements are in centimetres.

- $AC = BC$.
- The perimeter of the triangle is 72 cm.

Work out the area of the triangle.

Solution

Well,

$$\begin{aligned} AC = BC &\Rightarrow 2x + 11 = 4x - 4 \\ &\Rightarrow 2x = 15 \\ &\Rightarrow x = 7.5. \end{aligned}$$

Now,

$$2(7.5) + 11 = 26 \text{ and } 2(7.5) + 5 = 20.$$

Let D be the midpoint of AB:

$$\begin{aligned} \text{opp}^2 + \text{adj}^2 = \text{hyp}^2 &\Rightarrow AD^2 + CD^2 = AC^2 \\ &\Rightarrow 10^2 + CD^2 = 26^2 \\ &\Rightarrow CD^2 = 576 \\ &\Rightarrow CD = 24. \end{aligned}$$

Hence, the area of the triangle is

$$\begin{aligned} \tfrac{1}{2} \times AB \times CD &= \tfrac{1}{2} \times 20 \times 24 \\ &= \underline{\underline{240 \text{ cm}^2}}. \end{aligned}$$

8. (2)

$$1.25 \times 10^{-12} = k \times (4 \times 10^{-20}).$$

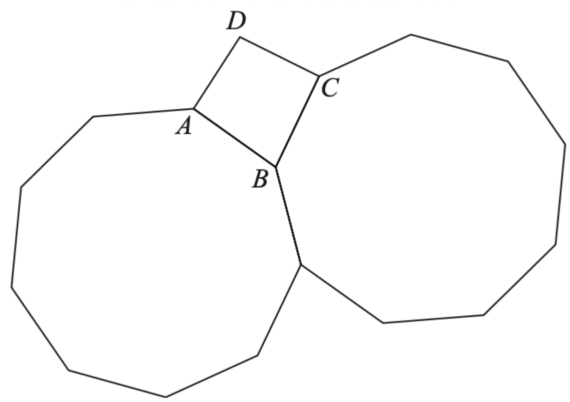
Work out the value of k.
Give your answer in standard form.

Solution

Well,

$$\begin{aligned} 1.25 \times 10^{-12} = k \times (4 \times 10^{-20}) &\Rightarrow k = \frac{1.25 \times 10^{-12}}{4 \times 10^{-20}} \\ &\Rightarrow k = 31\,250\,000 \\ &\Rightarrow \underline{\underline{k = 3.125 \times 10^7}}. \end{aligned}$$

9. The diagram shows two congruent regular 9-sided polygons. (3)
$ABCD$ is a quadrilateral.



Show that $ABCD$ is **not** a square.

Solution

Well,

$$(9 - 2) \times 180 = 1\,260^\circ$$

and each angle in the congruent regular 9-sided polygons is

$$\frac{1\,260}{9} = 140^\circ.$$

Now,

$$360 - (140 + 140) = 80^\circ < 90^\circ$$

if there was a square.
Hence, $ABCD$ is not a square.

10. Use algebra to solve the simultaneous equations: (4)

$$4x - 5y = 20$$
$$6x + 7y = -57.$$

You must show all your working.

Solution

$$4x - 5y = 20 \quad (1)$$
$$6x + 7y = -57 \quad (2)$$

E.g. do $7 \times (1)$ and $5 \times (2)$:

$$28x - 35y = 140 \quad (3)$$
$$30x + 35y = -285 \quad (4)$$

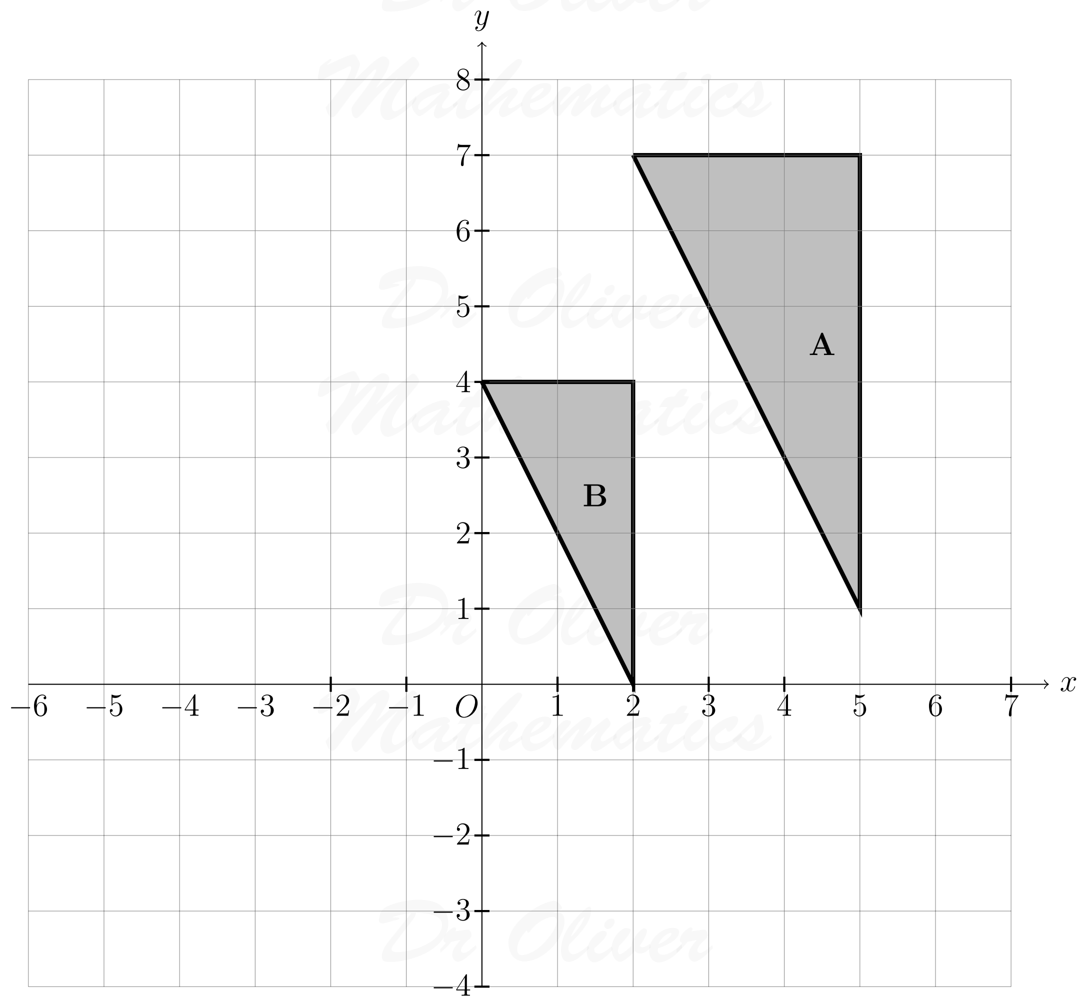
Do (3) + (4):

$$58x = -145 \Rightarrow \underline{\underline{x = -2.5}}$$

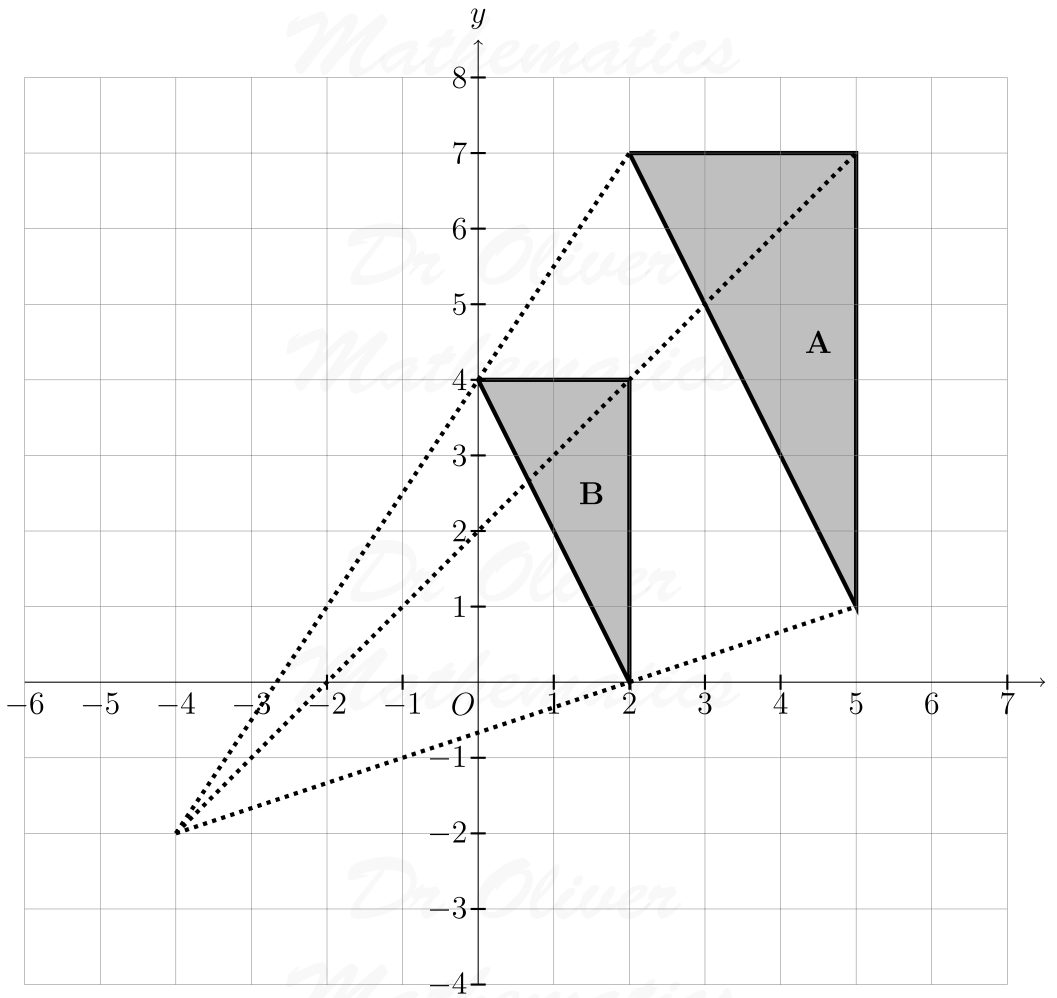
insert this value into (2):

$$\begin{aligned} &\Rightarrow 6(-2.5) + 7y = -57 \\ &\Rightarrow -15 + 7y = -57 \\ &\Rightarrow 7y = -42 \\ &\Rightarrow \underline{\underline{y = -6}}. \end{aligned}$$

11. Describe fully the single transformation that maps triangle **A** onto triangle **B**. (2)



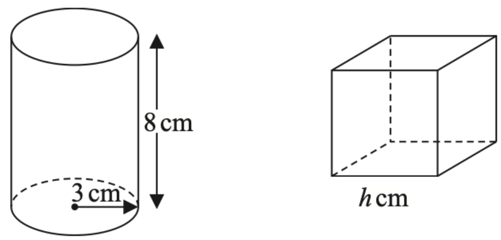
Solution



So, it is an enlargement, centre $(-4, -2)$, scale factor

$$\frac{4}{6} = \frac{2}{3}.$$

12. The diagram shows a solid cylinder with base radius 3 cm and height 8 cm. (5)
It also shows a solid cube with side length h cm.



- The cylinder is made from steel with a density of 7.86 g/cm^3.
- The cube is made from brass with a density of 8.5 g/cm^3.
- The mass of the cylinder is equal to the mass of the cube.

Work out the value of h.
Give your answer correct to 1 decimal place.

Solution

Well,

$$\begin{aligned} V_{\text{cylinder}} &= \pi \times 3^2 \times 8 \\ &= 72\pi \end{aligned}$$

and

$$\begin{aligned} V_{\text{cube}} &= h \times h \times h \\ &= h^3. \end{aligned}$$

Now, because the masses are equal,

$$\begin{aligned} \text{mass} = \text{density} \times \text{volume} &\Rightarrow 7.86 \times 72\pi = 8.5 \times h^3 \\ &\Rightarrow h^3 = \tfrac{5\,652}{85}\pi \\ &\Rightarrow h = 5.933\,501\,184 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{h = 5.9 \text{ cm (1 dp)}}}. \end{aligned}$$

13. Here is a table of values of x and y.

x	2	4	6	8
y	0	4	8	12

Nadia says that y is directly proportional to x because the value of y increases by 4 as the value of x increases by 2.

(a) Is Nadia correct? (1)
You must give a reason for your answer.

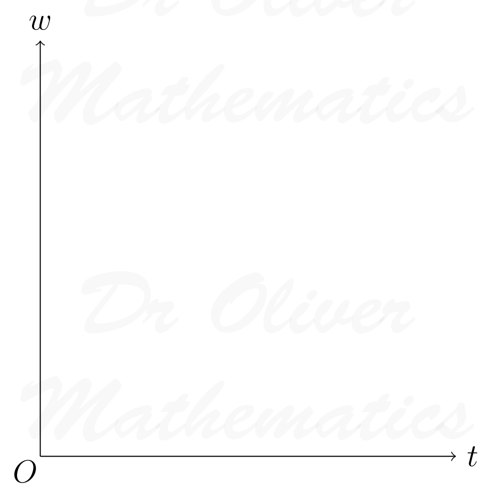
Solution
If that was the case,
$$y \propto x^2 \Rightarrow y = kx^2,$$
for some value of k. But
$$x = 2 \Rightarrow 0 = 4k \Rightarrow k = 0,$$
which is a nonsense.

w is directly proportional to the square root of t.
$w = 140$ when $t = 64$.

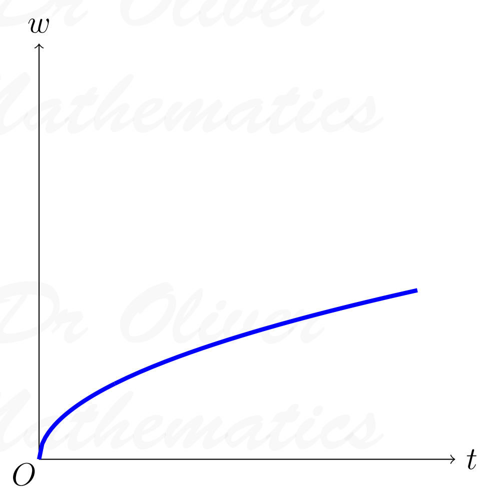
(b) (i) Calculate the value of w when $t = 7.84$. (3)

Solution
Now,
$$w \propto \sqrt{t} \Rightarrow w = l\sqrt{t},$$
for some value of l. Next,
$$\begin{aligned} w = 140, t = 64 &\Rightarrow 140 = l\sqrt{64} \\ &\Rightarrow 140 = 8l \\ &\Rightarrow l = \tfrac{35}{2}, \end{aligned}$$
and so
$$w = \tfrac{35}{2}\sqrt{t}.$$
Finally,
$$\begin{aligned} t = 7.84 &\Rightarrow w = \tfrac{35}{2}\sqrt{7.84} \\ &\Rightarrow w = \tfrac{35}{2} \times 2.8 \\ &\Rightarrow \underline{\underline{w = 49}}. \end{aligned}$$

(ii) On the axes below, sketch a graph to show the relationship between w and t. (1)



Solution



14. There are 10 football teams in a league. (2)
Each team plays every other team 4 times.

Work out the total number of games played.

Solution

$$4 \times 9 \times 5 = \underline{\underline{180}}.$$

15. Here are the first five terms of a quadratic sequence:

3 20 47 84 131

(a) Find an expression, in terms of n, for the nth term of this sequence. (3)

Solution
Let the

$$n\text{th term} = an^2 + bn + c.$$

We only need the second line of differences (why?):

$$\begin{array}{ccccc} 3 & & 20 & & 47 \\ & 17 & & 27 & \\ & & 10 & & \end{array}$$

$$\begin{array}{ccccc} a+b+c & & 4a+2b+c & & 9a+3b+c \\ & 3a+b & & 5a+b & \\ & & 2a & & \end{array}$$

We compare terms:

$$2a = 10 \Rightarrow a = 5,$$

$$\begin{aligned} 3a + b = 17 &\Rightarrow 3 \times 5 + b = 17 \\ &\Rightarrow b = 2, \end{aligned}$$

and

$$\begin{aligned} a + b + c = 3 &\Rightarrow 5 + 2 + c = 3 \\ &\Rightarrow c = -4; \end{aligned}$$

hence,

$$n\text{th term} = \underline{\underline{5n^2 + 2n - 4}}.$$

The terms of a different sequence are given by the rule

$$u_{n+1} = ku_n + k,$$

where k is a constant.

Given that

$$u_1 = 9 \text{ and } u_2 = 4,$$

(b) find the value of u_4. (3)

Solution

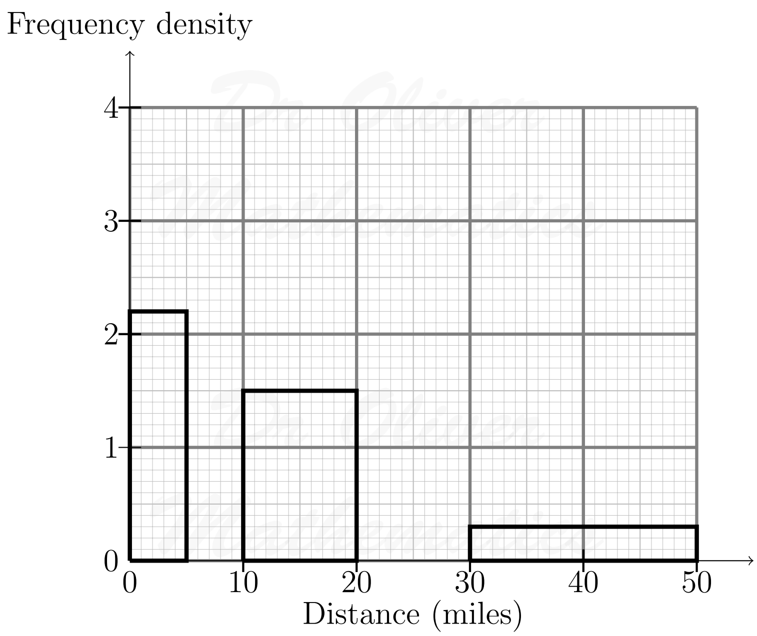
Well,

$$u_2 = ku_1 + k \Rightarrow 4 = 9k + k$$
$$\Rightarrow 4 = 10k$$
$$\Rightarrow k = \tfrac{2}{5}.$$

Finally,

$$u_3 = \tfrac{2}{5}(4) + \tfrac{2}{5}$$
$$= 2,$$
$$u_4 = \tfrac{2}{5}(2) + \tfrac{2}{5}$$
$$= \underline{\underline{\tfrac{6}{5}}}.$$

16. The histogram gives information about the distances that 60 teachers travelled to school on Monday. (4)
The histogram is incomplete.



- 11 of the teachers travelled between 0 miles and 5 miles.
- None of the teachers travelled a distance greater than 50 miles.

- The number of teachers who travelled between 5 miles and 10 miles is the same as the number of teachers who travelled between 20 miles and 30 miles.

Complete the histogram.

Solution

We make up a table:

Distance	Frequency	Width	Frequency Density
$0-5$	$5\times 2.2=11$	5	2.2
$5-10$		5	
$10-20$	$10\times 1.5=15$	10	1.5
$20-30$		10	
$30-50$	$20\times 0.3=6$	20	0.3

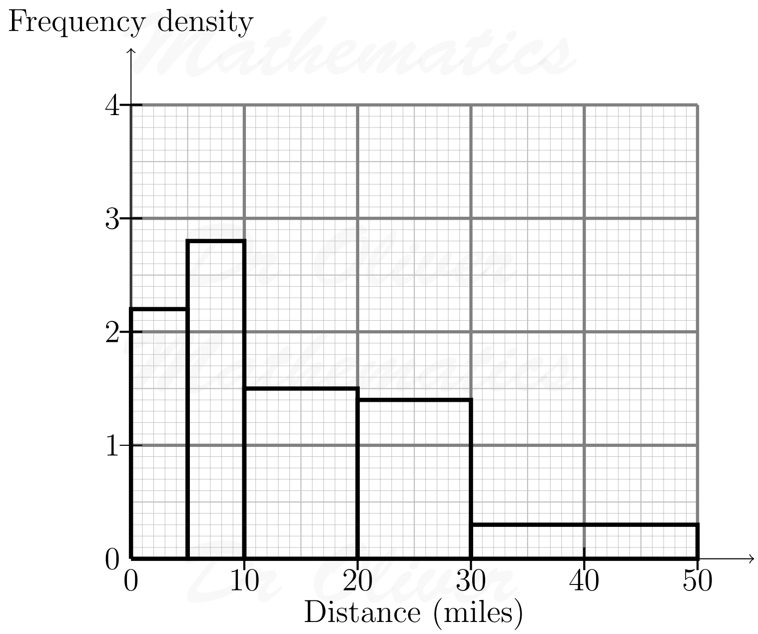
The number of teachers who travelled between 5 miles and 10 miles is the same as the number of teachers who travelled between 20 miles and 30 miles:

$$60-(11+15+6)=28$$

and so 14 teachers must have travelled in each interval:

Distance	Frequency	Width	Frequency Density
$0-5$	11	5	2.2
$5-10$	14	5	$\frac{14}{5}=2.8$
$10-20$	15	10	1.5
$20-30$	14	10	$\frac{14}{10}=1.4$
$30-50$	6	20	0.3

Finally, we complete the histogram:



17. Show that (3)

$$\frac{6x-y}{10xy}+\frac{1}{2x}-\frac{2y-7x}{5xy}$$

simplifies to

$$\frac{k}{y},$$

where k is an integer.

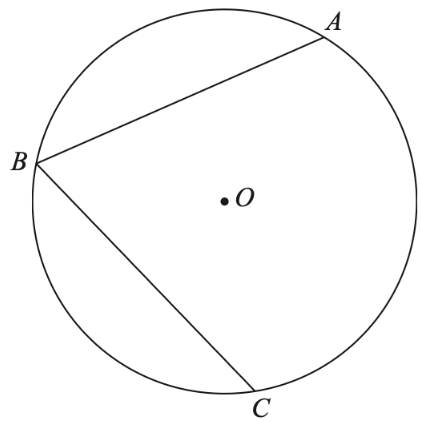
Solution

Well,

$$\begin{aligned}\frac{6x-y}{10xy}+\frac{1}{2x}-\frac{2y-7x}{5xy} &= \frac{6x-y}{10xy}+\frac{5y}{10xy}-\frac{2(2y-7x)}{10xy}\\ &= \frac{6x-y+5y-2(2y-7x)}{10xy}\\ &= \frac{6x-y+5y-4y+14x}{10xy}\\ &= \frac{20x}{10xy}\\ &= \underline{\underline{\frac{2}{y}}};\end{aligned}$$

so, $\underline{\underline{k = 2}}$.

18. A, B, and C are three points on a circle, centre O. (3)



$BA = BC$.

Prove that OB bisects angle ABC.

Solution

$OA = OB$ (radii)
$OB = OC$ (radii)
$BA = BC$ (given)
So $\triangle OAB = \triangle OBC$ (similar triangles but the same as OB is common).
Then $\angle OBA = \angle OBC$.
Hence, OB bisects angle ABC.

19. (5)

$$T = \frac{w}{a - c}.$$

- $w = 435$ correct to the nearest 5.
- $a = 9.8$ correct to 2 significant figures.
- $c = 2.5$ correct to 2 significant figures.

By considering bounds, calculate the value of T to a suitable degree of accuracy. You must show all your working and give a reason for your final answer.

Solution

Well,

$$\begin{aligned} 432.5 \leqslant w < 437.5 \\ 9.75 \leqslant a < 9.85 \\ 2.45 \leqslant c < 2.55, \end{aligned}$$

and

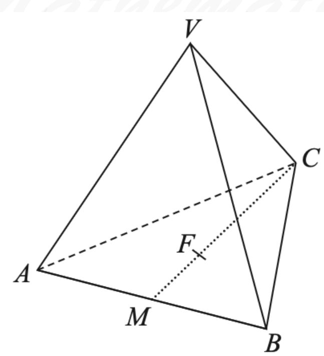
$$\begin{aligned} & \frac{432.5}{9.85 - 2.45} < T < \frac{437.5}{9.75 - 2.55} \\ \Rightarrow \quad & 58.4\dot{4}\dot{5}\dot{9} < T < 60.763\dot{8}. \end{aligned}$$

We draw up a table.

	Lower Bound	Upper Bound	Agree?
1 sf	60	60	Yes
2 sf	58	61	No

Hence, $\underline{\underline{T = 60 \text{ (1 sf)}}}$.

20. $VABC$ is a solid pyramid. (3)
ABC is an equilateral triangle.



- M is the midpoint of AB.
- F is the point on MC such that

$$MF : FC = 1 : 2.$$

- The vertex V is vertically above F.

- $VA = VB = VC$.
- $VF = 8$ cm.
- Angle $VCM = 52°$.

Work out the side length of the equilateral triangle ABC.
Give your answer correct to 1 decimal place.

Solution

Well,

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 52° &= \frac{8}{CF} \\ \Rightarrow CF &= \frac{8}{\tan 52°} \\ \Rightarrow CM &= 1.5 \times \frac{8}{\tan 52°} \\ \Rightarrow CM &= \frac{12}{\tan 52°}. \end{aligned}$$

Pythagoras' theorem:

$$\begin{aligned} CM^2 + MB^2 = CB^2 &\Rightarrow \left(\frac{12}{\tan 52°}\right)^2 + (\tfrac{1}{2}BC)^2 = BC^2 \\ &\Rightarrow \frac{144}{\tan^2 52°} + \tfrac{1}{4}BC^2 = BC^2 \\ &\Rightarrow \frac{144}{\tan^2 52°} = \tfrac{3}{4}BC^2 \\ &\Rightarrow BC^2 = \frac{192}{\tan^2 52°} \\ &\Rightarrow BC = \sqrt{\frac{192}{\tan^2 52°}} \\ &\Rightarrow BC = 10.825\,811\,2 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{BC = 10.8 \text{ (1 dp)}}}. \end{aligned}$$

21. The point P has coordinates $(-4, 5)$. (5)
The point Q has coordinates $(6, -6)$.
The point R has coordinates $(k, k + 3)$.

Given that angle PRQ is a right angle, find the possible values of k.
You must show all your working.

Solution

Well,

$$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{OQ} - \overrightarrow{OP}\\ &= \begin{pmatrix} 6 \\ -6 \end{pmatrix} - \begin{pmatrix} -4 \\ 5 \end{pmatrix}\\ &= \begin{pmatrix} 10 \\ -11 \end{pmatrix},\\ \overrightarrow{PR} &= \overrightarrow{OR} - \overrightarrow{OP}\\ &= \begin{pmatrix} k \\ k+3 \end{pmatrix} - \begin{pmatrix} -4 \\ 5 \end{pmatrix}\\ &= \begin{pmatrix} k+4 \\ k-2 \end{pmatrix},\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{QR} &= \overrightarrow{OR} - \overrightarrow{OQ}\\ &= \begin{pmatrix} k \\ k+3 \end{pmatrix} - \begin{pmatrix} 6 \\ -6 \end{pmatrix}\\ &= \begin{pmatrix} k-6 \\ k+9 \end{pmatrix}.\end{aligned}$$

Pythagoras' theorem:

$$\begin{aligned} & PR^2 + QR^2 = PQ^2\\ \Rightarrow\quad & \left[(k+4)^2 + (k-2)^2\right] + \left[(k-6)^2 + (k+9)^2\right] = 10^2 + (-11)^2\\ \Rightarrow\quad & (k^2+8k+16) + (k^2-4k+4) + (k^2-12k+36) + (k^2+18k+81) = 221\\ \Rightarrow\quad & 4k^2 + 10k - 84 = 0\\ \Rightarrow\quad & 2(2k^2+5k-42) = 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & +5\\ \text{multiply to:} & (+2)\times(-42) = -84\end{array}\right\} +12,\ -7$$

e.g.,

$$\begin{aligned}\Rightarrow\quad & 2[2k^2 + 12k - 7k - 42] = 0\\ \Rightarrow\quad & 2[2k(k+6) - 7(k+6)] = 0\\ \Rightarrow\quad & 2(2k-7)(k+6) = 0\\ \Rightarrow\quad & 2k-7 = 0 \text{ or } k+6 = 0\\ \Rightarrow\quad & \underline{\underline{k = 3\tfrac{1}{2} \text{ or } k = -6}}.\end{aligned}$$

22. There are only red counters and yellow counters in a box. (5)

$\frac{3}{5}$ of the counters are red.

Sophie takes at random two counters from the box.

The probability that the two counters are the same colour is $\frac{41}{80}$.

Work out the number of yellow counters in the box.

Solution

Let be number of red counters be $3n$.
Then the number of yellow counters will be $2n$.

P(RR):

- Pick any red counter — $3n$ — out of the total number — $5n$:
$$\frac{3n}{5n}.$$
- And then pick any red counter — $(3n-1)$ — out of the remaining counters — $(5n-1)$:
$$\frac{3n-1}{5n-1}.$$
- Multiply them:
$$\left(\frac{3n}{5n}\right) \times \left(\frac{3n-1}{5n-1}\right).$$

P(YY):

- Pick any yellow counter — $2n$ — out of the total number — $5n$:
$$\frac{2n}{5n}.$$
- And then pick any yellow counter — $(2n-1)$ — out of the remaining counters — $(5n-1)$:
$$\frac{2n-1}{5n-1}.$$
- Multiply them:
$$\left(\frac{2n}{5n}\right) \times \left(\frac{2n-1}{5n-1}\right).$$

Now,

$$\begin{aligned}
& \mathrm{P(same\ colour)} = \tfrac{41}{80} \\
\Rightarrow \quad & \mathrm{P}(RR) + \mathrm{P}(YY) = \tfrac{41}{80} \\
\Rightarrow \quad & \left(\frac{3n}{5n}\right) \times \left(\frac{3n-1}{5n-1}\right) + \left(\frac{2n}{5n}\right) \times \left(\frac{2n-1}{5n-1}\right) = \tfrac{41}{80}
\end{aligned}$$

cross-multiply:

$$\begin{aligned}
\Rightarrow \quad & 80[3n(3n-1) + 2n(2n-1)] = 41(5n)(5n-1) \\
\Rightarrow \quad & 80[9n^2 - 3n + 4n^2 - 2n] = 205n(5n-1) \\
\Rightarrow \quad & 80(13n^2 - 5n) = 205n(5n-1) \\
\Rightarrow \quad & 1\,040n^2 - 400n = 1\,025n^2 - 205n \\
\Rightarrow \quad & 15n^2 - 195n = 0 \\
\Rightarrow \quad & 15n(n-13) = 0 \\
\Rightarrow \quad & n = 0 \text{ (cannot happen) or } n - 13 = 0 \\
\Rightarrow \quad & n = 13 \\
\Rightarrow \quad & 2n = 26;
\end{aligned}$$

hence, the number of yellow counters in the box is $\underline{\underline{26}}$.