Dr Oliver Mathematics
Worked Examples
Find a 1

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1. (a) Expand

$$(a+1)(a+4).$$

Solution

$\times$	a	$+1$
a	a^2	$+a$
$+4$	$+4a$	$+4$

So

$$(a+1)(a+4) = \underline{\underline{a^2+5a+4}}.$$

(b) Expand

$$(a+2)(a+3).$$

Solution

$\times$	a	$+2$
a	a^2	$+2a$
$+3$	$+3a$	$+6$

So

$$(a+2)(a+3) = \underline{\underline{a^2+5a+6}}.$$

You are given that

$$c^2 - d^2 = (c-d)(c+d).$$

Hence, or otherwise,

(c) solve

$$(a+1)(a+2)(a+3)(a+4) = 120.$$

Solution

Well,

$$\begin{aligned}
& (a+1)(a+2)(a+3)(a+4) = 120 \\
\Rightarrow\ & (a+1)(a+4)\cdot(a+2)(a+3) = 120 \\
\Rightarrow\ & (a^2+5a+4)\cdot(a^2+5a+6) = 120 \\
\Rightarrow\ & [(a^2+5a+5)-1]\cdot[(a^2+5a+5)+1] = 120 \\
\Rightarrow\ & (b-1)(b+1) = 120,
\end{aligned}$$

where $b = a^2+5a+5$.

$\times$	b	-1
b	b^2	$-b$
$+1$	$+b$	-1

so

$$\begin{aligned}
(b-1)(b+1) = 120 &\Rightarrow b^2 - 1 = 120 \\
&\Rightarrow b^2 = 121 \\
&\Rightarrow b = \pm 11.
\end{aligned}$$

Case 1: $a^2+5a+5 = 11$:

$$a^2+5a+5 = 11 \Rightarrow a^2+5a-6 = 0$$

$$\left.\begin{array}{ll} \text{add to:} & +5 \\ \text{multiply to:} & -6 \end{array}\right\} -1,\ +6$$

$$\begin{aligned}
&\Rightarrow (a-1)(a+6) = 0 \\
&\Rightarrow a-1 = 0 \text{ or } a+6 = 0 \\
&\Rightarrow a = 1 \text{ or } a = -6.
\end{aligned}$$

Case 2: $a^2+5a+5 = -11$:

$$a^2+5a+5 = -11 \Rightarrow a^2+5a+16.$$

Now,

$$\begin{aligned}\text{discriminant} &= 5^2 - 4 \times 1 \times 16 \\ &= 25 - 64 \\ &= -39 \\ &< 0\end{aligned}$$

and so there are no real solutions to the quadratic equation.

Hence, the solutions are

$$\underline{\underline{a = -6}} \text{ or } \underline{\underline{a = 1}}.$$