

Dr Oliver Mathematics
Mathematics: National Qualifications N5
2017 Paper 2: Calculator
1 hour 20 minutes

The total number of marks available is 50.

You must write down all the stages in your working.

1. Find $|\mathbf{v}|$, the magnitude of vector

(2)

$$\mathbf{v} = \begin{pmatrix} 18 \\ -14 \\ 3 \end{pmatrix}.$$

Solution

$$\begin{aligned} |\mathbf{v}| &= \sqrt{18^2 + (-14)^2 + 3^2} \\ &= \sqrt{324 + 196 + 9} \\ &= \sqrt{529} \\ &= \underline{\underline{23}}. \end{aligned}$$

2. A necklace is valued at £1 200.

(3)

Its value is expected to increase by 4.5% per year over the next 3 years.

Calculate the expected value of the necklace after this time.

Give your answer to the nearest pound.

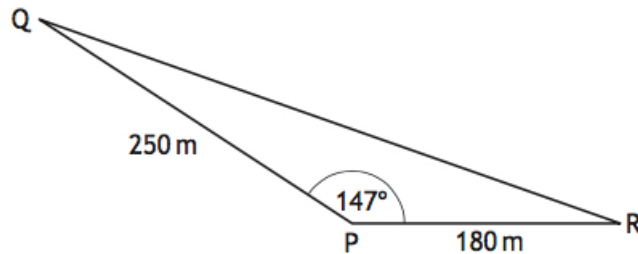
Solution

$$\begin{aligned} \text{Value} &= 1\,200 \times (1.045)^3 \\ &= 1\,369.399\,35 \text{ (FCD)} \\ &= \underline{\underline{£1\,369}} \text{ (nearest pound)}. \end{aligned}$$

3. A piece of land is in the shape of a triangle as shown.

(3)

- $PQ = 250$ metres,
- $PR = 180$ metres, and
- angle $QPR = 147^\circ$.



The owner wishes to build a fence along the side QR .
Calculate the length of the fence.

Solution

$$\begin{aligned}
 QR &= \sqrt{PQ^2 + PR^2 - 2 \cdot PQ \cdot PR \cdot \cos QPR} \\
 &= \sqrt{250^2 + 180^2 - 2 \cdot 250 \cdot 180 \cdot \cos 147^\circ} \\
 &= 412.771\,548\,3 \text{ (FCD)} \\
 &= \underline{\underline{413 \text{ m (3 sf)}}}.
 \end{aligned}$$

4. Solve the equation

$$2x^2 + 5x - 4 = 0.$$

(3)

Give your answers correct to one decimal place.

Solution

$a = 2$, $b = 5$, and $c = -4$:

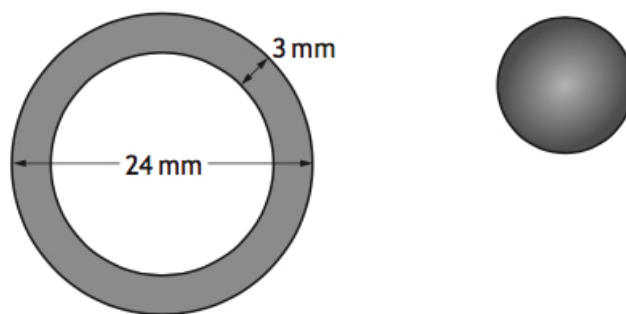
$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times (-4)}}{2 \times 2} \\&= \frac{-5 \pm \sqrt{57}}{4} \\&= -3.137\ 458\ 609, 0.637\ 458\ 608\ 8 \text{ (FCD)} \\&= \underline{\underline{-3.1, 0.6 \text{ (1 dp)}}}.\end{aligned}$$

5. A theatre group sold 4 830 tickets for their show. (3)
This was 15% more than they sold last year.
How many tickets did they sell last year?

Solution

$$\begin{aligned}4\ 830 &= 1.15 \times \text{last year} \Rightarrow \text{last year} = \frac{4\ 830}{1.15} \\&\Rightarrow \text{last year} = \underline{\underline{4\ 200 \text{ tickets}}}.\end{aligned}$$

6. A spherical sweet is made by coating a caramel sphere evenly with chocolate. (5)
A cross-section of the sweet is shown below.



The diameter of the sweet is 24 millimetres and the thickness of the chocolate coating is 3 millimetres.

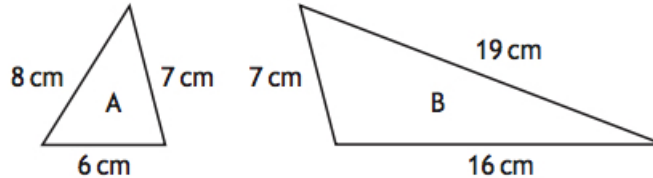
Calculate the volume of the chocolate coating.
Give your answer correct to 3 significant figures.

Solution

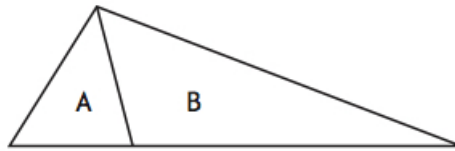
$$\begin{aligned}
 \text{Volume} &= \frac{4}{3}\pi(12^3 - 9^3) \\
 &= \frac{4}{3}\pi \times 999 \\
 &= 4\,184.601\,415 \text{ (FCD)} \\
 &= \underline{\underline{4\,180 \text{ mm}^3 \text{ (3 sf)}}}.
 \end{aligned}$$

7. Triangles A and B are shown below.

(3)



The triangles are placed together to form the larger triangle shown below.



Is this larger triangle right-angled?

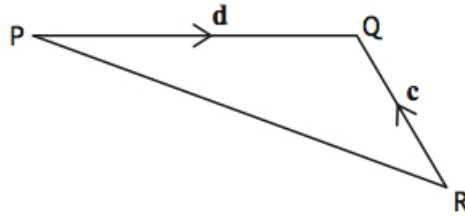
Justify your answer.

Solution

$$\begin{aligned}
 &\text{Top angle at } A + \text{top angle at } B \\
 &= \cos^{-1} \left(\frac{8^2 + 7^2 - 6^2}{2 \times 8 \times 7} \right) + \cos^{-1} \left(\frac{19^2 + 7^2 - 16^2}{2 \times 19 \times 7} \right) \\
 &= \cos^{-1} \left(\frac{11}{16} \right) + \cos^{-1} \left(\frac{11}{19} \right) \\
 &= 46.567\,463\,44 + 54.623\,459\,85 \text{ (FCD)} \\
 &= 101.190 \dots;
 \end{aligned}$$

so, no, it is not because the angle is greater than 90° .

8. In the diagram below, \overrightarrow{RQ} and \overrightarrow{PQ} represent the vectors \mathbf{c} and \mathbf{d} respectively.



- (a) Express \overrightarrow{PR} in terms of \mathbf{c} and \mathbf{d} .

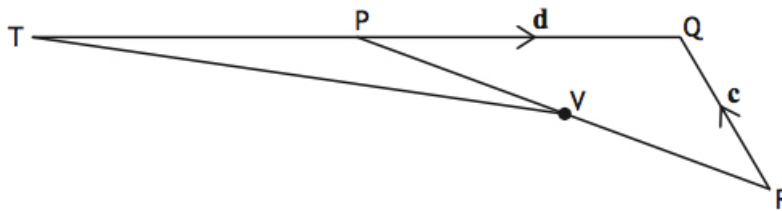
(1)

Solution

$$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{PQ} + \overrightarrow{QR} \\ &= \underline{\underline{\mathbf{d} - \mathbf{c}}}.\end{aligned}$$

The line QP is extended to T .

- $TP = PQ$ and
- V is the midpoint of PR .



- (b) Express \overrightarrow{TV} in terms of \mathbf{c} and \mathbf{d} .
Give your answer in simplest form.

(2)

Solution

$$\begin{aligned}\overrightarrow{TV} &= \overrightarrow{TP} + \overrightarrow{PV} \\ &= \overrightarrow{TP} + \frac{1}{2}\overrightarrow{PR} \\ &= \mathbf{d} + \frac{1}{2}(\mathbf{d} - \mathbf{c}) \\ &= \mathbf{d} + \left(\frac{1}{2}\mathbf{d} - \frac{1}{2}\mathbf{c}\right) \\ &= \frac{3}{2}\mathbf{d} - \frac{1}{2}\mathbf{c} \\ &= \underline{\underline{\frac{1}{2}(3\mathbf{d} - \mathbf{c})}}.\end{aligned}$$

9. (a) Factorise (1)
 $4x^2 - 25.$

Solution

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to: } (+4) \times (-25) = -100 \end{array} \right\} -10, +10$$
$$\begin{aligned}4x^2 - 25 &= 4x^2 - 10x + 10x - 25 \\ &= 2x(2x - 5) + 5(2x - 5) \\ &= \underline{\underline{(2x + 5)(2x - 5)}}.\end{aligned}$$

(b) Hence simplify (3)
 $\frac{4x^2 - 25}{2x^2 - x - 10}.$

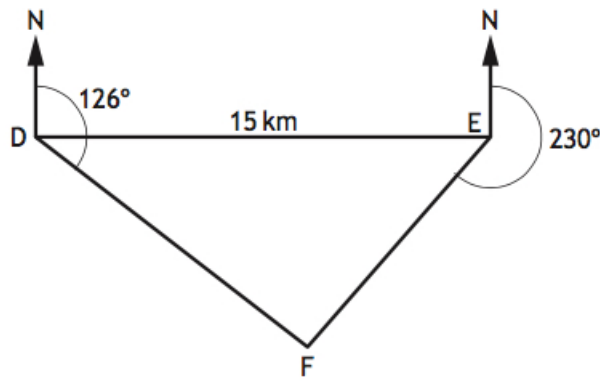
Solution

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to: } (+2) \times (-10) = -20 \end{array} \right\} -5, +4$$
$$\begin{aligned}2x^2 - x - 10 &= 2x^2 - 5x + 4x - 10 \\ &= x(2x - 5) + 2(2x - 5) \\ &= (x + 2)(2x - 5).\end{aligned}$$

So

$$\begin{aligned}\frac{4x^2 - 25}{2x^2 - x - 10} &= \frac{(2x + 5)(2x - 5)}{(x + 2)(2x - 5)} \\ &= \frac{2x + 5}{x + 2}.\end{aligned}$$

10. In the diagram below D , E , and F represent the positions of Dunbridge, Earlsford, and Fairtown respectively. (4)



Dunbridge is 15 kilometres west of Earlsford.
From Dunbridge, the bearing of Fairtown is 126° .
From Earlsford, the bearing of Fairtown is 230° .

Calculate the distance between Dunbridge and Fairtown.
Do not use a scale drawing.

Solution

$$\angle FDE = 126 - 90 = 36^\circ, \angle FED = 270 - 230 = 40^\circ, \text{ and}$$

$$\angle DFE = 180 - 36 - 40 = 104^\circ.$$

Now,

$$\begin{aligned}\frac{DF}{\sin FED} &= \frac{DE}{\sin DFE} \Rightarrow \frac{DF}{\sin 40^\circ} = \frac{15}{\sin 104^\circ} \\ \Rightarrow DF &= \frac{15 \sin 40^\circ}{\sin 104^\circ} \\ \Rightarrow DF &= 9.936\,985\,07 \text{ (FCD)} \\ \Rightarrow DF &= \underline{\underline{9.94 \text{ km (3 sf)}}}.\end{aligned}$$

11. A straight line has equation

$$3x - 5y - 10 = 0.$$

(2)

Find the gradient of this line.

Solution

$$\begin{aligned}3x - 5y - 10 = 0 &\Rightarrow 5y = 3x - 10 \\ \Rightarrow y &= \frac{3}{5}x - 2;\end{aligned}$$

hence, the gradient is $\frac{3}{5}$.

12. Express

$$\frac{1}{\sqrt[3]{x}}$$

(2)

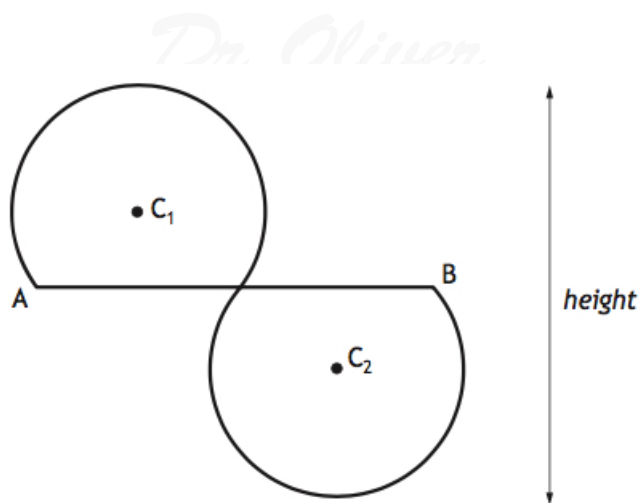
in the form x^n .

Solution

$$\begin{aligned}\frac{1}{\sqrt[3]{x}} &= \frac{1}{x^{\frac{1}{3}}} \\ &= \underline{\underline{x^{-\frac{1}{3}}}}.\end{aligned}$$

13. Two identical shapes are used to form a logo.
Each shape is part of a circle.

(4)



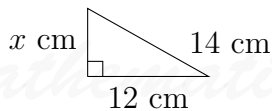
- The circles have centres C_1 and C_2 .
- The radius of each circle is 14 centimetres.
- The logo has half-turn symmetry about the mid-point of AB .
- AB is 48 centimetres long.

Calculate the height of the logo.

Solution

$$AB = \frac{48}{4} = 12 \text{ cm.}$$

Now,



Next,

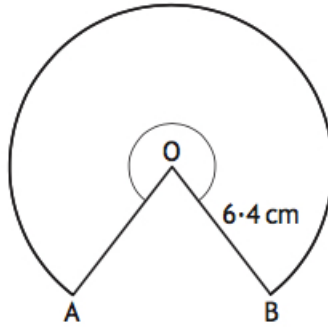
$$\begin{aligned} x &= \sqrt{14^2 - 12^2} \\ &= \sqrt{196 - 144} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \end{aligned}$$

and, finally,

$$\begin{aligned} \text{height} &= 2(14 + 2\sqrt{13}) \\ &= \underline{\underline{28 + 4\sqrt{13} \text{ or } 42.4 \text{ cm (3 sf)}}}. \end{aligned}$$

14. The diagram below shows part of a circle, centre O .

(3)

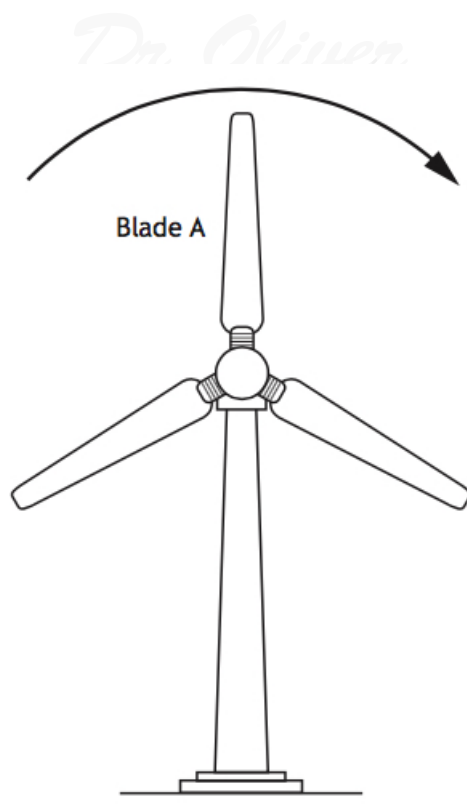


The radius of the circle is 6.4 centimetres.
Major arc AB has length 31.5 centimetres.
Calculate the size of the reflex angle AOB .

Solution

$$\begin{aligned} 31.5 &= \frac{\text{reflex angle } AOB}{360} \times 2 \times \pi \times 6.4 \\ \Rightarrow \text{reflex angle } AOB &= \frac{31.5 \times 360}{12.8\pi} \\ \Rightarrow \text{reflex angle } AOB &= 282.002\,664\,8 \text{ (FCD)} \\ \Rightarrow \text{reflex angle } AOB &= \underline{\underline{282^\circ}} \text{ (3 sf)}. \end{aligned}$$

15. A wind turbine has three blades as shown below.



The height, h metres, of the tip of blade A above the ground in each rotation is given by

$$h = 40 + 23 \cos x^\circ, \quad 0 \leq x < 360,$$

where x is the angle blade A has turned clockwise from its vertical position.

- (a) Calculate the height of the tip of blade A after it has turned through an angle of 60° . (1)

Solution

$$\begin{aligned} \text{Height} &= 40 + 23 \cos 60^\circ \\ &= \underline{\underline{51\frac{1}{2} \text{ m.}}} \end{aligned}$$

- (b) Find the minimum height of the tip of blade A above the ground. (1)

Solution

$$\text{Minimum height} = 40 - 23 = \underline{\underline{17 \text{ m.}}}$$

- (c) Calculate the values of x for which the tip of blade A is 61 metres above the ground. (4)

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Mathematics

Solution

$$40 + 23 \cos x^\circ = 61 \Rightarrow 23 \cos x^\circ = 21$$

$$\Rightarrow \cos x^\circ = \frac{21}{23}$$

$$\Rightarrow x = 24.070\ 614\ 62, 335.929\ 385\ 4 \text{ (FCD)}$$

$$\Rightarrow \underline{\underline{x = 24.1, 336 \text{ (3 sf)}}}$$

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