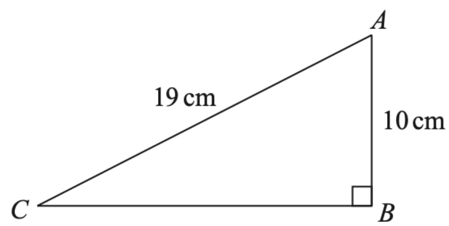
Dr Oliver Mathematics
GCSE Mathematics
2024 June Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. ABC is a right-angled triangle. (2)



Work out the length of CB.
Give your answer correct to 3 significant figures.

Solution

Pythagoras' theorem:

$$\begin{aligned} AB^2 + BC^2 = AC^2 &\Rightarrow 10^2 + BC^2 = 19^2 \\ &\Rightarrow 100 + BC^2 = 361 \\ &\Rightarrow BC^2 = 261 \\ &\Rightarrow BC = 16.155\,494\,42 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{BC = 16.2 \text{ cm (3 sf)}}}. \end{aligned}$$

2. (a) Write 90 as a product of its prime factors. (2)

Solution

$$\begin{array}{r|l} & 90 \\ \hline 2 & 45 \\ \hline 3 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

and so

$$90 = \underline{\underline{2 \times 3^2 \times 5}}.$$

$A = 2^2 \times 3$ and $B = 2 \times 3^2$.

(b) Write down the lowest common multiple (LCM) of A and B. (1)

Solution

Well,

$$\begin{aligned} \text{LCM} &= 2^2 \times 3^2 \\ &= 4 \times 9 \\ &= \underline{\underline{36}}. \end{aligned}$$

3. The number of hours, H, that some machines take to make 5 000 bottles is given by (2)

$$H = \frac{72}{n},$$

where n is the number of machines.

- On Monday, 6 machines made 5 000 bottles.
- On Tuesday, 9 machines made 5 000 bottles.

The machines took more time to make the bottles on Monday than on Tuesday.

How much more time?

Solution

On Monday,

$$H_1 = \frac{72}{6} = 12 \text{ hours},$$

and, on Tuesday,

$$H_2 = \frac{72}{9} = 8 \text{ hours}.$$

Hence,

$$\text{difference} = 12 - 8 = \underline{\underline{4 \text{ hours}}}.$$

4. There are only red discs, blue discs, and yellow discs in a bag. (4)
There are 24 yellow discs in the bag.

Mel is going to take at random a disc from the bag.

The probability that the disc will be yellow is 0.16.

$$\text{The number of red discs} : \text{the number of blue discs} = 5 : 4.$$

Work out the number of red discs in the bag.

Solution

Let there be r red discs and let there be b blue discs in the bag. Now,

$$\begin{aligned} \mathrm{P}(Y) = 0.16 &\Rightarrow \frac{24}{r+b+24} = 0.16 \\ &\Rightarrow \frac{24}{0.16} = r + b + 24 \\ &\Rightarrow 150 = r + b + 24 \\ &\Rightarrow r + b = 126. \end{aligned}$$

Finally,

$$\begin{aligned} \text{the number of red counters} &= \left(\tfrac{5}{5+4}\right) \times 126 \\ &= \underline{\underline{70}}. \end{aligned}$$

5. (a) Complete the table of values for (2)

$$y = x^2 - x.$$

x	-2	-1	0	1	2	3
y	6		0		2	

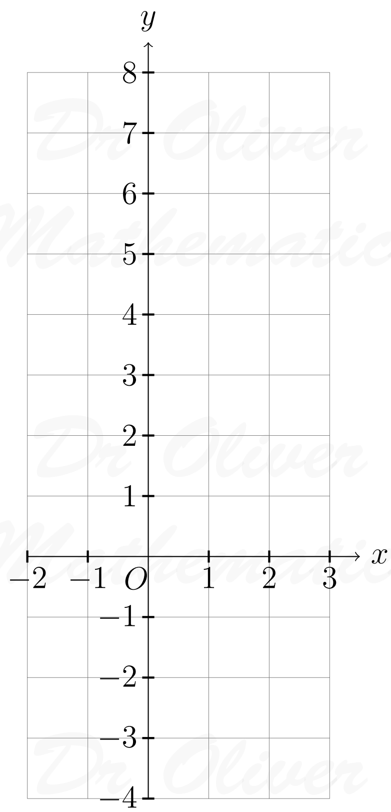
Solution

x	-2	-1	0	1	2	3
y	6	$\underline{\underline{2}}$	0	$\underline{\underline{0}}$	2	$\underline{\underline{6}}$

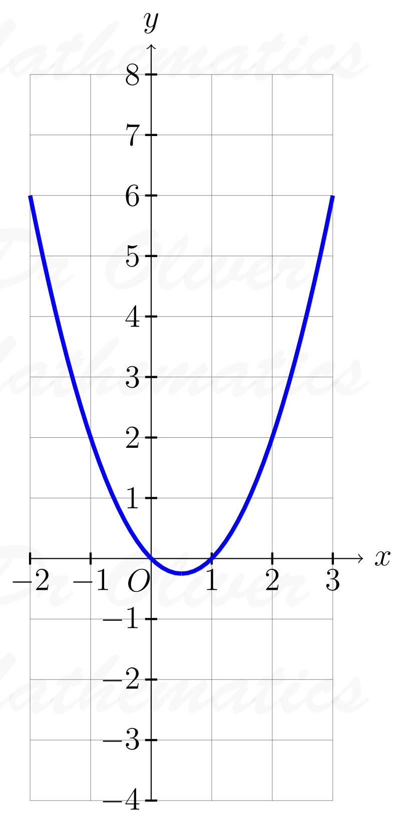
(b) On the grid, draw the graph of (2)

$$y = x^2 - x$$

for values of x from -2 to 3.



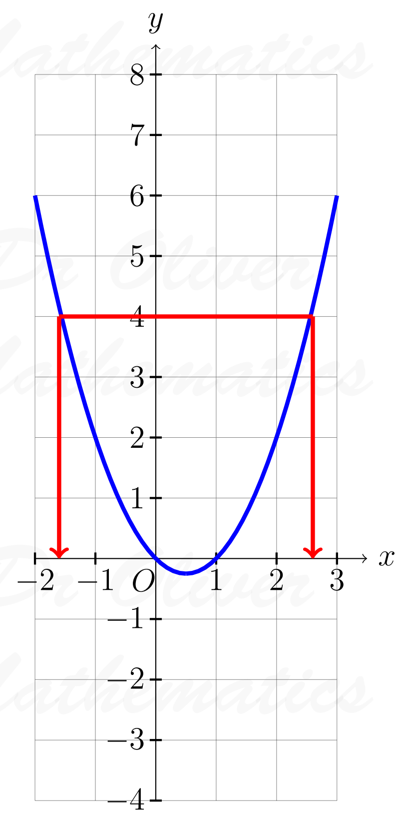
Solution



(c) Use your graph to find estimates for the solutions of the equation (2)

$$x^2 - x = 4.$$

Solution



Correct read-off: approximately $x = -1.6$ or $x = 2.6$.

6. Andy, Luke, and Tina share some sweets in the ratio 1 : 6 : 14. (4)

Tina gives $\frac{3}{7}$ of her sweets to Andy.

Tina then gives $12\frac{1}{2}\%$ of the rest of her sweets to Luke.

Tina says, "Now all three of us have the same number of sweets."

Is Tina correct?
You must show how you get your answer.

Solution

Well, she gives

$$\tfrac{3}{7} \times 14 = 6$$

and after, the sweets are in ratio

$$(1+6):6:(14-6)=7:6:8.$$

Now,

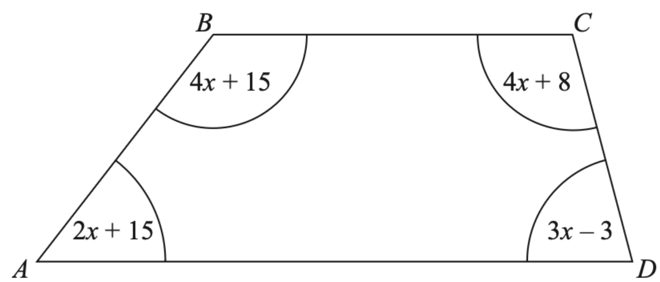
$$12\tfrac{1}{2}\% \times 8 = 1$$

and after, the sweets are in ratio

$$7:(6+1):(8-1)=7:7:7.$$

Hence, $\underline{\underline{\text{Tina is correct}}}$.

7. $ABCD$ is a quadrilateral. (4)



All angles are measured in degrees.

Show that $ABCD$ is a trapezium.

Solution

Well,

$$\begin{aligned}(2x+15)+(4x+15)+(4x+8)+(3x-3)=360 &\Rightarrow 13x+35=360\\ &\Rightarrow 13x=325\\ &\Rightarrow x=25;\end{aligned}$$

so, the angles are 65°, 115°, 108°, and 72°.

Now,

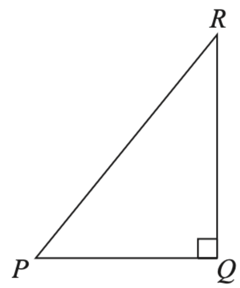
$$65+115=180^\circ$$

and

$$108 + 72 = 180°;$$

hence, $\underline{\underline{\text{interior angles}}}$ add to 180°.

8. A playground is in the shape of a right-angled triangle. (3)



Dan makes a scale drawing of the playground.
He uses a scale of 1 cm represents 5 m.

The area of the playground on the scale drawing is 28 cm^2.

The real length of QR is 40 m.

Work out the real length of PQ.

Solution

The area scale factor, ASF, is

$$5^2 = 25.$$

Now, area of the actual playground is

$$28 \times 25 = 700 \text{ m}^2.$$

Next,

$$\begin{aligned} \tfrac{1}{2} \times PQ \times 40 = 700 \Rightarrow PQ &= \frac{2 \times 700}{40} \\ \Rightarrow \underline{\underline{PQ}} &\underline{\underline{= 35 \text{ m}}}. \end{aligned}$$

9. A number N is rounded to 2 significant figures.
The result is 7.3.

(a) Write down the least possible value of N. (1)

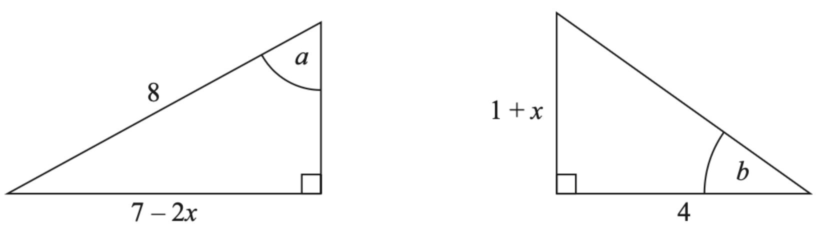
> **Solution**
> $\underline{\underline{7.25}}$.

Leila says, "The value of N cannot be greater than 7.349 because 7.350 would round up to 7.4."

(b) Is Leila correct? (1)
You must give a reason for your answer.

> **Solution**
> No, Leila is $\underline{\underline{\text{not correct}}}$: the 9 should be recurring, $7.34\dot{9}$.

10. The diagram shows two right-angled triangles. (3)



All lengths are measured in centimetres.

Given that

$$\sin a = \tan b,$$

work out the value of x.

> **Solution**
> Cross-multiply:
>
> $$\frac{7-2x}{8} = \frac{1+x}{4} \Rightarrow 4(7-2x) = 8(1+x)$$

divide by 4:

$$\begin{aligned} &\Rightarrow 7 - 2x = 2(1 + x) \\ &\Rightarrow 7 - 2x = 2 + 2x \\ &\Rightarrow 5 = 4x \\ &\Rightarrow \underline{\underline{x = \tfrac{5}{4}}}. \end{aligned}$$

11. The frequency table gives information about the weights of 60 parcels.

Weight (w kg)	Frequency
$0 < w \leqslant 2$	7
$2 < w \leqslant 4$	21
$4 < w \leqslant 6$	15
$6 < w \leqslant 8$	11
$8 < w \leqslant 10$	6

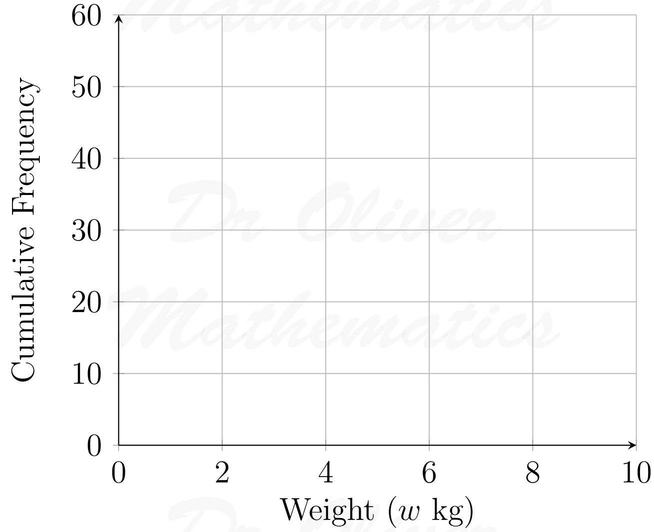
(a) Complete the cumulative frequency table. (1)

Weight (w kg)	Cumulative frequency
$0 < w \leqslant 2$	
$2 < w \leqslant 4$	
$4 < w \leqslant 6$	
$6 < w \leqslant 8$	
$8 < w \leqslant 10$	

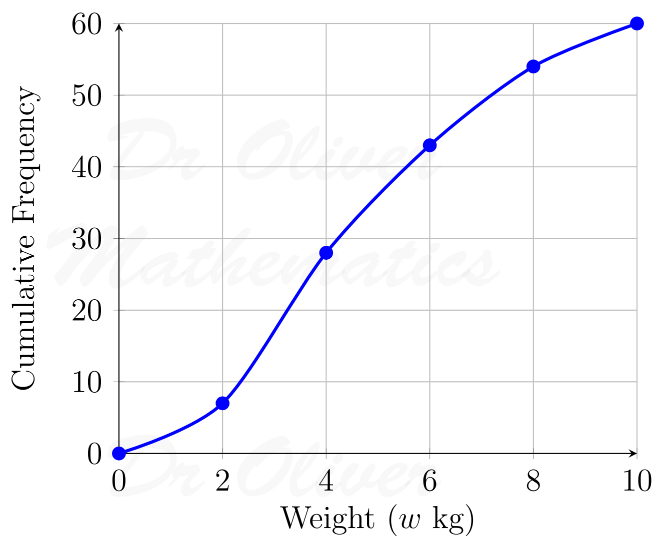
Solution

Weight (w kg)	Cumulative frequency
$0 < w \leqslant 2$	$\underline{\underline{7}}$
$2 < w \leqslant 4$	$7 + 21 = \underline{\underline{28}}$
$4 < w \leqslant 6$	$28 + 15 = \underline{\underline{43}}$
$6 < w \leqslant 8$	$43 + 11 = \underline{\underline{54}}$
$8 < w \leqslant 10$	$54 + 6 = \underline{\underline{60}}$

(b) On the grid opposite, draw a cumulative frequency graph for your table. (2)

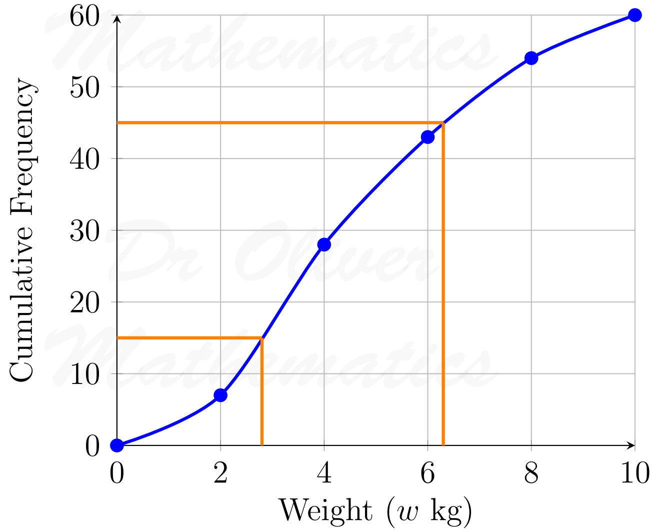


Solution



(c) Use your graph to find an estimate for the interquartile range. (2)

Solution

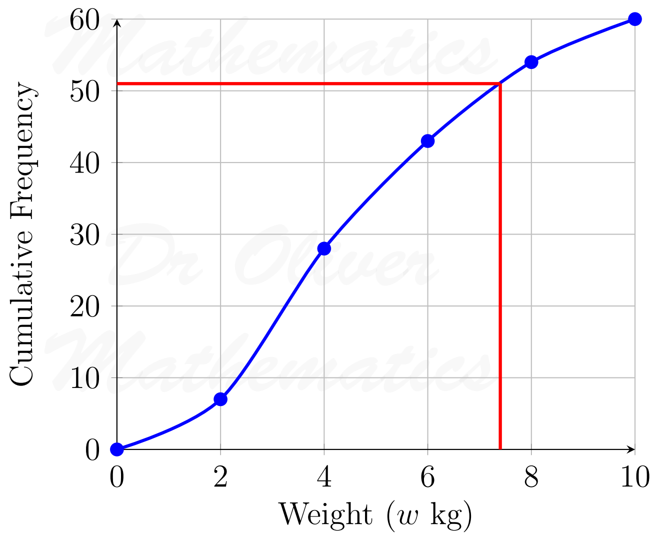


Well,

$$\begin{aligned} \text{IQR} &= \text{UQ} - \text{LQ} \\ &= 6.3 - 2.8 \\ &= \underline{\underline{3.5 \text{ kg}}}. \end{aligned}$$

(d) Use your graph to find an estimate for the number of these parcels with a weight greater than 7.4 kg. (2)

Solution



So,

$$60 - 51 = \underline{\underline{9 \text{ parcels}}}.$$

12. f is inversely proportional to d^2.
    $f = 3.5$ when $d = 8$.

    (a) Find an equation for f in terms of d. (2)

Solution

Well,

$$f \propto d^2 \Rightarrow f = \frac{k}{d^2},$$

for some constant d. Now,

$$\begin{aligned} f = 3.5, d = 8 \Rightarrow 3.5 &= \frac{k}{8^2} \\ \Rightarrow k &= 3.5 \times 8^2 \\ \Rightarrow k &= 224; \end{aligned}$$

hence,

$$\underline{\underline{f = \frac{224}{d^2}}}.$$

(b) Find the positive value of d when $f = 10$.
Give your answer correct to 3 significant figures. (2)

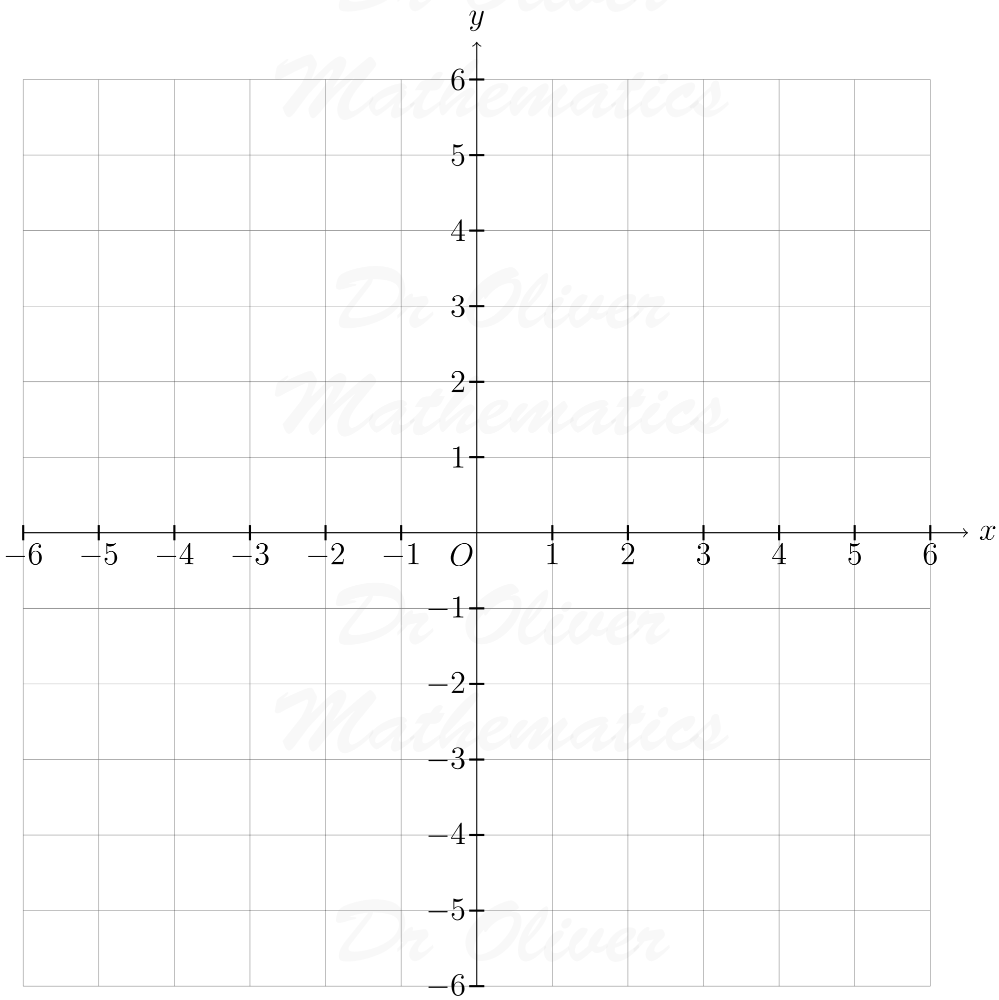
Solution

Well,

$$\begin{aligned} f = 10 \Rightarrow 10 &= \frac{224}{d^2} \\ \Rightarrow d^2 &= \frac{224}{10} \\ \Rightarrow d^2 &= 22.4 \\ \Rightarrow d &= 4.732\,863\,826 \text{ (FCD)} \\ \Rightarrow \underline{\underline{d = 4.73 \text{ (3 sf)}}}&. \end{aligned}$$

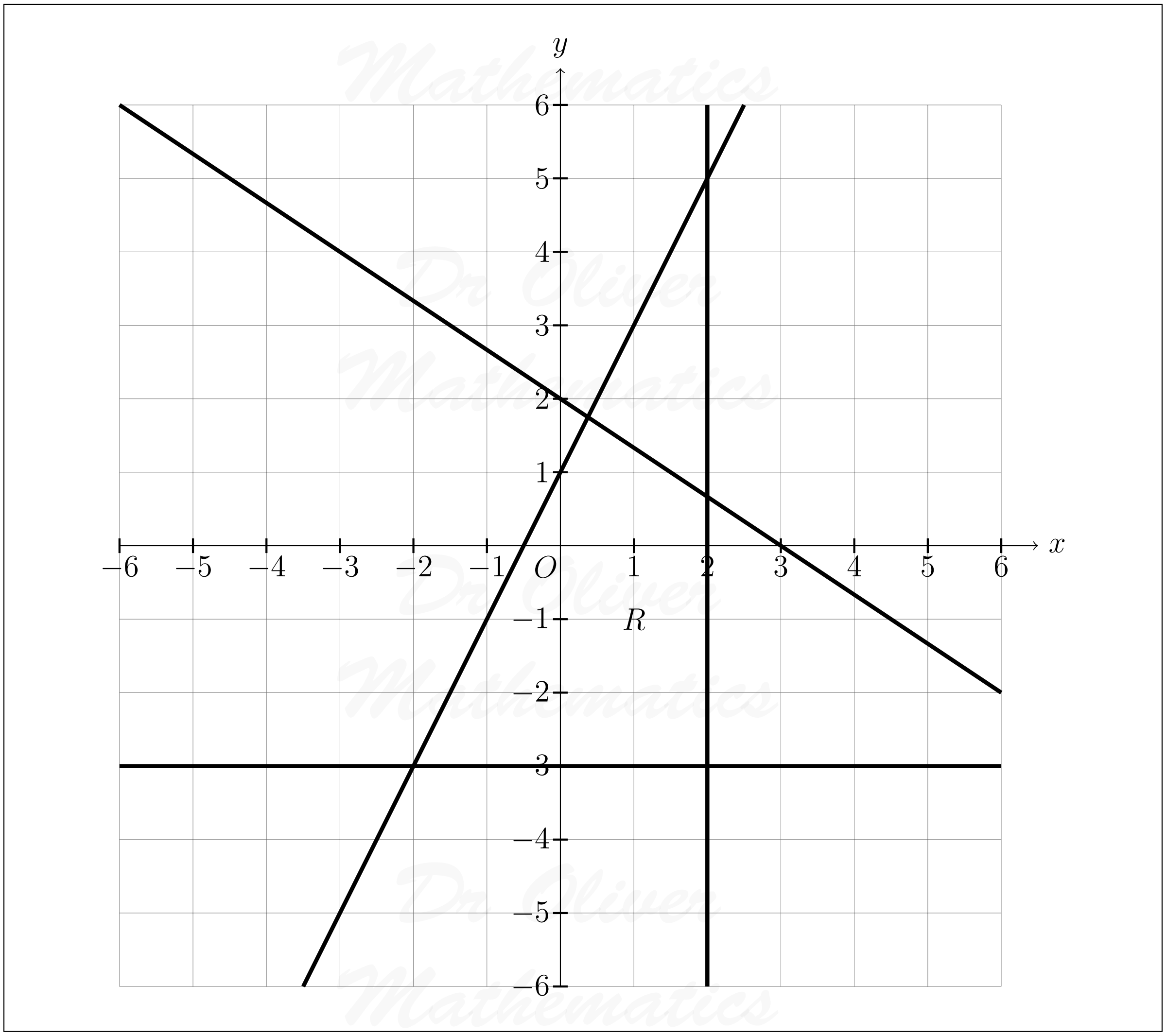
13. On the grid, shade the region R that satisfies all the following inequalities: (3)

$$x \leqslant 2 \quad y \geqslant -3 \quad y \leqslant 2x + 1 \quad 3x + 2y \leqslant 6.$$

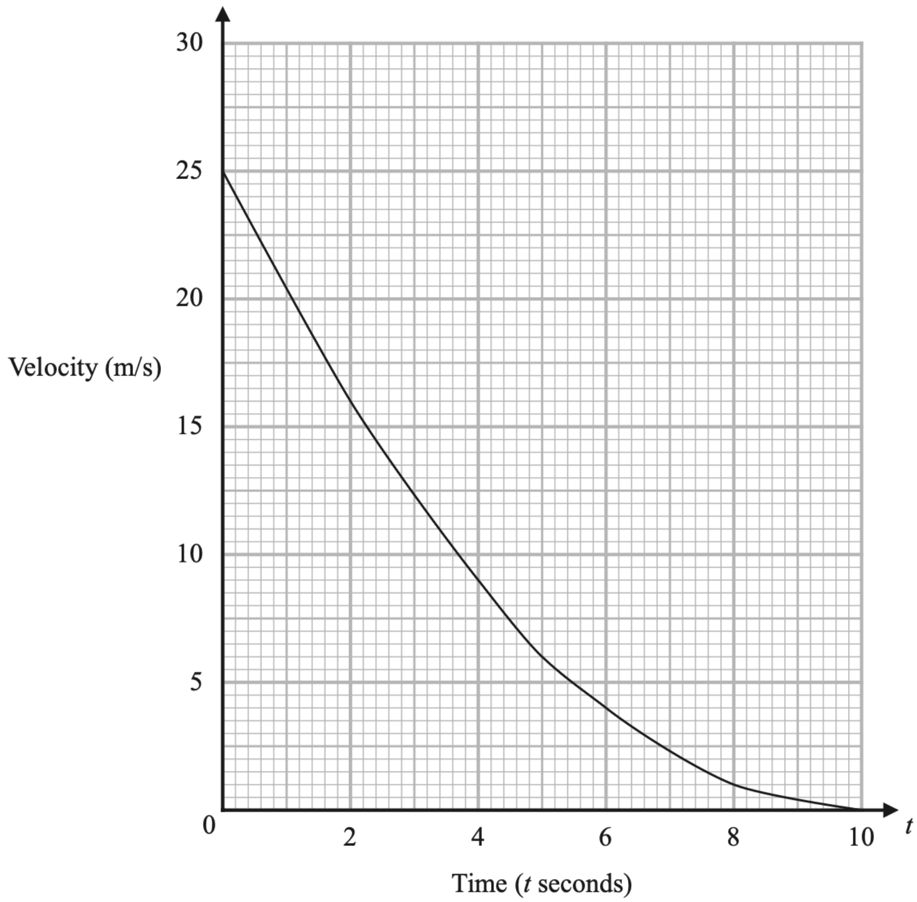


Label the region R.

Solution



14. The graph shows the velocity of a car, in metres per second, t seconds after it starts to slow down.



(a) Calculate an estimate for the acceleration of the car when $t = 5$. (3)
You must show all your working.

Solution

The tangent to the curve when $t = 5$ passes through $(7.8, 0)$ and $(0, 16.5)$:

$$\begin{aligned} \text{gradient} &= \frac{16.5 - 0}{7.8 - 0} \\ &= \underline{\underline{-2\tfrac{3}{26} \text{ ms}^{-2}}}. \end{aligned}$$

(b) Work out an estimate for the distance the car travels in the first 6 seconds after it starts to slow down. (3)
Use 3 strips of equal width.

Solution

Well,

$$\begin{aligned} \text{distance} &= \tfrac{1}{2} \times 2 \times [25 + 2(16 + 9) + 4] \\ &= \underline{\underline{79 \text{ m}}}. \end{aligned}$$

15. Given that a is a prime number, rationalise the denominator of (2)

$$\frac{1}{\sqrt{a} + 1}.$$

Give your answer in its simplest form.

Solution

Well,

$\times$	$\sqrt{a}$	$+1$
$\sqrt{a}$	a	$+\sqrt{a}$
-1	$-\sqrt{a}$	-1

and

$$(\sqrt{a} + 1)(\sqrt{a} - 1) = a - 1.$$

Now,

$$\begin{aligned} \frac{1}{\sqrt{a} + 1} &= \frac{1}{\sqrt{a} + 1} \times \frac{\sqrt{a} - 1}{\sqrt{a} - 1} \\ &= \underline{\underline{\frac{\sqrt{a} - 1}{a - 1}}}. \end{aligned}$$

16. Solve (2)

$$(4x - 3)(x + 5) < 0.$$

Solution

Well,

$$\begin{aligned}(4x-3)(x+5)=0 &\Rightarrow 4x-3=0 \text{ or } x+5=0\\ &\Rightarrow x=\tfrac{3}{4} \text{ or } x=-5.\end{aligned}$$

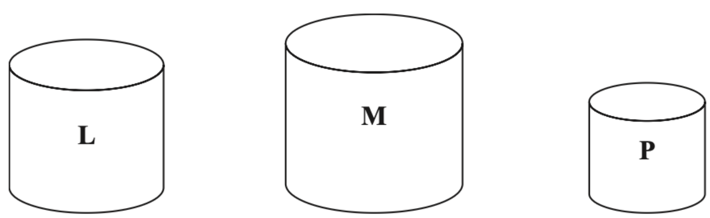
We need a 'table of signs':

	$x<-5$	$x=-5$	$-5<x<\frac{3}{4}$	$x=\frac{3}{4}$	$x>\frac{3}{4}$
$(x+5)$	$-$	0	$+$	$+$	$+$
$(4x-3)$	$-$	$-$	$-$	0	$+$
$(x+5)(4x-3)$	$+$	0	$-$	0	$+$

Hence,

$$(4x-3)(x+5)<0 \Rightarrow \underline{\underline{-5<x<\tfrac{3}{4}}}.$$

17. **L**, **M**, and **P** are three similar solid cylinders made from the same material. (4)



- **L** has a mass of 64 g.
- **M** has a mass of 125 g.
- **M** has a total surface area of 144 cm^2.
- **P** has a total surface area of 16 cm^2.

Work out

height of cylinder **L** : height of cylinder **M** : height of cylinder **P**.

Solution

The Volume Scale Ratio (VSR) between **L** and **M** is

$$64 : 125 = 4^3 : 5^3$$

so the Length Scale Ratio (LSR) between **L** and **M** is

$$4 : 5 = 12 : 15.$$

The Area Scale Ratio (ASR) between **M** and **P** is

$$144 : 16 = 12^2 : 4^2$$

so the Length Scale Ratio (LSR) between **M** and **P** is

$$12 : 4 = 3 : 1 = 15 : 5.$$

Hence,

$$\text{height of cylinder } \mathbf{L} : \text{height of cylinder } \mathbf{M} : \text{height of cylinder } \mathbf{P} = \underline{\underline{12 : 15 : 5}}.$$

18. There are only 4 red counters, 3 yellow counters and 1 green counter in a bag. (5)

Tony takes at random three counters from the bag.

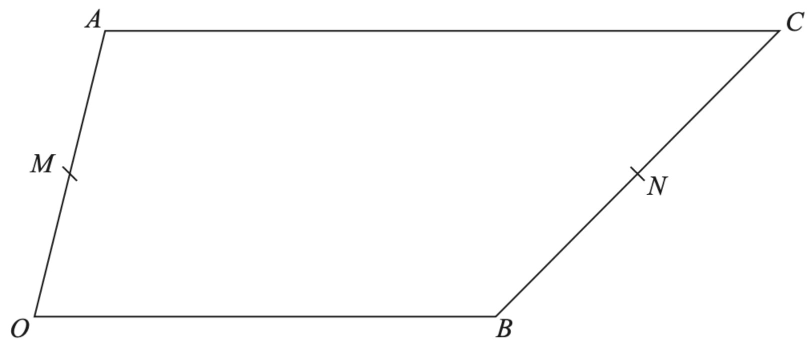
Work out the probability that there are now more yellow counters than red counters in the bag.
You must show all your working.

Solution

Well,

$$\begin{aligned} \text{P(more yellow than red)} &= \text{P}(RRR) + \text{P}(2R, G) \\ &= (\tfrac{4}{8} \times \tfrac{3}{7} \times \tfrac{2}{6}) + (3 \times \tfrac{4}{8} \times \tfrac{3}{7} \times \tfrac{1}{6}) \\ &= \tfrac{24}{336} + \times \tfrac{36}{333} \\ &= \tfrac{60}{336} \\ &= \underline{\underline{\tfrac{5}{28}}}. \end{aligned}$$

19. The diagram shows quadrilateral $OACB$.



- M is the midpoint of OA.
- N is the point on BC such that

$$BN : NC = 4 : 5.$$

- $\overrightarrow{OA} = \mathbf{a}$.
- $\overrightarrow{OB} = \mathbf{b}$.
- $\overrightarrow{AC} = k\mathbf{b}$, where k is a positive integer.

(a) Express $\overrightarrow{MN}$ in terms of k, $\mathbf{a}$, and $\mathbf{b}$. (4)
Give your answer in its simplest form.

Solution
Well,

$$\begin{aligned}
\overrightarrow{MN} &= \overrightarrow{MO} + \overrightarrow{OB} + \overrightarrow{BN} \\
&= \tfrac{1}{2}\overrightarrow{AO} + \overrightarrow{OB} + \tfrac{4}{9}\overrightarrow{BC} \\
&= -\tfrac{1}{2}\overrightarrow{OA} + \overrightarrow{OB} + \tfrac{4}{9}(\overrightarrow{BO} + \overrightarrow{OA} + \overrightarrow{AC}) \\
&= -\tfrac{1}{2}\overrightarrow{OA} + \overrightarrow{OB} + \tfrac{4}{9}(-\overrightarrow{OB} + \overrightarrow{OA} + \overrightarrow{AC}) \\
&= -\tfrac{1}{2}\mathbf{a} + \mathbf{b} + \tfrac{4}{9}(-\mathbf{b} + \mathbf{a} + k\mathbf{b}) \\
&= -\tfrac{1}{2}\mathbf{a} + \mathbf{b} - \tfrac{4}{9}\mathbf{b} + \tfrac{4}{9}\mathbf{a} + \tfrac{4}{9}k\mathbf{b} \\
&= \underline{\underline{-\tfrac{1}{18}\mathbf{a} + (\tfrac{5}{9} + \tfrac{4}{9}k)\mathbf{b}}}.
\end{aligned}$$

(b) Is MN parallel to OB? (1)
Give a reason for your answer.

Solution

No: $-\frac{1}{18}\mathbf{a} + (\frac{5}{9} + \frac{4}{9}k)\mathbf{b}$ is not a multiple of $\mathbf{b}$.

20. The curve $\mathbf{C}$ has equation (3)

$$y = 2x^2 - 12x + 7.$$

Find the coordinates of the turning point on $\mathbf{C}$.

Solution

Well,

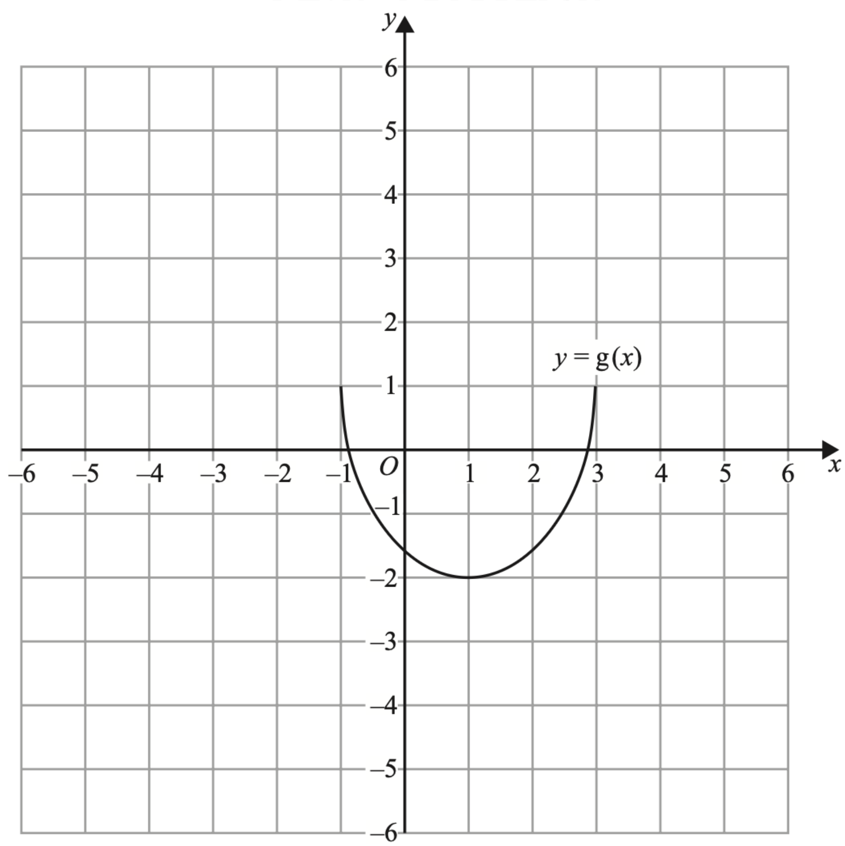
$$\begin{aligned} y &= 2x^2 - 12x + 7 \\ &= 2(x^2 - 6x) + 7 \end{aligned}$$

coefficient of x :	-6
half it:	-3
square it:	$(-3)^2 = +9$

$$\begin{aligned} &= 2[(x^2 - 6x + 9) - 9] + 7 \\ &= 2[(x-3)^2 - 9] + 7 \\ &= 2(x-3)^2 - 18 + 7 \\ &= 2(x-3)^2 - 11; \end{aligned}$$

so, the coordinates of the turning point are $(3, -11)$.

21. The graph of $y = \mathrm{g}(x)$ is shown on the grid. (2)



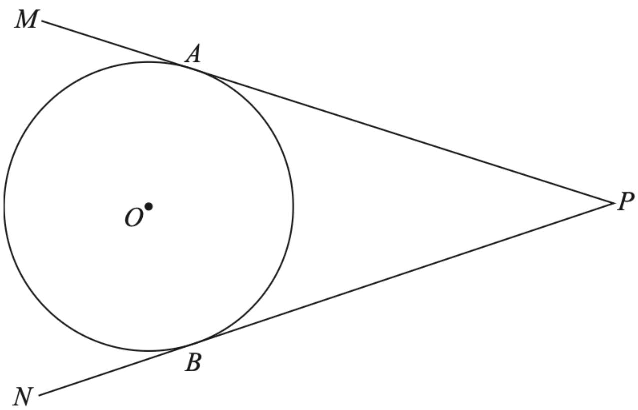
On the grid, draw the graph of

$$y = \mathrm{g}(-x) + 2.$$

Solution

It goes through $(-3, 3)$, $(-2.5, 1)$, $(-2, 0.5)$ $(-1, 0)$, $(0, 0.5)$ $(0.5, 1)$, $(1, 3)$.

22. A and B are points on a circle, centre O. (4)



MAP and NBP are tangents to the circle.

Prove that

$$AP = BP.$$

Solution

$OA = OB$ (radii).
$\angle OAP = \angle OBP$ (right-angle).
OP is common.
So, the triangles OAP and OBP are congruent (SAS).
Hence, $\underline{\underline{AP = BP}}$, as required.