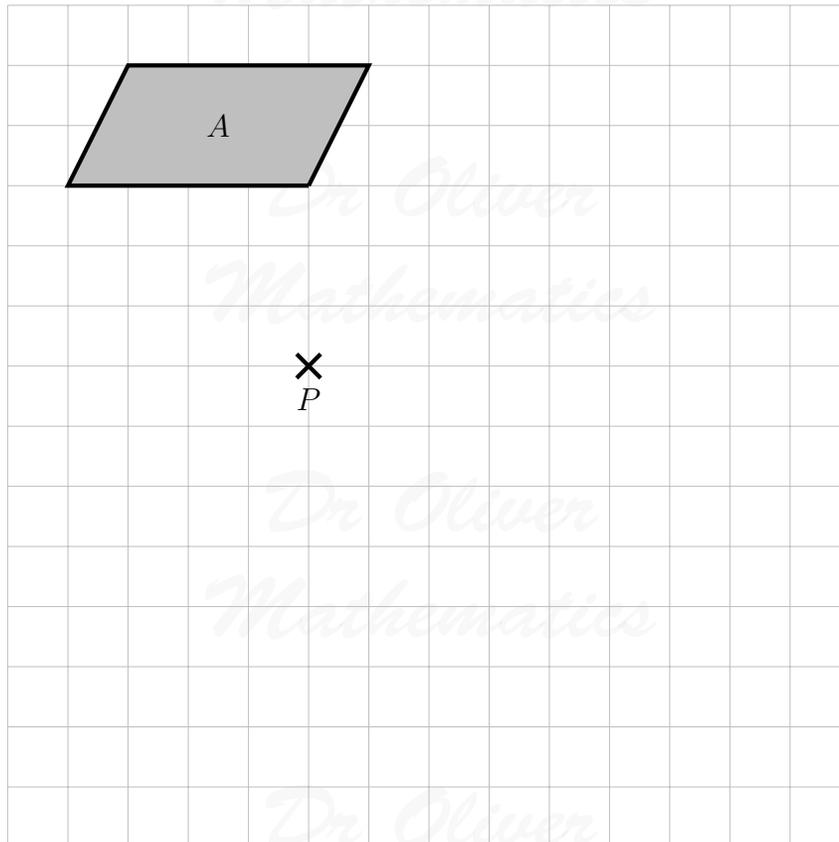


Dr Oliver Mathematics
AQA GCSE Mathematics
2016 November Paper 2: Calculator
2 hours

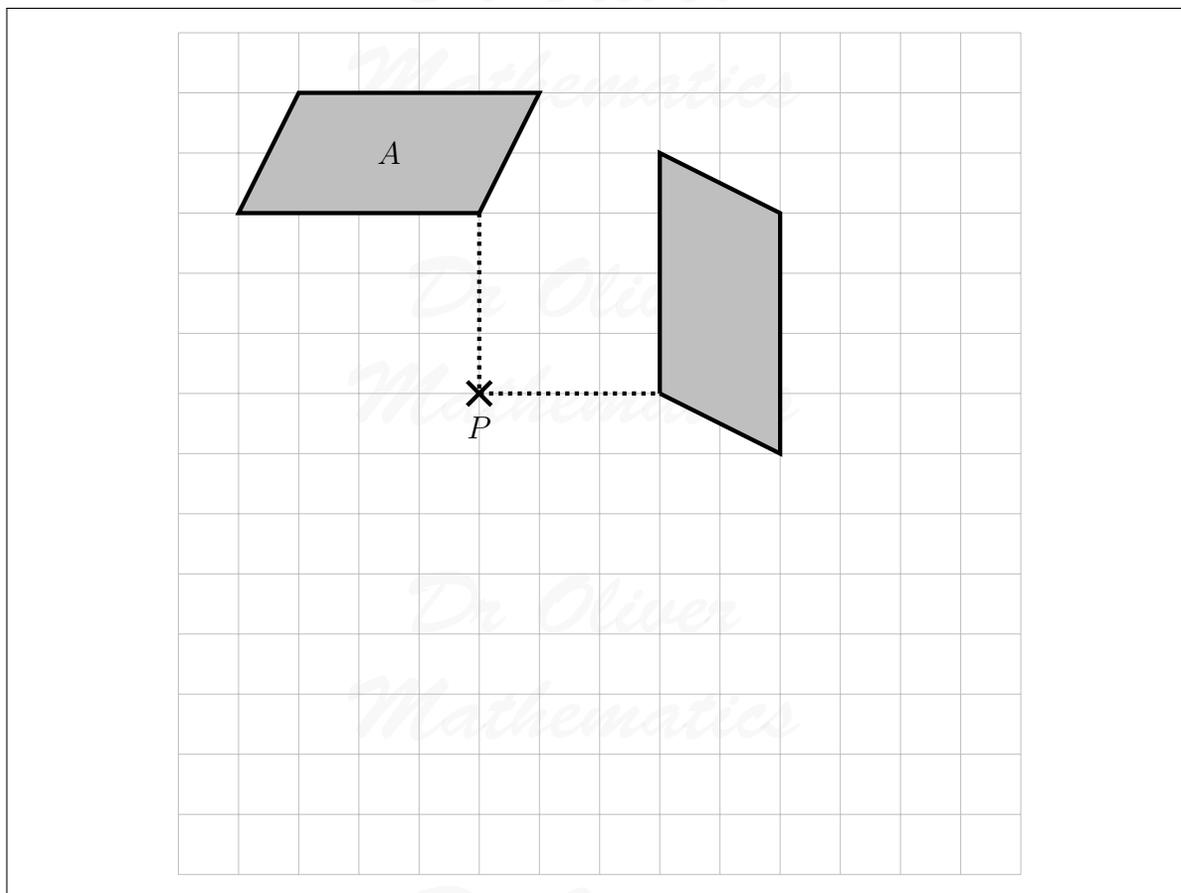
The total number of marks available is 105.
You must write down all the stages in your working.

1. On this grid, rotate shape A by 90° clockwise about point P .

(3)



Solution



2. 100 people are asked about their work.
Here are some of the results.

	Full-Time	Part-Time	Not Working	Total
Men	24	9		60
Women	18			40
Total	42			100

- (a) The total number working **part-time** is the same as the total number of people **not working**. (4)

Complete the table.

Solution

	Full-Time	Part-Time	Not Working	Total
Men	24	9	<u>27</u>	60
Women	18	<u>20</u>	<u>2</u>	40
Total	42	<u>29</u>	<u>29</u>	100

In this survey, there are 60 men and 40 women.

(b) Which is greater:

the percentage of the men who work full time

or

the percentage of the women who work full time?

You **must** show your working.

Solution

$$\text{Men : } \frac{24}{60} \times 100 = 40\%$$

$$\text{Women : } \frac{18}{40} \times 100 = 45\%.$$

The percentage of the women who work full time is 5 percentage points higher.

3. This hexagon has two lines of symmetry.

Not drawn accurately



Work out the size of angle y .

Solution

Now, the angle sum of the hexagon is

$$180(6 - 2) = 720^\circ.$$

Finally,

$$\begin{aligned} 2[y + (2 \times 152)] &= 720 \Rightarrow y + 304 = 360 \\ &\Rightarrow \underline{y = 56^\circ}. \end{aligned}$$

4. A builder mixes sand and cement in the ratio 4 : 1.

(a) Altogether he mixes 250 kg.

(2)

How much sand and cement does he use?

Solution

Well,

$$4 + 1 = 5$$

and

$$\text{sand : } \frac{4}{5} \times 250 = \underline{\underline{200 \text{ kg}}}$$

$$\text{cement : } \frac{1}{5} \times 250 = \underline{\underline{50 \text{ kg}}}$$

Cement is sold in 25 kg bags.

(b) Work out the **maximum** amount of mix that the builder can make with 3 bags of cement.

(3)

Solution

Well,

$$\begin{aligned} 4 : 1 &= 100 : 25 \\ &= 300 : 75. \end{aligned}$$

Hence, the maximum amount of mix that the builder can make is

$$300 + 75 = \underline{\underline{375 \text{ kg}}}.$$

5. (a) Complete the table of values for

(2)

$$y = x^2 - 5$$

for values of x from -3 to 3 .

x	-3	-2	-1	0	1	2	3
y	4		-4			-1	4

Solution

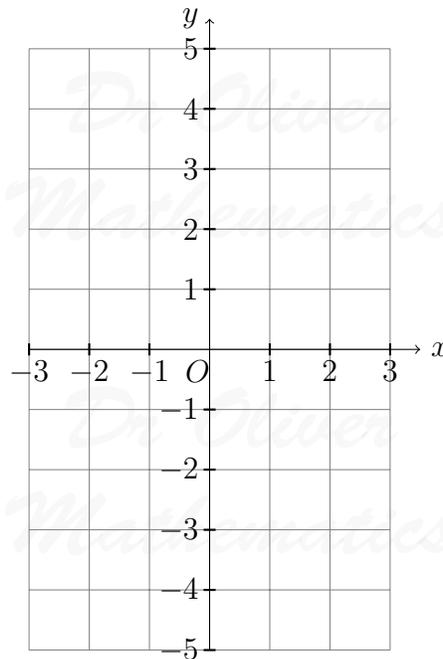
x	-3	-2	-1	0	1	2	3
y	4	<u>-1</u>	-4	<u>-5</u>	<u>-4</u>	-1	4

(b) Draw the graph of

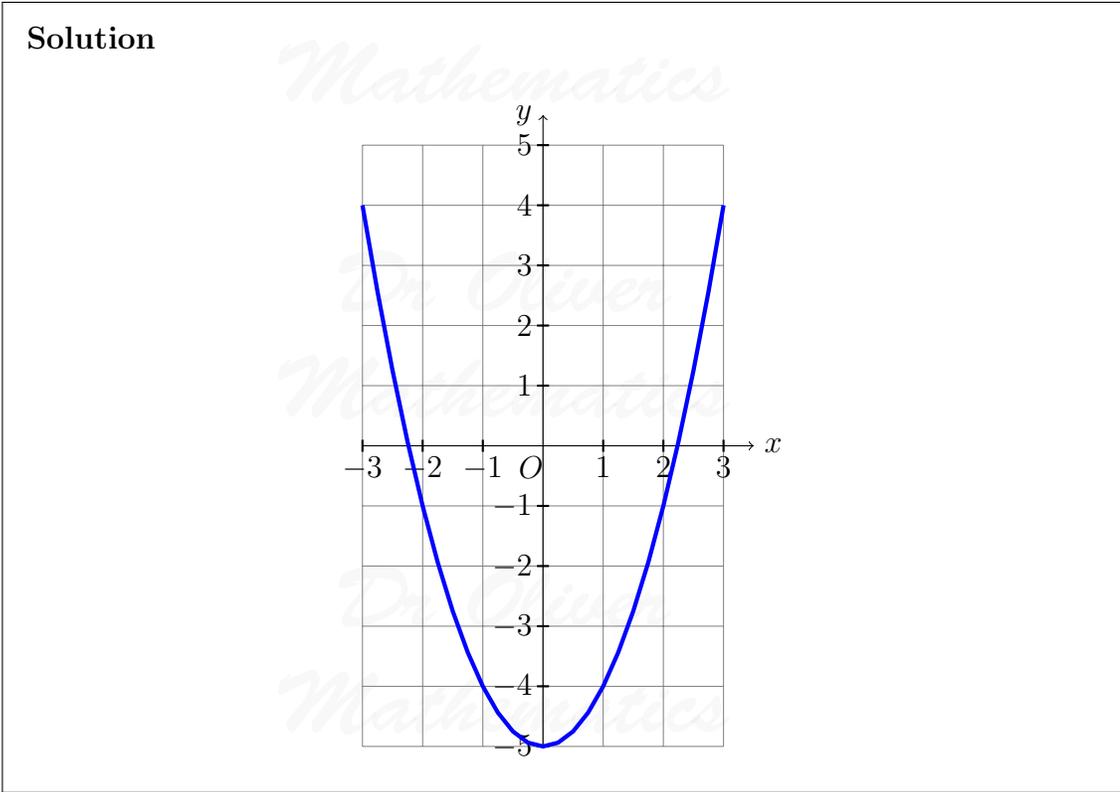
(2)

$$y = x^2 - 5$$

for values of x from -3 to 3 .



$$y = x^2 - 5$$



- (c) Use the graph of $y = x^2 - 5$ to write down the values of x when $y = 0$. (1)

Solution
Correct read-off: approximately $x = -2.2$ and $x = 2.2$.

6. The table shows the proportions of left-handed and right-handed students in a school.

	Left-handed	Right-handed
Boys	15%	85%
Girls	12%	88%

- (a) 20 boys and 10 girls are chosen at random from the school. (3)
Estimate the number of left-handed students chosen.

Solution

$$\begin{aligned}\text{Estimate} &= (20 \times 0.15) + (10 \times 0.12) \\ &= 3 + 1.2 \\ &= 4.2;\end{aligned}$$

so, about four left-handers.

- (b) There are an equal number of boys and girls in the school. (2)
A student is chosen at random.

Work out the probability that the student is right-handed.

Solution

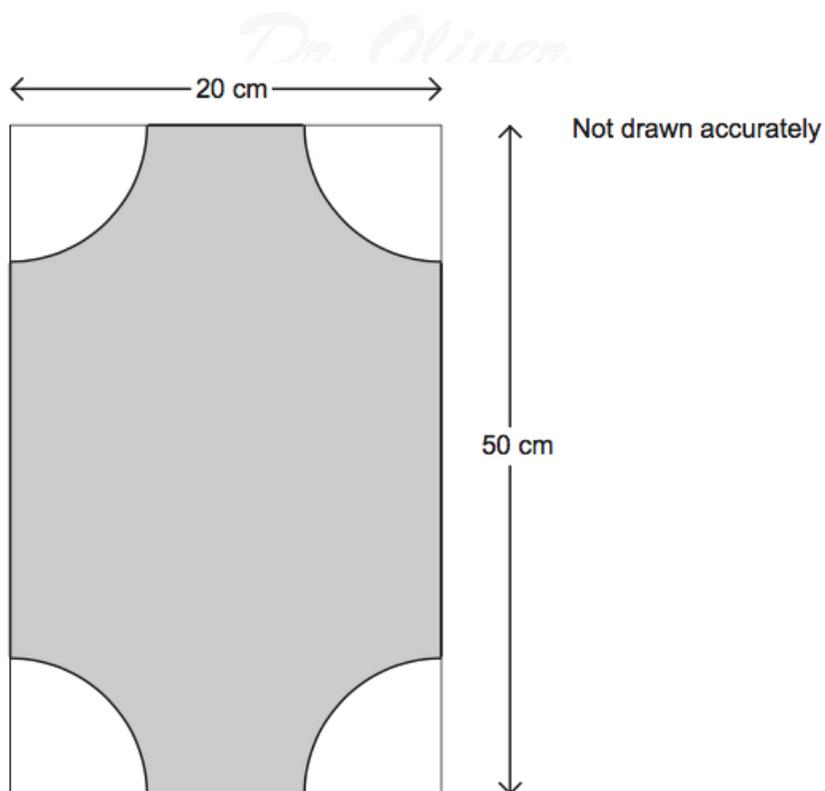
$$\begin{aligned}P(\text{right-handed}) &= P(\text{boy, right-handed}) + P(\text{girl, right-handed}) \\ &= (0.5 \times 0.85) + (0.5 \times 0.88) \\ &= 0.425 + 0.44 \\ &= \underline{0.865}.\end{aligned}$$

7. (a) Work out the area of a circle of radius 6 cm. (2)

Solution

$$\begin{aligned}\text{Area} &= \pi \times 6^2 \\ &= \underline{\underline{36\pi \text{ or } 113 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

- (b) Quarter circles of radius 6 cm are cut from the corners of a rectangle as shown. (3)



Work out the shaded area.

Solution

Well,

$$20 \times 50 = 1\,000 \text{ cm}^2$$

and we have four quarter-circles, i.e., one full circle! Hence,

$$\begin{aligned} \text{shaded area} &= \text{rectangle} - \text{circle} \\ &= \underline{\underline{(1\,000 - 36\pi) \text{ or } 887 \text{ cm}^2 \text{ (3 sf)}}}. \end{aligned}$$

8. In 1981, the population of England was 46 million.
In 2011, the population of England was 53 million.

(3)

Work out the increase in population as a percentage of the 1981 figure.

Solution

Now,

$$53 - 46 = 7 \text{ million}$$

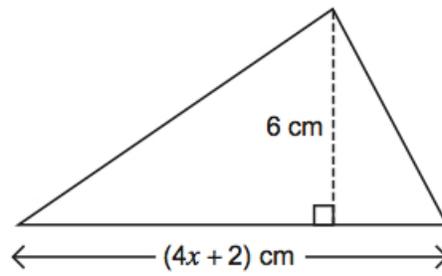
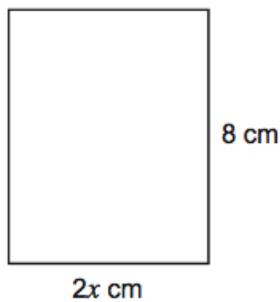
and

$$\begin{aligned}\text{percentage} &= \frac{7}{46} \times 100 \\ &= 15\frac{5}{23} \text{ (exact)} \\ &= \underline{\underline{15.2\% \text{ (3 sf)}}}.\end{aligned}$$

9. The area of the rectangle and the area of the triangle are equal.

(4)

Not drawn accurately



Work out the value of x .

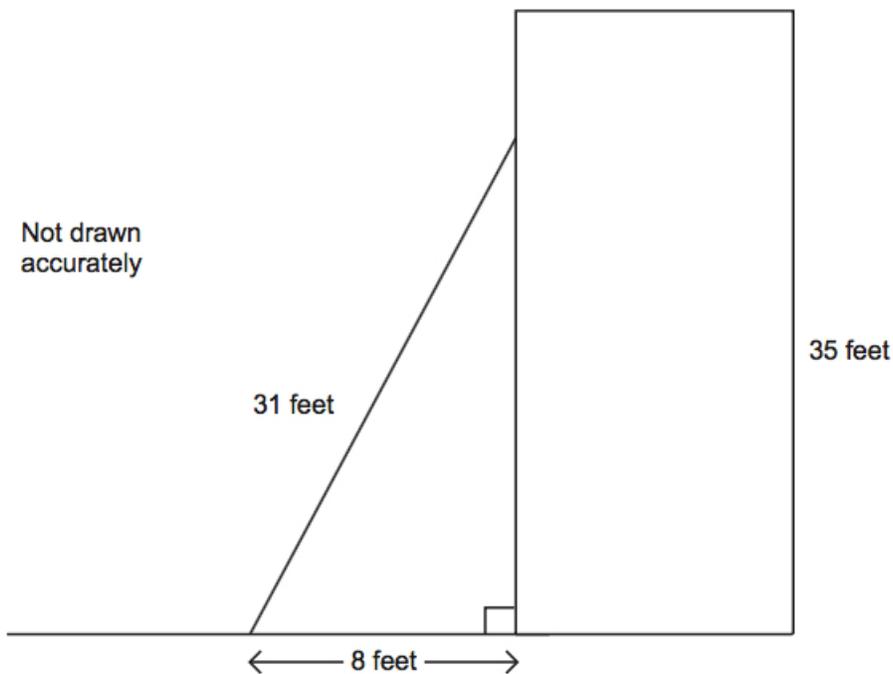
Solution

Now,

$$\begin{aligned}8(2x) &= \frac{1}{2} \times (4x + 2) \times 6 \Rightarrow 16x = 3(4x + 2) \\ &\Rightarrow 16x = 12x + 6 \\ &\Rightarrow 4x = 6 \\ &\Rightarrow \underline{\underline{x = 1\frac{1}{2}}}.\end{aligned}$$

10. A ladder of length 31 feet is leaning against a wall as shown.
The foot of the ladder is 8 feet from the wall.
The wall is 35 feet tall.

(4)



Work out the distance from the top of the ladder to the top of the wall.

Solution

Well,

$$\begin{aligned} \text{opp}^2 + \text{adj}^2 &= \text{hyp}^2 \Rightarrow \text{opp}^2 + 8^2 = 31^2 \\ &\Rightarrow \text{opp}^2 + 64 = 961 \\ &\Rightarrow \text{opp}^2 = 897 \\ &\Rightarrow \text{opp} = \sqrt{897} \end{aligned}$$

and

$$\begin{aligned} \text{distance} &= 35 - \sqrt{897} \\ &= 5.050\,041\,736 \text{ (FCD)} \\ &= \underline{\underline{5.05 \text{ m (3 sf)}}}. \end{aligned}$$

11. Bag *A* contains 3 red balls and 7 blue balls.
Bag *B* contains 8 red balls and 2 blue balls.



Bag A

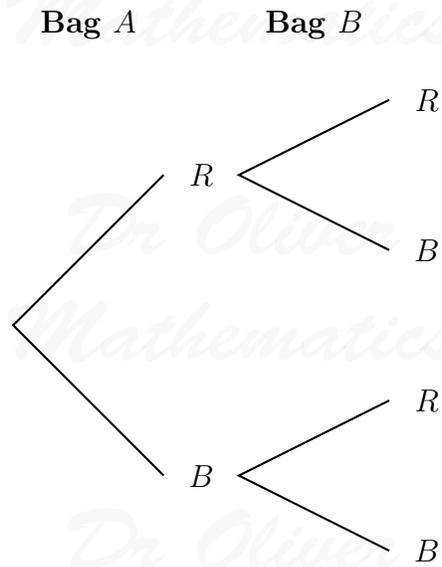


Bag B

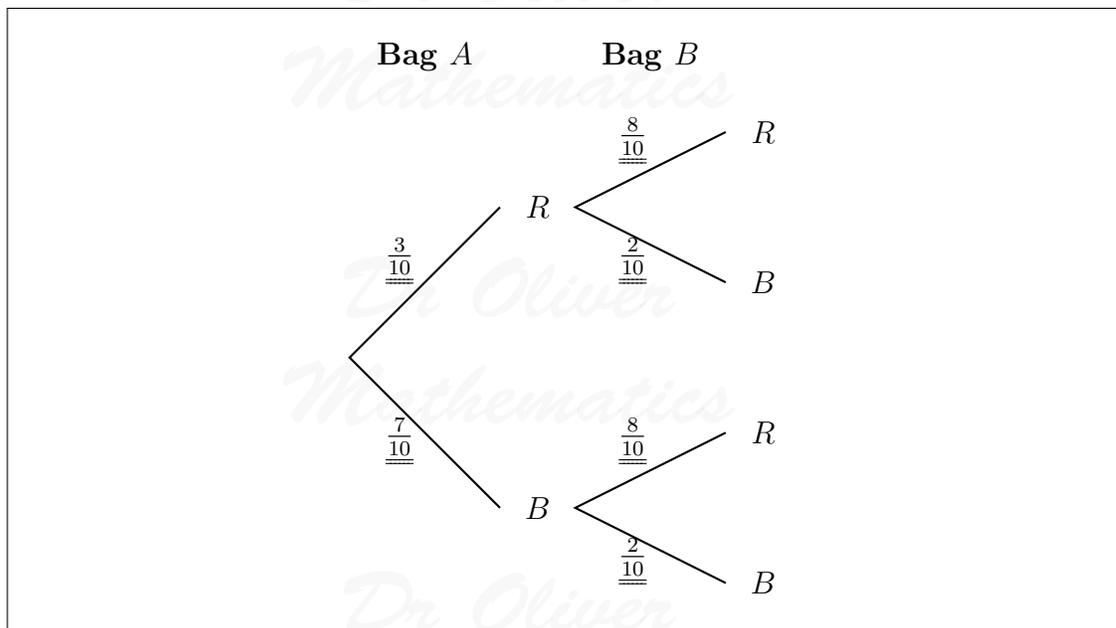
A ball is picked at random from each bag.

(a) Complete the tree diagram to show all the probabilities.

(3)



Solution

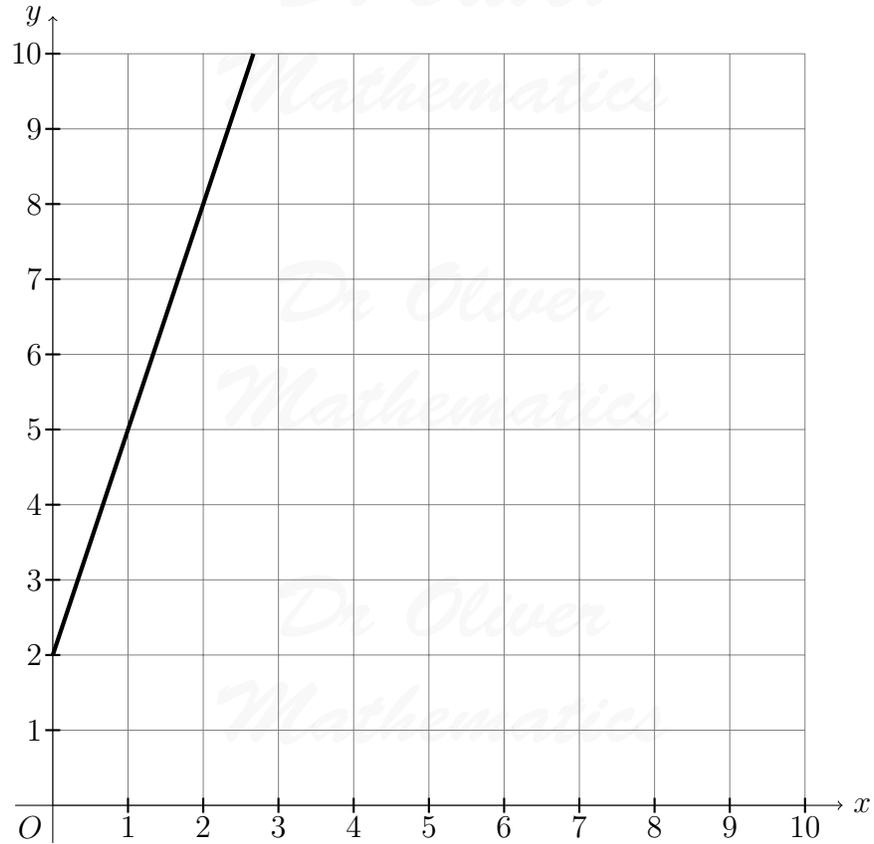


- (b) Work out the probability of picking a **red** ball from Bag A and a **blue** ball from Bag B. (2)

Solution

$$\begin{aligned}
 P(\text{red ball, blue ball}) &= \frac{3}{10} \times \frac{2}{10} \\
 &= \underline{\underline{0.06}}.
 \end{aligned}$$

12. The straight line passes through points (0, 2) and (2, 8).



(a) Work out the equation of the straight line.

(3)

Solution

Well,

$$\begin{aligned} \text{gradient} &= \frac{8 - 2}{2 - 0} \\ &= \frac{6}{2} \\ &= 3 \end{aligned}$$

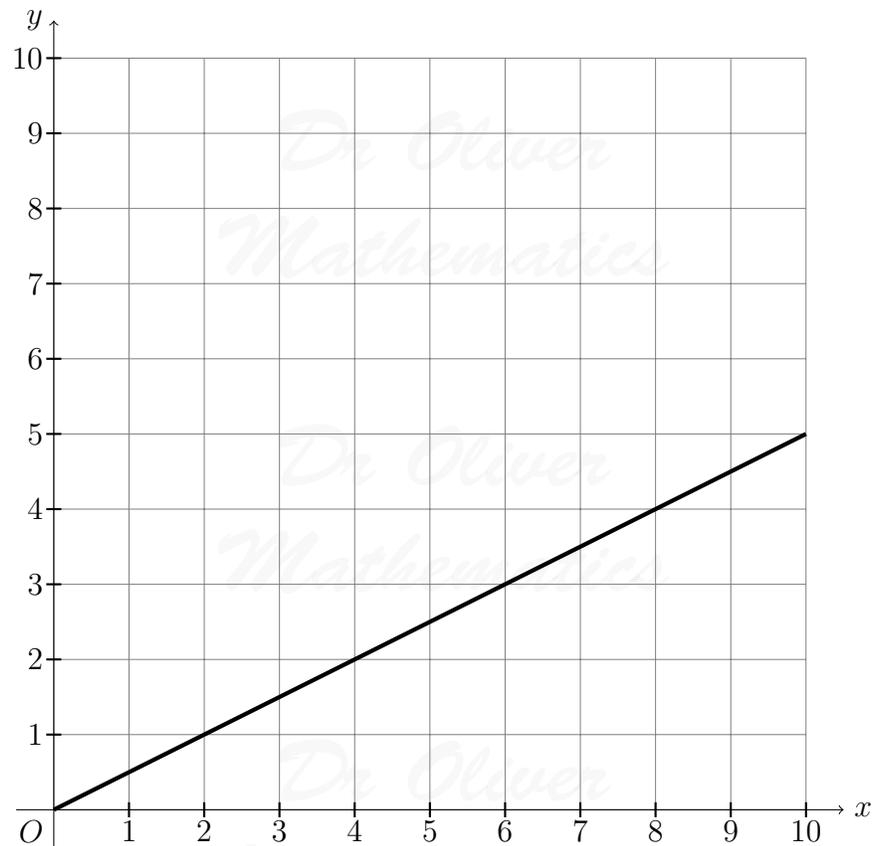
and, clearly, the y -intercept is 2. Hence, the equation of the straight line is

$$\underline{\underline{y = 3x + 2.}}$$

On this grid the line

$$y = \frac{1}{2}x$$

is shown.



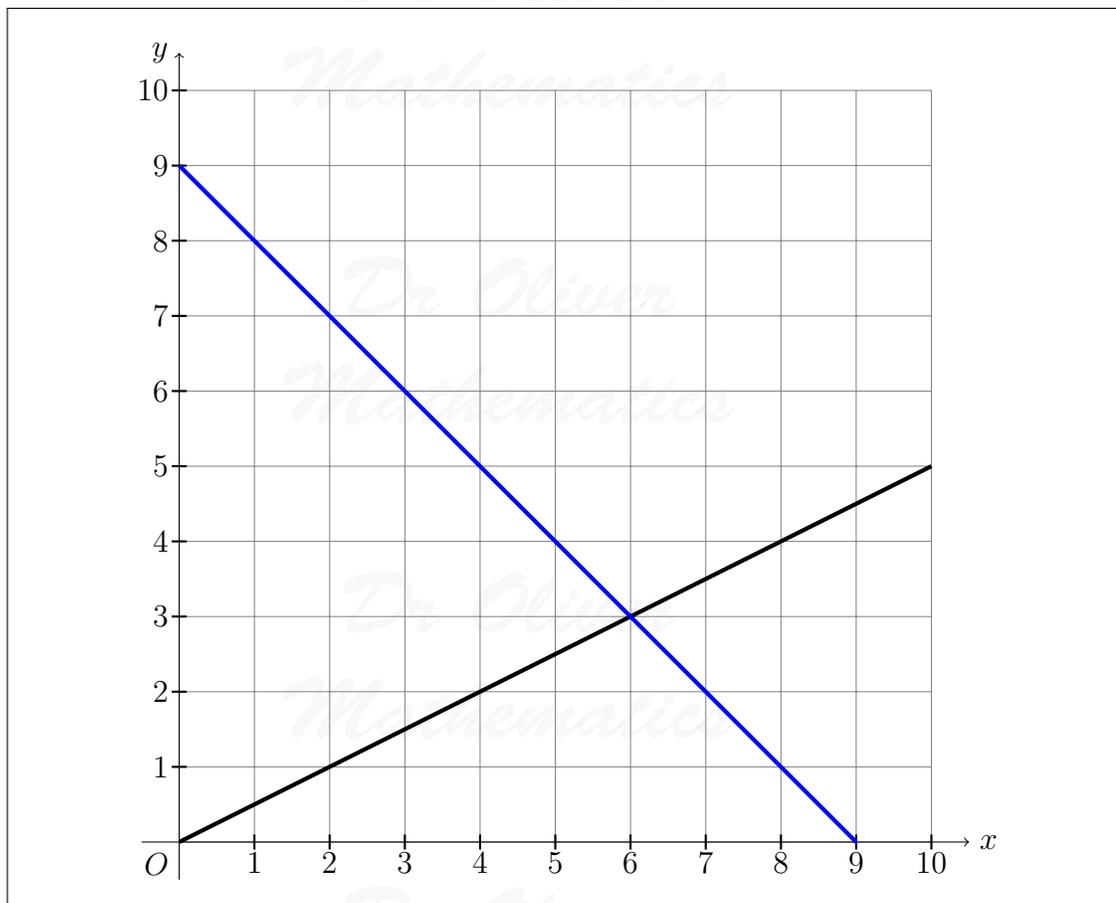
(b) On the same grid, draw the line

(2)

$$x + y = 9$$

for values of x from 0 to 9.

Solution



(c) Solve the simultaneous equations

(2)

$$y = \frac{1}{2}x \text{ and } x + y = 9.$$

Solution

Find the point of intersection:

$$\underline{x = 6, y = 3.}$$

13. (a) Simplify fully

(2)

$$5x^2 \times 3y^4 \times 2x \times y^3.$$

Solution

$$\begin{aligned}5x^2 \times 3y^4 \times 2x \times y^3 &= (5x^2 \times 2x) \times (3y^4 \times y^3) \\ &= 10x^3 \times 3y^7 \\ &= \underline{\underline{30x^3y^7}}.\end{aligned}$$

(b) Expand and simplify

$$(x + 7)(x - 3).$$

(2)

Solution

$$\begin{array}{r|rr} \times & x & +7 \\ \hline x & x^2 & +7x \\ -3 & -3x & -21 \\ \hline\end{array}$$

Hence,

$$(x + 7)(x - 3) = \underline{\underline{x^2 + 4x - 21}}.$$

(c) Solve

$$(x - 8)(x + 2) = 0.$$

(1)

Solution

$$\begin{aligned}(x - 8)(x + 2) = 0 &\Rightarrow x - 8 = 0 \text{ or } x + 2 = 0 \\ &\Rightarrow \underline{\underline{x = 8 \text{ or } x = -2}}.\end{aligned}$$

(d) Factorise

$$8x^2y + 6xy^2.$$

(2)

Solution

$$8x^2y + 6xy^2 = \underline{\underline{2xy(4x + 3y)}}.$$

14. In a sale the normal price of a dress is reduced by 25%.
The sale price is then reduced by £10.

(5)

The dress is now priced at £80.

The manager says, "The price is now one-third less than the normal price."

Show that he is correct.

Solution

Now,

$$80 = (1 - 0.25)\text{normal price} - 10 \Rightarrow 90 = 0.75\text{normal price}$$

$$\Rightarrow \text{normal price} = \frac{90}{0.75}$$

$$\Rightarrow \text{normal price} = 120$$

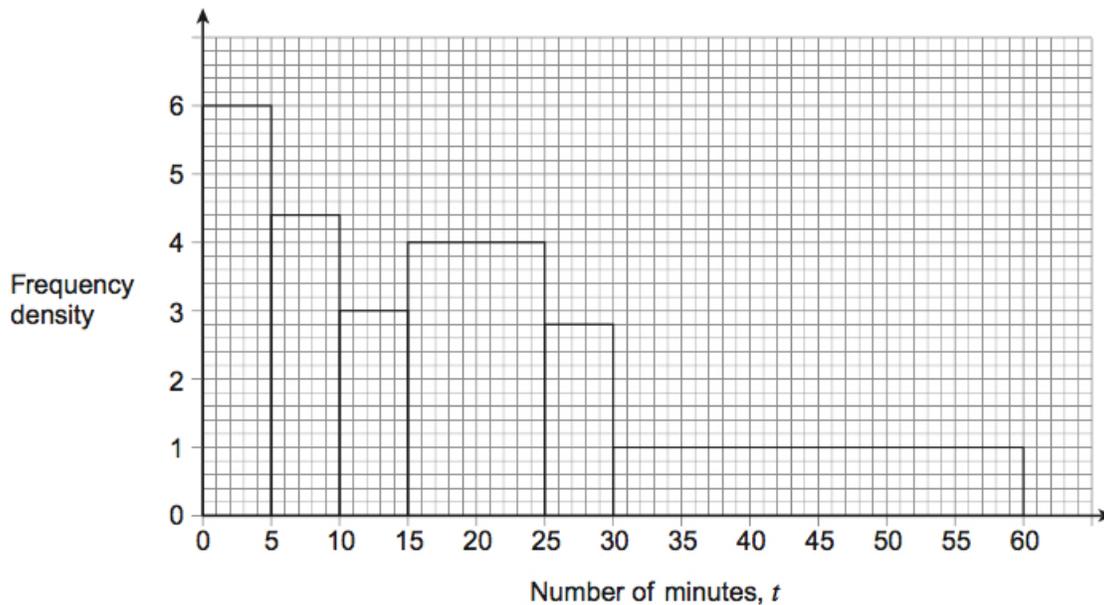
and

$$\frac{120 - 80}{120} = \frac{40}{120}$$
$$= \frac{1}{3}.$$

Hence, he is correct.

15. A train company records the number of minutes, t , some trains were late in one month.

The histogram summarises the results.



- (a) How many trains were **more** than 15 minutes late? (3)

Solution

Interval	Frequency Density	Width	Frequency
0 - 5	6	5	$6 \times 5 = 30$
5 - 10	4.2	5	$4.2 \times 5 = 21$
10 - 15	3	5	$3 \times 5 = 15$
15 - 25	4	10	$4 \times 10 = 40$
25 - 30	2.8	5	$3 \times 5 = 14$
30 - 60	1	30	$1 \times 30 = 30$
Total			141

Hence,

$$\begin{aligned} \text{more than 15 minutes late} &= 40 + 14 + 30 \\ &= \underline{\underline{84 \text{ trains}}} \end{aligned}$$

- (b) Which is the modal class?
Circle your answer. (1)

$$0 < t \leq 5 \quad 15 < t \leq 25 \quad 25 < t \leq 30 \quad 30 < t \leq 60$$

Solution

$$0 < t \leq 5 \quad \underline{\underline{15 < t \leq 25}} \quad 25 < t \leq 30 \quad 30 < t \leq 60$$

16. Which of these when converted to decimals are recurring decimals? (2)

Circle your answers.

$$\frac{1}{3} \quad \pi \quad \sqrt{3} \quad \frac{3}{16} \quad \frac{5}{7}$$

Solution

$$\underline{\underline{\frac{1}{3}}} \quad \pi \quad \sqrt{3} \quad \frac{3}{16} \quad \underline{\underline{\frac{5}{7}}}$$

17. The surface area of a solid cylinder is given by the formula

$$S = 2\pi rh + 2\pi r^2.$$

- (a) Rearrange the formula to make h the subject. (2)

Solution

$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 \Rightarrow S - 2\pi r^2 = 2\pi rh \\ &\Rightarrow h = \underline{\underline{\frac{S - 2\pi r^2}{2\pi r}}}. \end{aligned}$$

- (b) Work out the value of h when (4)

$$S = 95\pi \text{ cm}^2 \text{ and } r = 5.3 \text{ cm}.$$

Give your answer to a suitable degree of accuracy.

Solution

$$\begin{aligned} h &= \frac{95\pi - 2\pi(5.3)^2}{2\pi \times 5.3} \\ &= \frac{95 - 56.18}{10.6} \\ &= 3.662\,264\,151 \text{ (FCD);} \end{aligned}$$

hence, to a suitable degree of accuracy, the height is 3.7 cm.

18. y is inversely proportional to x^2 where $x > 0$.

When $x = 2$, $y = 20$.

(a) Form an equation for y in terms of x .

(3)

Solution

$$y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2},$$

for some scalar k . Now,

$$\begin{aligned}x = 2, y = 20 &\Rightarrow 20 = \frac{k}{2^2} \\ &\Rightarrow 20 = \frac{k}{4} \\ &\Rightarrow k = 80\end{aligned}$$

and so

$$y = \frac{80}{x^2}.$$

(b) Work out the value of x when $y = 5$.

(2)

Solution

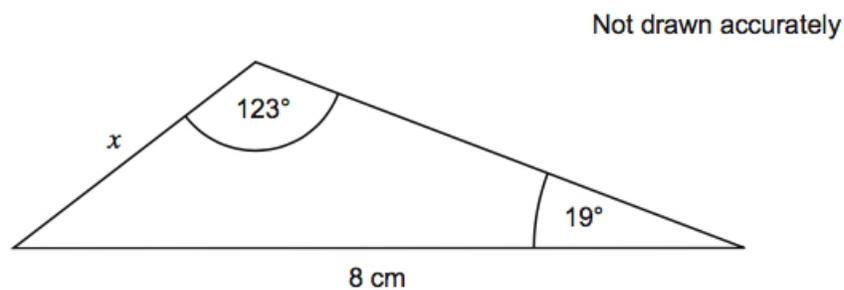
Well,

$$\begin{aligned}5 &= \frac{80}{x^2} \Rightarrow x^2 = \frac{80}{5} \\ &\Rightarrow x^2 = 16 \\ &\Rightarrow \underline{\underline{x = 4}},\end{aligned}$$

because $x > 0$.

19. (a) Work out the length x .

(3)



Solution

$$\begin{aligned}\frac{x}{\sin 19^\circ} &= \frac{8}{\sin 123^\circ} \Rightarrow x = \frac{8 \sin 19^\circ}{\sin 123^\circ} \\ &\Rightarrow x = 3.105\,564\,134 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 3.11 \text{ cm (3 sf)}}}.\end{aligned}$$

(b) Circle the statements that are true.

(2)

$$\sin 123^\circ = \sin 57^\circ$$

$$\sin 123^\circ = \cos 57^\circ$$

$$\cos 123^\circ = \cos 57^\circ$$

$$\cos 123^\circ = -\cos 57^\circ$$

Solution

$$\underline{\underline{\sin 123^\circ = \sin 57^\circ}}$$

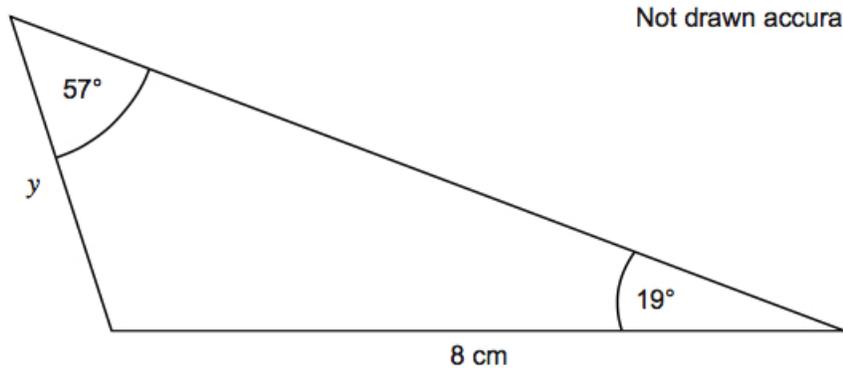
$$\sin 123^\circ = \cos 57^\circ$$

$$\cos 123^\circ = \cos 57^\circ$$

$$\underline{\underline{\cos 123^\circ = -\cos 57^\circ}}$$

(c) Work out the length y .

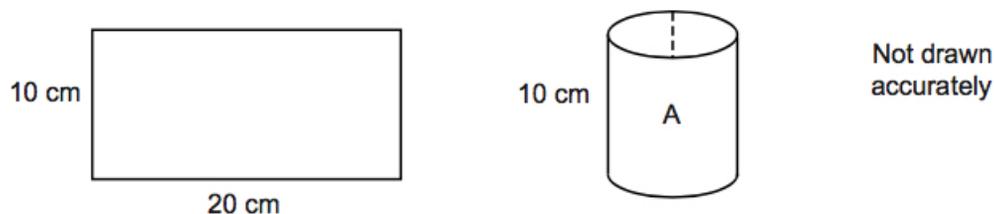
(1)



Solution

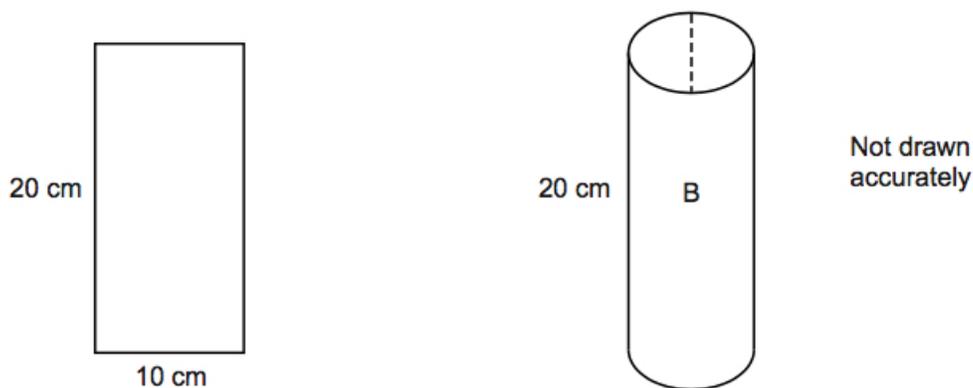
$$\frac{y}{\sin 19^\circ} = \frac{8}{\sin 57^\circ} \Rightarrow y = \frac{8 \sin 19^\circ}{\sin 57^\circ}$$
$$\Rightarrow y = 3.105\,564\,134 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{y = 3.11 \text{ cm (3 sf)}}}$$

20. A rectangle of card, 20 cm by 10 cm, is used to make a cylindrical tube A, as shown. (4)
The card does **not** overlap.



Another rectangle of card, 20 cm by 10 cm, is used to make a cylindrical tube B, as shown.

The card does **not** overlap.



The tubes are filled with clay.

Which tube uses more clay?

You **must** show your working.

Solution

Cylindrical tube A:

$$2\pi r = 20 \Rightarrow r = \frac{10}{\pi}$$

and

$$\begin{aligned} \text{volume}_A &= \pi \times \left(\frac{10}{\pi}\right)^2 \times 10 \\ &= \frac{1000}{\pi}. \end{aligned}$$

Cylindrical tube B:

$$2\pi r = 10 \Rightarrow r = \frac{5}{\pi}$$

and

$$\begin{aligned} \text{volume}_B &= \pi \times \left(\frac{5}{\pi}\right)^2 \times 20 \\ &= \frac{500}{\pi}. \end{aligned}$$

Hence, cylindrical tube A has more clay (by a factor of 2).

21. Use algebra to work out the x -coordinates of the points of intersection of

(5)

$$y = 3x^2 \text{ and } y = 4x + 2.$$

Give your answers to 1 decimal place.

Solution

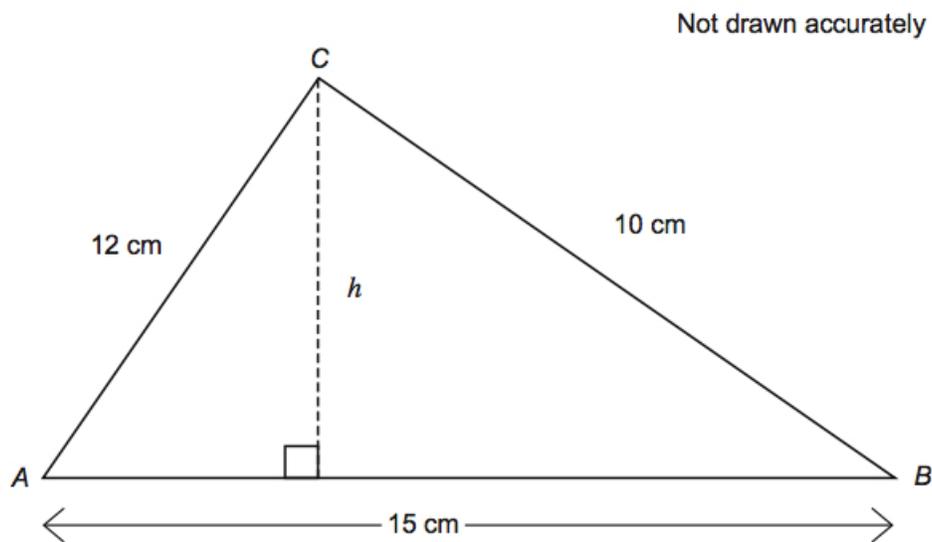
$$3x^2 = 4x + 2 \Rightarrow 3x^2 - 4x - 2 = 0$$

$a = 3$, $b = -4$, and $c = -2$:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{4 \pm \sqrt{(-4)^2 - 4 \times 3 \times (-2)}}{2 \times 3} \\&= \frac{4 \pm \sqrt{40}}{6} \\&= -0.387\,425\,886, -1.720\,759\,22 \text{ (FCD)} \\&= \underline{\underline{-0.4, 1.7}} \text{ (1 dp)}.\end{aligned}$$

22. Work out the height h of the triangle ABC .

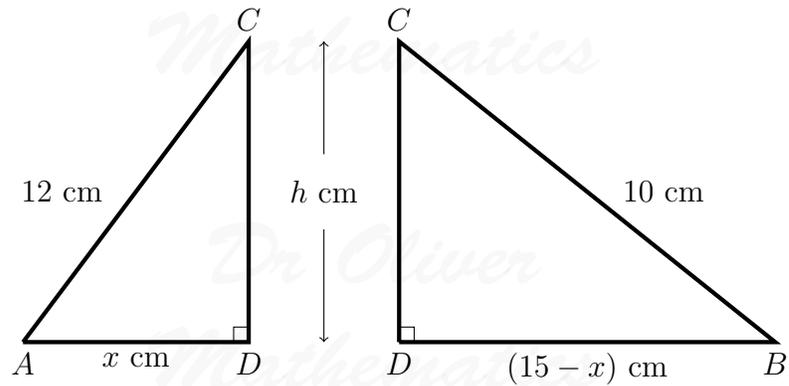
(5)



Solution

Let D be the point takes $\angle ADC$ be a right-angle.

Let $AD = x$ cm and $DB = (15 - x)$ cm, as shown below



Then

$$h^2 + x^2 = 12^2 \Rightarrow h^2 = 144 - x^2 \quad (1)$$

and

$$h^2 + (15 - x)^2 = 10^2 \Rightarrow h^2 = 100 - (15 - x)^2$$

\times	15	$-x$
15	225	$-15x$
$-x$	$-15x$	$+x^2$

$$\Rightarrow h^2 = 100 - (225 - 30x + x^2)$$

$$\Rightarrow h^2 = -125 + 30x - x^2 \quad (2).$$

Do (1) = (2):

$$144 - x^2 = -125 + 30x - x^2 \Rightarrow 269 = 30x$$

$$\Rightarrow x = 8\frac{29}{30}$$

$$\Rightarrow x^2 = 80\frac{361}{900}$$

Finally, we apply Pythagoras' theorem:

$$h^2 = 144 - 80\frac{361}{900} \Rightarrow h^2 = 63\frac{539}{900}$$

$$\Rightarrow h = 7.974891152 \text{ (FCD)}$$

$$\Rightarrow \underline{\underline{h = 7.97 \text{ cm (3 sf)}}}$$