

Dr Oliver Mathematics
AQA GCSE Mathematics
2019 June Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

1. Circle the point that lies on the curve

(1)

$$y = x^2 - 4x + 1.$$

(-1, 4) (-1, -4) (-1, -2) (-1, 6)

Solution

Well,

$$x = -1 \Rightarrow y = (-1)^2 - 4(-1) + 1$$

$$\Rightarrow y = 1 + 4 + 1$$

$$\Rightarrow y = 6$$

so

(-1, 4) (-1, -4) (-1, -2) (-1, 6)

2. The height of a tree is 12 metres, correct to the nearest metre.
Circle the error interval.

(1)

$$11.5 \text{ m} \leq \text{height} < 12.5 \text{ m}$$

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Solution

$$\underline{\underline{11.5 \text{ m} \leq \text{height} < 12.5 \text{ m}}}$$

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$$11.5 \text{ m} < \text{height} < 12.5 \text{ m}$$

3. $2a$ is five times bigger than b .
Circle the ratio $a : b$

(1)

10 : 1 1 : 10 5 : 2 2 : 5

Solution

Well,

$$2a : b = 5 : 1 \Rightarrow a : b = 2.5 : 1$$

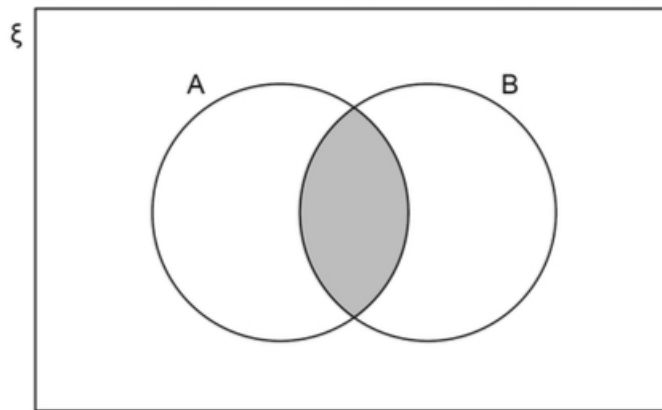
$$\Rightarrow a : b = 5 : 2$$

so

10 : 1 1 : 10 5 : 2 2 : 5

4. Which of these represents the shaded region?

(1)



Circle your answer.

$A \cup B$ $(A \cap B)'$ $A \cap B$ $A' \cup B'$

Solution

$A \cup B$ $(A \cap B)'$ $A \cap B$ $A' \cup B'$

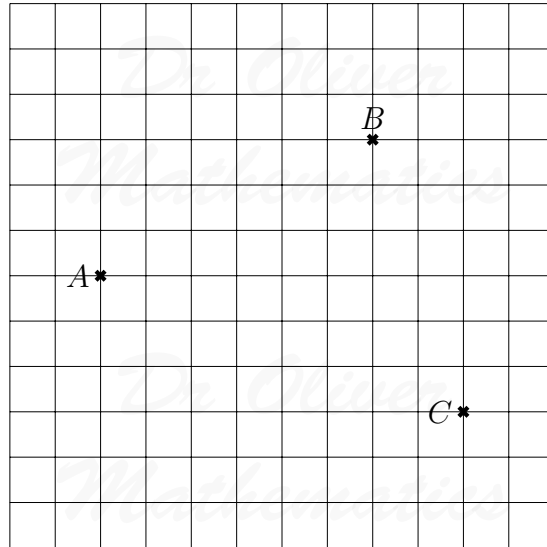
5. Using ruler and compasses, show the region inside the grid that is

(3)

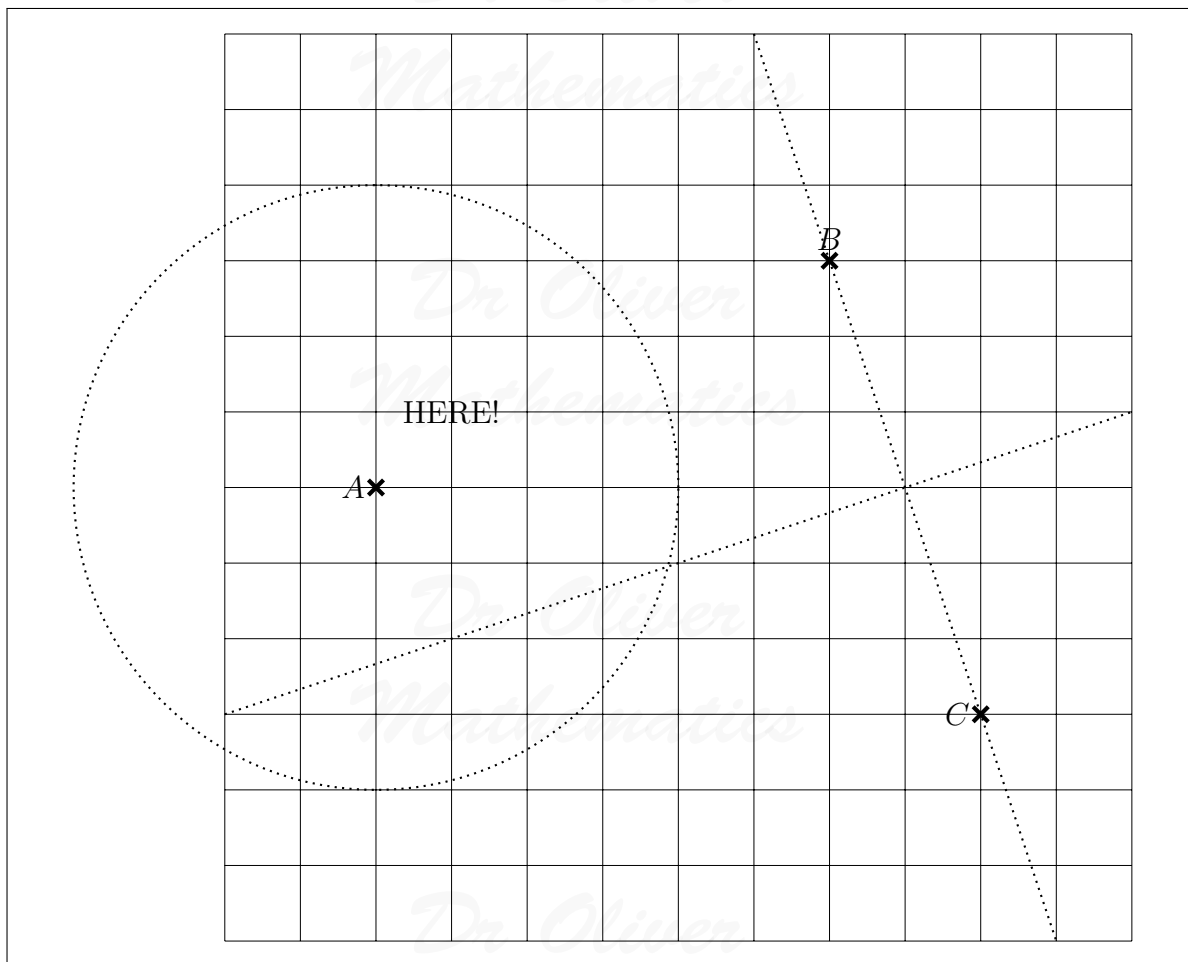
- less than 4 cm from A and
- nearer to B than to C .

Label the region R .

Show all your construction lines.



Solution



6. Beth drives 200 miles in 4 hours.

(3)

She drives the first 18 miles at an average speed of 36 mph.

Work out her average speed for the rest of the journey.

Solution

Well, on the first leg,

$$\begin{aligned} \text{time} &= \frac{18}{36} \\ &= 0.5 \text{ hour} \end{aligned}$$

so, she still has to go

$$200 - 18 = 182 \text{ miles}$$

in

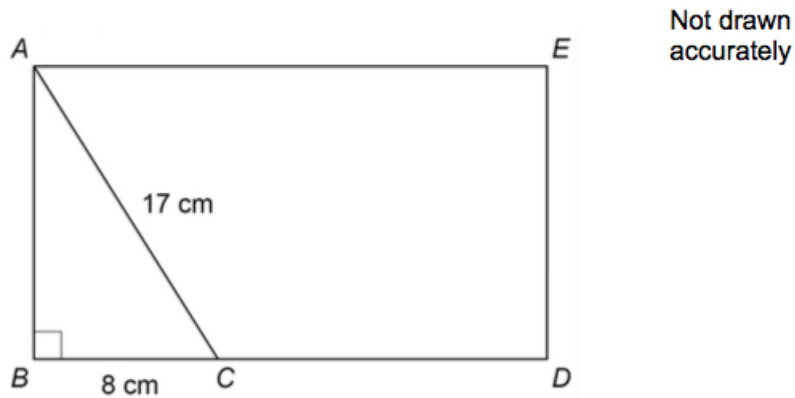
$$4 - 0.5 = 3.5 \text{ hours.}$$

Finally, on the second leg,

$$\begin{aligned} \text{average speed} &= \frac{182}{3.5} \\ &= \underline{\underline{52 \text{ mph.}}} \end{aligned}$$

7. The diagram shows rectangle $ABDE$ and right-angled triangle ABC .

(4)



- $AC = 17 \text{ cm}$
- $BC = 8 \text{ cm.}$
- $BC : CD = 1 : 2.$

Work out the area of rectangle $ABDE$.

Solution

Pythagoras' theorem:

$$\begin{aligned} AB^2 + BC^2 &= AC^2 \Rightarrow AB^2 + 8^2 = 17^2 \\ &\Rightarrow AB^2 = 289 - 64 \\ &\Rightarrow AB^2 = 225 \\ &\Rightarrow AB = 15 \text{ cm.} \end{aligned}$$

Now,

$$\begin{aligned}\frac{CD}{BC} &= \frac{2}{1} \Rightarrow CD = 2 \times 8 \\ &\Rightarrow CD = 16 \\ &\Rightarrow BD = 24 \text{ cm.}\end{aligned}$$

Finally,

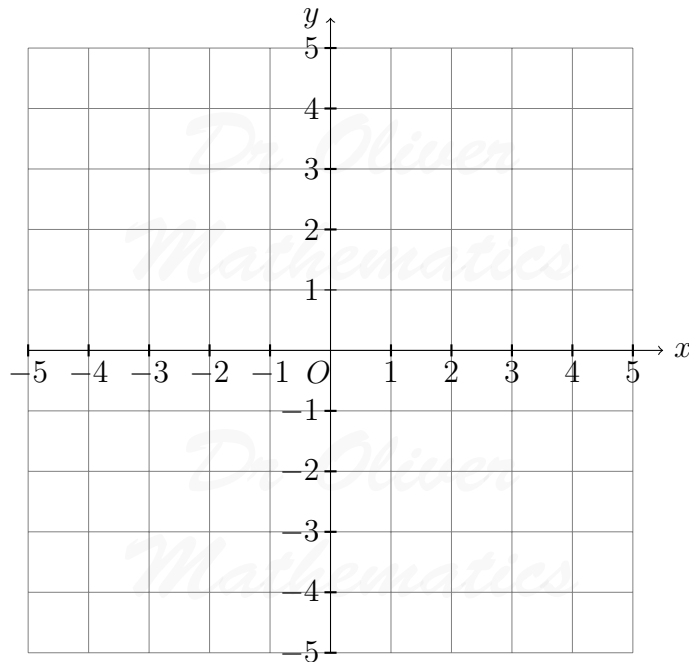
$$\begin{aligned}\text{area} &= AB \times BC \\ &= 15 \times 24 \\ &= \underline{\underline{360 \text{ cm}^2}}.\end{aligned}$$

8. On the axes, sketch the curve

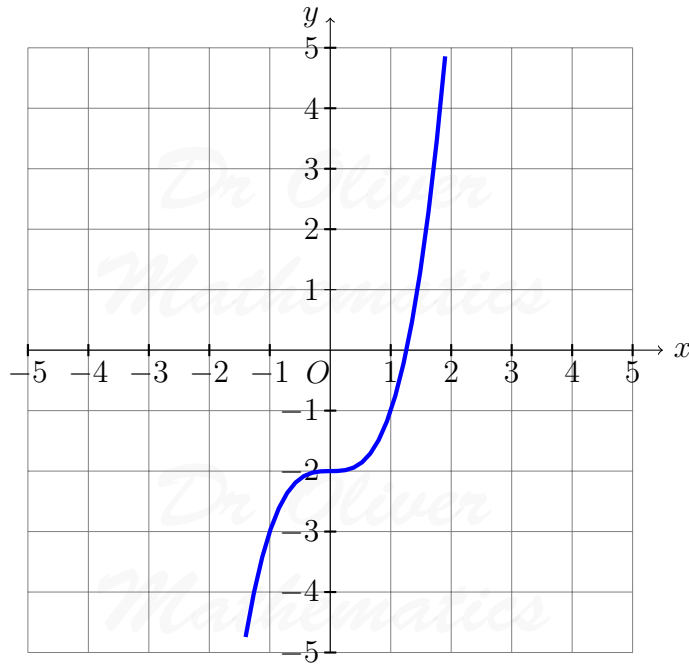
$$y = x^3 - 2.$$

(2)

You must show the coordinates of the y -intercept.



Solution



The y -intercept is $(0, -2)$.

9. In a sport, injury time is added time played at the end of a match.

The table shows the injury time, t (minutes) played in 380 matches.

Injury time, t (minutes)	Frequency
$0 < t \leq 2$	59
$2 < t \leq 4$	158
$4 < t \leq 6$	106
$6 < t \leq 8$	45
$8 < t \leq 10$	12

(a) Circle the two words that describe the data.

(1)

continuous discrete grouped ungrouped

Solution

continuous discrete grouped ungrouped

(b) Which class interval contains the median?

You **must** show your working.

(2)

Solution

Injury time, t (minutes)	Frequency	Cumulative Frequency
$0 < t \leq 2$	59	59
$2 < t \leq 4$	158	$59 + 158 = 217$
$4 < t \leq 6$	106	$217 + 106 = 323$
$6 < t \leq 8$	45	$323 + 45 = 368$
$8 < t \leq 10$	12	$368 + 12 = 380$

The median is at

$$\frac{380 + 1}{2} = 190\frac{1}{2}\text{th place}$$

and, hence, the median is $2 < t \leq 4$.

(c) What percentage of the matches had more than **6 minutes** of injury time?

(2)

Solution

$$\begin{aligned}\text{Percentage} &= \left(\frac{45 + 12}{380} \right) \times 100\% \\ &= \frac{57}{380} \times 100\% \\ &= \underline{\underline{15\%}}.\end{aligned}$$

10. x is an integer.

$$-4 < x \leq 2$$

(3)

and

$$2 \leq x + 3 < 9.$$

Work out all the possible values of x .

Solution

Well,

$$2 \leq x + 3 < 9 \Rightarrow -1 \leq x < 6$$

and the values of x are

$$\underline{\underline{-1, 0, 1, \text{ and } 2.}}$$

11. Joe and Kyle share an amount of money in the ratio $7 : n$.

(2)

Joe gets 35% of the money.

Work out the value of n .

Solution

Well,

$$\begin{aligned} \frac{\text{Kyle}}{\text{Joe}} &= \frac{65\%}{35\%} \Rightarrow \frac{n}{7} = \frac{13}{7} \\ &\Rightarrow \underline{\underline{n = 13.}} \end{aligned}$$

12. A biased coin is thrown 250 times.

(1)

The relative frequency of Heads is worked out after every 50 throws.

Total number of throws	50	100	150	200	250
Relative frequency	0.4	0.29	0.4	0.32	0.3

Circle the best estimate of the probability of Heads.

0.3 0.32 0.342 0.4

Solution

0.3 0.32 0.342 0.4

13. The amounts spent on clothes by 40 boys and 40 girls in one month were recorded. (5)

The table shows information about the amounts spent by the boys.

Amount, x , (£)	Midpoint	Number of boys
$0 \leq x < 20$		22
$20 \leq x < 40$		9
$40 \leq x < 60$		6
$60 \leq x < 80$		3
		Total = 40

The mean for the girls was £35.

Estimate the mean for the girls as a percentage of the mean for the boys.

Solution

Amount, x , (£)	Midpoint	Number of boys	Midpoint \times Number
$0 \leq x < 20$	10	22	$10 \times 22 = 220$
$20 \leq x < 40$	30	9	$30 \times 9 = 270$
$40 \leq x < 60$	50	6	$50 \times 6 = 300$
$60 \leq x < 80$	70	3	$70 \times 3 = 210$
		Total = 40	Total = 1 000

Now,

$$\begin{aligned} \text{mean for boys} &= \frac{\sum fx}{\sum x} \\ &\approx \frac{1\,000}{40} \\ &= \text{£}25. \end{aligned}$$

Finally, the mean for the girls as a percentage of the mean for the boys is

$$\left(\frac{35}{25}\right) \times 100\% = \underline{\underline{140\%}}.$$

14. Ali and Mel are making 3-digit codes.

- The digit 0 is not used.
 - Ali only uses odd digits.
 - Mel only uses even digits.
- (a) Ali can make x more codes than Mel. (3)

Assume that digits **cannot** be repeated.

Work out the value of x .

Solution

Well, Ali can make

$$5 \times 4 \times 3 = 60 \text{ codes}$$

and Mel can make

$$4 \times 3 \times 2 = 24 \text{ codes.}$$

So, the difference is

$$60 - 24 = \underline{36}.$$

In fact, digits **can** be repeated.

- (b) What does this tell you about the actual value of x ? (1)
 Tick one box.

- It is bigger than my answer to part (a)
- It is smaller than my answer to part (a)
- It is the same as my answer to part (a)

Solution

Now, Ali can make

$$5 \times 5 \times 5 = 125 \text{ codes}$$

and Mel can make

$$4 \times 4 \times 4 = 64 \text{ codes.}$$

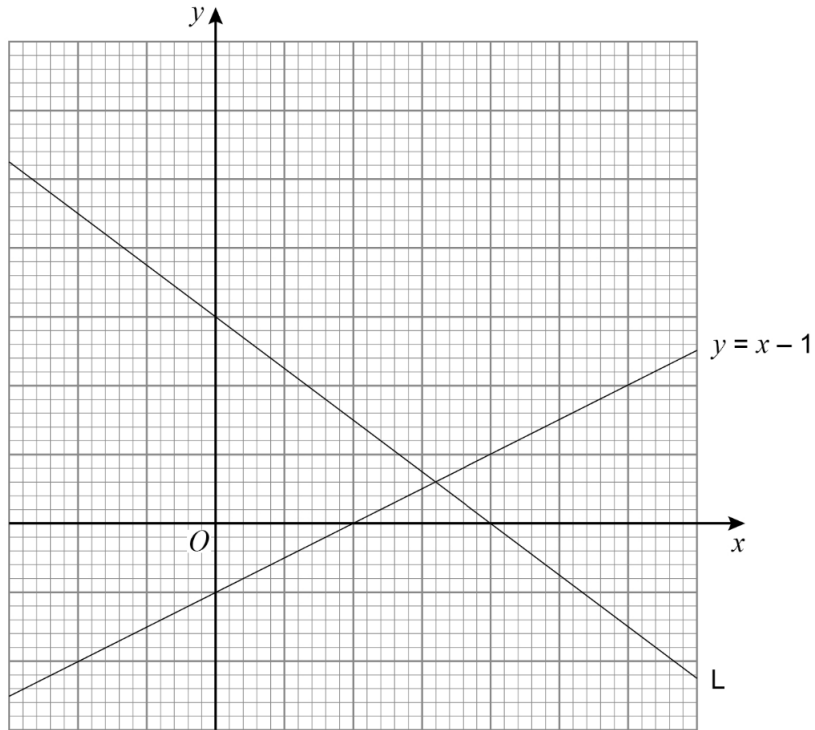
Hence, It is bigger than my answer to part (a).

15. Here is line L and the graph of

$$y = x - 1.$$

(4)

The scales of the axes are not shown.



Work out the equation of line L .

Solution

Well, the x -axis is stretched by a factor of 2 and the y -axis is left alone.

So, the line L goes through $(0, 3)$ and $(2, 0)$.

Now,

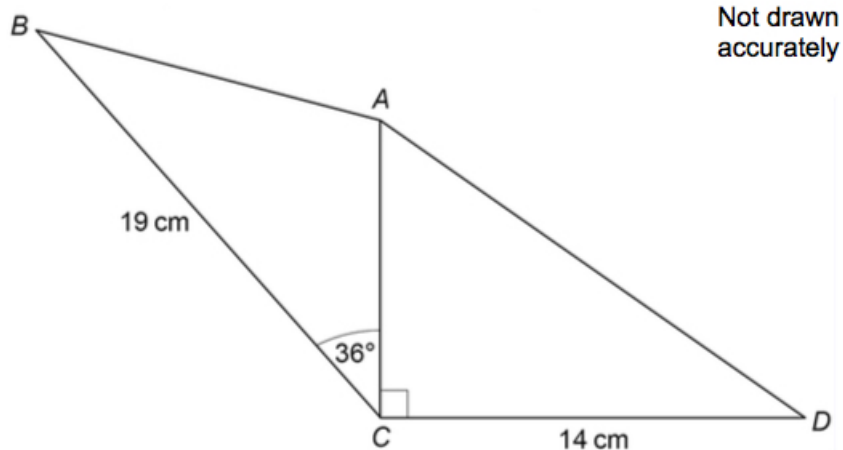
$$\begin{aligned} m &= \frac{3 - 0}{0 - 2} \\ &= -\frac{3}{2} \end{aligned}$$

and the equation of line L is

$$\underline{\underline{y = -\frac{3}{2}x + 3.}}$$

16. ABC and ACD are triangles.

(4)



The area of ACD is 80.5 cm^2 .

Work out the area of ABC .

Give your answer to 3 significant figures.

Solution

Well,

$$\begin{aligned} A &= \frac{1}{2}bh \Rightarrow 80.5 = \frac{1}{2} \times AC \times 14 \\ &\Rightarrow 161 = 14AC \\ &\Rightarrow AC = 11.5 \text{ cm.} \end{aligned}$$

Finally,

$$\begin{aligned} \text{area of } ABC &= \frac{1}{2} \times 11.5 \times 19 \times \sin 36^\circ \\ &= 64.215\ 538\ 81 \text{ (FCD)} \\ &= \underline{\underline{64.2 \text{ cm}^2}} \text{ (3 sf).} \end{aligned}$$

17.

$$m = \frac{p - 2b}{2}.$$

(3)

- $p = 68.3$, correct to 1 decimal place.
- $b = 8.7$, correct to 1 decimal place.

Work out the lower bound for m .

Solution

Now,

$$68.25 \leq p < 68.35$$

and

$$8.65 \leq b < 8.75.$$

So, the lower bound for m is

$$\frac{68.25 - 2(8.75)}{2} = \underline{\underline{23.375}}.$$

18. In a bag there are blue discs, green discs, and white discs.

(3)

- There are four times as many blue discs as green discs.
- Number of blue discs : number of white discs = 3 : 5.

One disc is selected at random.

Work out the probability that the disc is either blue or white.

Solution

Now,

$$B : G = 4 : 1 = 12 : 3$$

and

$$B : W = 3 : 5 = 12 : 20.$$

So,

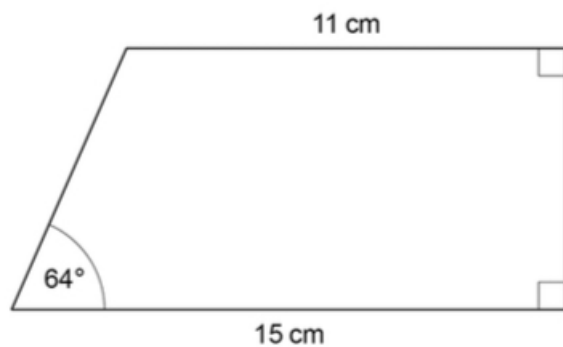
$$B : G : W = 12 : 3 : 20.$$

Finally,

$$\begin{aligned} P(\text{either blue or white}) &= 1 - P(\text{green}) \\ &= 1 - \frac{3}{35} \\ &= \underline{\underline{\frac{32}{35}}}. \end{aligned}$$

19. Work out the area of the trapezium.

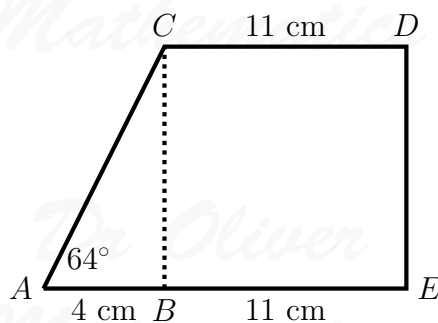
(4)

Not drawn
accurately**Solution**

Well,

$$15 - 11 = 4 \text{ cm}$$

and



Now,

$$\begin{aligned} \tan &= \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 64^\circ = \frac{BC}{4} \\ &\Rightarrow BC = 4 \tan 64^\circ. \end{aligned}$$

Finally,

$$\begin{aligned} \text{area} &= \text{area of } ABC + \text{area of } BCDE \\ &= \left(\frac{1}{2} \times 4 \times 4 \tan 64^\circ\right) + (4 \tan 64^\circ \times 11) \\ &= 8 \tan 64^\circ + 44 \tan 64^\circ \\ &= \underline{52 \tan 64^\circ} \\ &= 106.6157998 \text{ (FCD)} \\ &= \underline{\underline{107 \text{ cm}^2}} \text{ (3 sf)}. \end{aligned}$$

20. Expressions for consecutive triangular numbers are (4)

$$\frac{1}{2}n(n+1) \text{ and } \frac{1}{2}(n+1)(n+2).$$

Prove that the sum of two consecutive triangular numbers is always a square number.

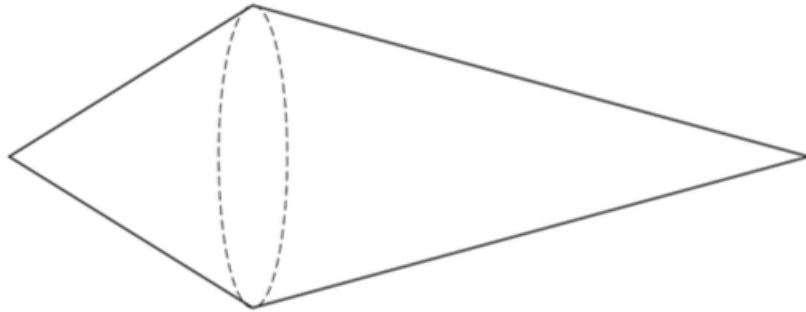
Solution

$$\begin{aligned} \text{Sum} &= \frac{1}{2}n(n+1) + \frac{1}{2}(n+1)(n+2) \\ &= \frac{1}{2}(n+1)[n + (n+2)] \\ &= \frac{1}{2}(n+1)(2n+2) \\ &= \frac{1}{2}(n+1)[2(n+1)] \\ &= (n+1)^2; \end{aligned}$$

so, the sum of two consecutive triangular numbers is always a square number.

21. A solid shape is made by joining two cones. (3)

Each cone has the same radius.



One cone has

$$\text{slant height} = 2 \times \text{radius}.$$

The other cone has

$$\text{slant height} = 3 \times \text{radius}.$$

The total surface area of the shape is $57.8\pi \text{ cm}^2$.

Curved surface area of a cone = $\pi r l$ where r is the radius and l is the slant height

Work out the radius.

Solution

Let r cm be the radius. Then

$$\begin{aligned}(\pi \times r \times 2r) + (\pi \times r \times 3r) &= 57.8\pi \Rightarrow 5\pi r^2 = 57.8\pi \\ &\Rightarrow r^2 = 11.56 \\ &\Rightarrow \underline{\underline{r = 3.4}}.\end{aligned}$$

22. Show that

$$(5\sqrt{3} - \sqrt{12})^2$$

(3)

simplifies to an integer.

Solution

Well,

$$\begin{aligned}\sqrt{12} &= \sqrt{4 \times 3} \\ &= \sqrt{4} \times \sqrt{3} \\ &= 2\sqrt{3}\end{aligned}$$

and so

$$\begin{aligned}(5\sqrt{3} - \sqrt{12})^2 &= (5\sqrt{3} - 2\sqrt{3})^2 \\ &= (3\sqrt{3})^2 \\ &= 9 \times 3 \\ &= \underline{\underline{27}},\end{aligned}$$

as required.

23. A and B are similar cuboids.

(1)

Surface area of A : surface area of $B = 16 : 25$.

Work out

volume of A : volume of B .

Circle your answer.

4 : 5 16 : 25 64 : 125 256 : 625

Solution

Well, the ratio of the surface area is

$$16 : 25 = 4^2 : 5^2.$$

Now, the ratio of the lengths area is

$$4 : 5$$

and the ratio of the volumes area is

$$4^3 : 5^3 = 64 : 125.$$

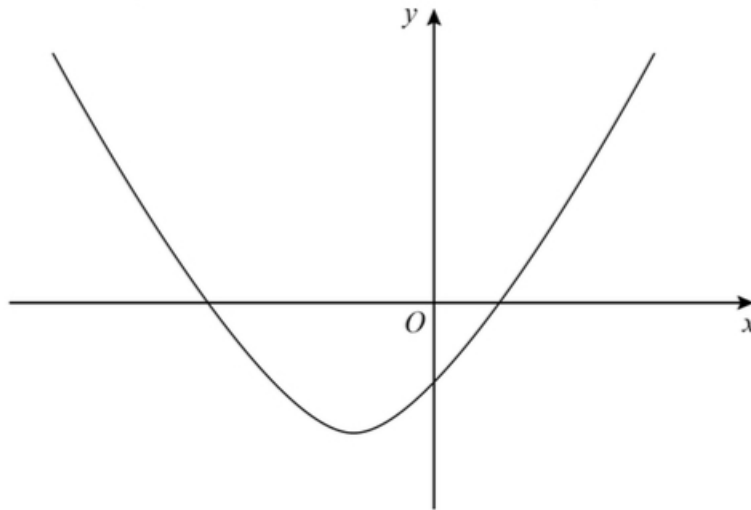
So

$$4 : 5 \quad 16 : 25 \quad \underline{64 : 125} \quad 256 : 625$$

24. Here is a sketch of the curve

$$y = x^2 + 4x - 12.$$

(3)



Work out the values of x for which

$$x^2 + 4x - 12 < 0.$$

Give your answer as an inequality.

Solution

$$\left. \begin{array}{l} \text{add to: } +4 \\ \text{multiply to: } -12 \end{array} \right\} + 6, -2$$

Now,

$$x^2 + 4x - 12 < 0 \Rightarrow (x + 6)(x - 2) < 0$$

We need a 'table of signs':

	$x < -6$	$x = -6$	$-6 < x < 2$	$x = 2$	$x > 2$
$x + 6$	-	0	+	+	+
$x - 2$	-	-	-	0	+
$(x + 6)(x - 2)$	+	0	-	0	+

$$\Rightarrow \underline{\underline{-6 < x < 2.}}$$

25. A sample of 50 eggs is taken from Farm A.

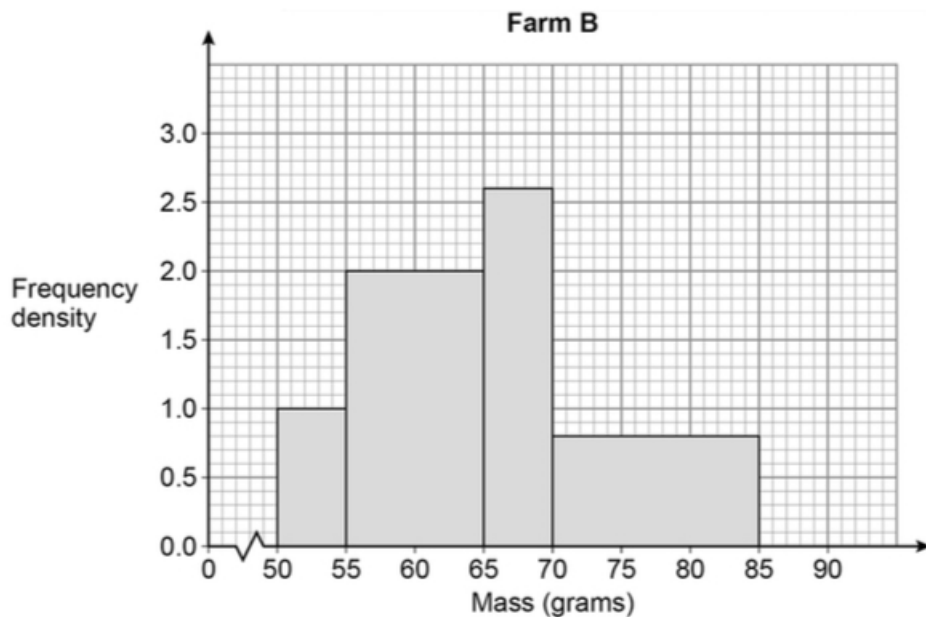
(4)

The table shows information about the masses of the eggs from Farm A.

Mass, m (grams)	Frequency
$53 < m \leq 58$	8
$58 < m \leq 63$	19
$63 < m \leq 68$	15
$68 < m \leq 73$	8

A sample of 50 eggs is taken from Farm B.

The histogram shows information about the masses of the eggs from Farm B.



For medium eggs, $53 \text{ g} < \text{mass} \leq 63 \text{ g}$.

The Farm A sample has more medium eggs than the Farm B sample.

Using the table and the histogram, estimate how many more.

You **must** show your working.

Solution

Farm B:

Mass, m (grams)	Frequency Density	Width	Frequency
$50 < m \leq 55$	1	5	$1 \times 5 = 5$
$55 < m \leq 65$	2	10	$2 \times 10 = 20$
$65 < m \leq 70$	2.6	5	$2.6 \times 5 = 13$
$70 < m \leq 85$	0.8	15	$0.8 \times 15 = 12$

Now, Farm B's $53 \text{ g} < \text{mass} \leq 63 \text{ g}$:

$$\left(\frac{2}{5} \times 5\right) + \left(\frac{8}{10} \times 20\right) = 2 + 16 = 18.$$

Farm A has

$$8 + 19 = 27$$

and so I would estimate Farm A has

$$27 - 18 = \underline{\underline{9 \text{ more.}}}$$

26.

$$(x + 5)(x + 2)(x + a) \equiv x^3 + bx^2 + cx - 30. \quad (3)$$

Work out the values of the integers a , b , and c .

Solution

$$\begin{array}{r|rr} \times & x & +5 \\ \hline x & x^2 & +5x \\ +2 & +2x & +10 \end{array}$$

and so

$$(x + 5)(x + 2) = x^2 + 7x + 10.$$

$$\begin{array}{r|rrr} \times & x^2 & +7x & +10 \\ \hline x & x^3 & +7x^2 & +10x \\ +a & +ax^2 & +7ax & +10a \end{array}$$

and so

$$(x + 5)(x + 2)(x + a) = x^3 + (a + 7)x^2 + (7a + 10)x + 10a.$$

Well, we take the constant term:

$$10a = -30 \Rightarrow \underline{\underline{a = -3.}}$$

x^2 -term:

$$b = -3 + 7 = \underline{\underline{4.}}$$

x -term:

$$c = 7(-3) + 10 = \underline{\underline{-11.}}$$

27.

$$f(x) = \frac{2}{5}x - 1. \quad (5)$$

Work out the value of

$$f^{-1}(3) + f(-0.5).$$

Solution

$$\begin{aligned}y = \frac{2}{5}x - 1 &\Rightarrow y + 1 = \frac{2}{5}x \\ &\Rightarrow \frac{5}{2}(y + 1) = x\end{aligned}$$

and so

$$f^{-1}(x) = \frac{5}{2}(x + 1).$$

Finally,

$$\begin{aligned}[f^{-1}(3) + f(-0.5)] &= \frac{5}{2}(3 + 1) + \left[\frac{2}{5}(-0.5) - 1\right] \\ &= 10 + \left(-1\frac{1}{5}\right) \\ &= \underline{\underline{8\frac{4}{5}}}.\end{aligned}$$

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