Dr Oliver Mathematics Worked Examples Two Tangent Circles 1

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1. Two tangent circles are drawn, as shown in Figure 1.



Figure 1: two tangent circles

The circle, centred at A, has a radius of 7 cm. The circle, centred at B, has a radius of 21 cm. C and D are the points of tangency. Let x cm be the horizontal distance CD.

Find x.

Solution

Connect the centres A and B with their respective points of tangency D and C. The angle between the radius and the tangent will be 90°. Connect the centres of A and B: the centres and the point of contact are collinear. Let E the the point of contact. Then AEB is a straight line. How long is it? AEB = 7 + 21 = 28 cm. Now, draw the auxiliary line AF parallel to the DC. So

$$BF = 21 - 7 = 14 \text{ cm}$$

and AF = x, as shown in Figure 2.



Figure 2: with the dimensions

We invoke Pythagoras' theorem:

$$x^{2} + 14^{2} = (21 + 7)^{2} \Rightarrow x^{2} + 14^{2} = 28^{2}$$
$$\Rightarrow x^{2} + 196 = 784$$
$$\Rightarrow x^{2} = 588$$
$$\Rightarrow \underline{x} = 14\sqrt{3}.$$

