Dr Oliver Mathematics
Integration: Part 1

1. Find

$$\int_0^1 \sqrt{x^2 - 2x + 1}\,\mathrm{d}x$$

Solution

$$\begin{aligned}\sqrt{x^2 - 2x + 1} &= \sqrt{(x-1)^2} \\ &= \begin{cases} 1 - x & \text{if } x \leqslant 1, \\ x - 1 & \text{if } x > 1. \end{cases}\end{aligned}$$

Thus,

$$\begin{aligned}\int_0^1 \sqrt{x^2 - 2x + 1}\,\mathrm{d}x &= \int_0^1 (1 - x)\,\mathrm{d}x \\ &= \left[x - \tfrac{1}{2}x^2\right]_{x=0}^1 \\ &= \left(1 - \tfrac{1}{2}\right) - (0 - 0) \\ &= \tfrac{1}{2}.\end{aligned}$$