Dr Oliver Mathematics
GCSE Mathematics
2019 November Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Find the Lowest Common Multiple (LCM) of 108 and 120. (3)

Solution

$$\begin{array}{r|l} & 108 \\ \hline 2 & 54 \\ \hline 2 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline 3 & 1 \\ \hline \end{array}$$

So
$$108 = 2^2 \times 3^3.$$

$$\begin{array}{r|l} & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 2 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

So
$$120 = 2^3 \times 3 \times 5.$$

Hence, Lowest Common Multiple (LCM) is

$$\begin{aligned} 2^3 \times 3^3 \times 5 &= 8 \times 27 \times 5 \\ &= 8 \times 135 \\ &= \underline{\underline{1\,080}}. \end{aligned}$$

2. There are 60 people in a choir. (4)
Half of the people in the choir are women.

The number of women in the choir is 3 times the number of men in the choir.
The rest of the people in the choir are children.

$$\text{The number of children in the choir} : \text{the number of men in the choir} = n : 1.$$

Work out the value of n.

You must show how you get your answer.

Solution

"Half of the people in the choir are women" which means 30 women and the rest is divided between the men and the children. Now,

$$\text{women} : \text{men} = 3 : 1$$

which means there are 10 men and 20 children. Next,

$$\text{children} : \text{men} = 20 : 10 = 2 : 1;$$

hence $\underline{\underline{n = 2}}$.

3. Work out (3)

$$1\tfrac{3}{4} \times 1\tfrac{1}{3}.$$

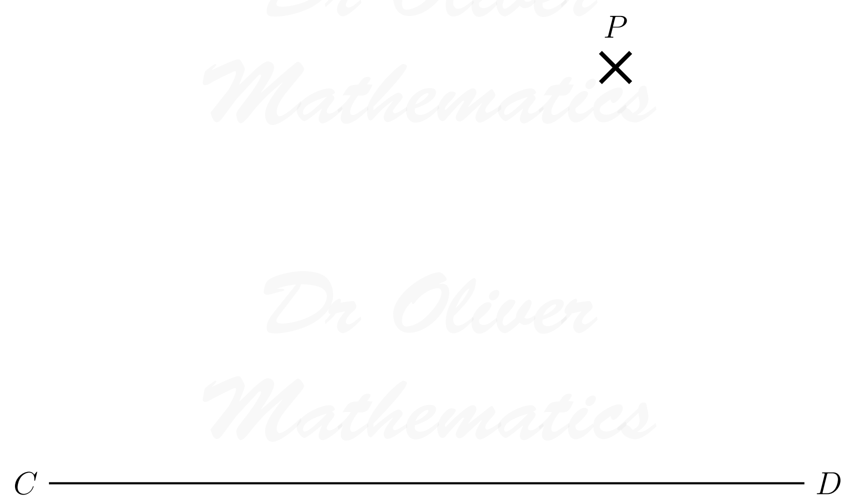
Give your answer as a mixed number.

Solution

$$\begin{aligned} 1\tfrac{3}{4} \times 1\tfrac{1}{3} &= \tfrac{7}{4} \times \tfrac{4}{3} \\ &= \tfrac{7}{\cancel{4}} \times \tfrac{\cancel{4}}{3} \\ &= \tfrac{7}{3} \\ &= \underline{\underline{2\tfrac{1}{3}}}. \end{aligned}$$

4. Use a ruler and compasses to construct the line from the point P perpendicular to the line CD. (2)
You must show all construction lines.

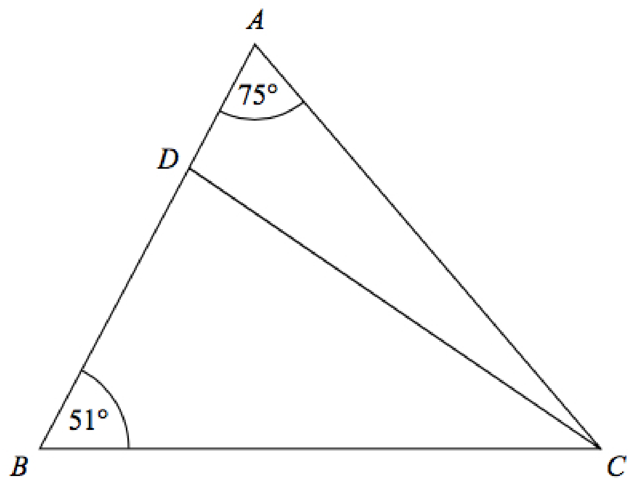
Dr Oliver Mathematics



Solution

Make your compass about 6 cm long.
From P, use your compasses to draw a faint (but visible!) arc of a circle so that it cuts CD in two places: we will call that P_1 (to the left) and P_2 (to the right).
At P_1, centre your compasses so that they pass through the point P_2. At P_1, make a faint (but visible!) arc of a circle, going from P_2 down in a quarter-circle.
At P_2, make a faint (but visible!) arc of a circle, going from P_1 down in a quarter-circle.
Where P_1 and P_2 cross, label that P_3.
Finally, draw a line between P and P_3: that's it!

5. The diagram shows triangle ABC. (4)



ADB is a straight line.

The size of angle DCB : the size of angle $ACD = 2 : 1$.

Work out the size of angle BDC.

Solution

Well,

$$\angle ACB = 180 - (75 + 51) = 180 - 126 = 54°.$$

Now,

$$\begin{aligned}\angle DCB &= \tfrac{2}{3} \times 54 \\ &= \tfrac{2}{\cancel{3}} \times \cancel{54}^{18} \\ &= 2 \times 18 \\ &= 36°\end{aligned}$$

and

$$\begin{aligned}\angle BDC &= 180 - (51 + 36) \\ &= 180 - 87 \\ &= \underline{\underline{93°}}.\end{aligned}$$

6. 4 red bricks have a mean weight of 5 kg. (3)
   5 blue bricks have a mean weight of 9 kg.

1 green brick has a weight of 6 kg.

Donna says, "The mean weight of the 10 bricks is less than 7 kg."

Is Donna correct?
You must show how you get your answer.

Solution

$$\begin{aligned}\text{Mean} &= \frac{(4 \times 5) + (5 \times 9) + 6}{10} \\ &= \frac{20 + 45 + 6}{10} \\ &= \frac{71}{10} \\ &= 7.1 \text{ kg.}\end{aligned}$$

Hence, Donna is $\underline{\underline{\text{wrong}}}$.

7. (a) Simplify (1)

$$(p^2)^5.$$

Solution

$$(p^2)^5 = p^{2 \times 5} = \underline{\underline{p^{10}}}.$$

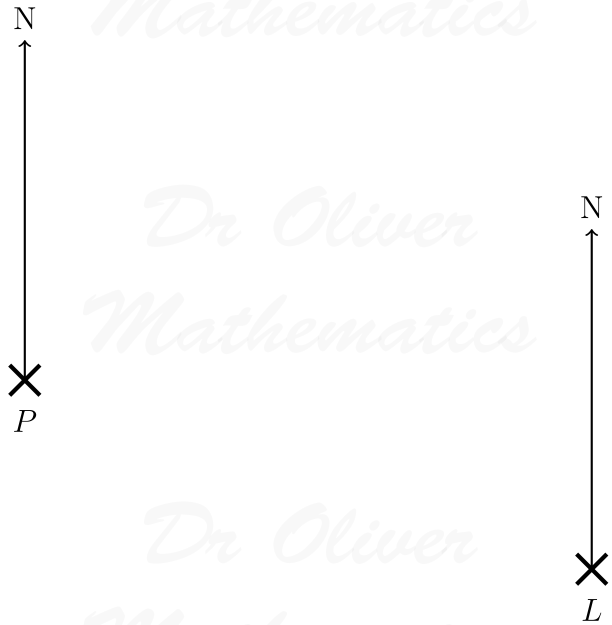
(b) Simplify (2)

$$12x^7y^3 \div 6x^3y.$$

Solution

$$\begin{aligned}12x^7y^3 \div 6x^3y &= \frac{12x^7y^3}{6x^3y} \\ &= \frac{\cancel{12}^{2} x^{\cancel{7}\,4} y^{\cancel{3}\,2}}{\cancel{6}\cancel{x^3}\cancel{y}} \\ &= \underline{\underline{2x^4y^2}}.\end{aligned}$$

8. The accurate scale drawing shows the positions of port P and a lighthouse L.



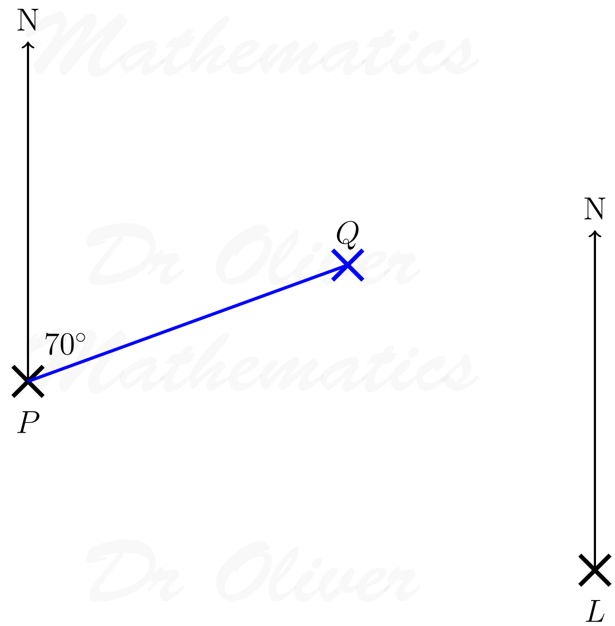
Scale: 1 cm represents 4 km.
(Note: on the actual drawing, the lines are 7.5 cm apart.)
Aleena sails her boat from port P on a bearing of 070°.
She sails for $1\frac{1}{2}$ hours at an average speed of 12 km/h to a port Q.

Find

(a) the distance, in km, of port Q from lighthouse L, (3)

Solution
She sails a distance of
$$1\tfrac{1}{2} \times 12 = 18 \text{ km}.$$

Dr Oliver Mathematics



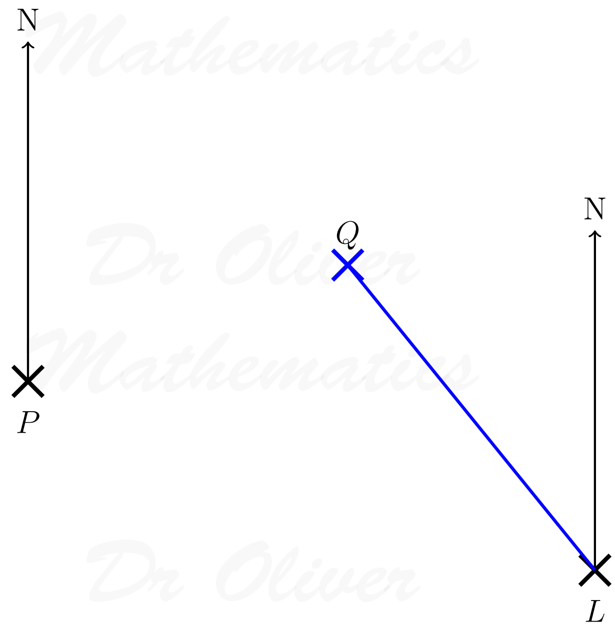
Now, get out you ruler: about

$$5 \times 4 = \underline{\underline{20 \text{ km}}}.$$

(b) the bearing of port Q from lighthouse L. (2)

Solution

Dr Oliver Mathematics



So the bearing of port Q from lighthouse L is

$$360 - 40 = \underline{\underline{320^\circ}}.$$

9. A car travels for 18 minutes at an average speed of 72 km/h.

   (a) How far will the car travel in these 18 minutes? (2)

Solution

$$\begin{aligned} \text{Distance} &= 18 \text{ minutes} \times 72 \text{ km/h} \\ &= \frac{18}{60} \text{ hours} \times 72 \text{ km/h} \\ &= \frac{18}{\cancel{60}^{5}} \text{ hours} \times \cancel{72}^{6} \\ &= \frac{108}{5} \\ &= \underline{\underline{21.6 \text{ km}}}. \end{aligned}$$

David says, "72 kilometres per hour is faster than 20 metres per second."

   (b) Is David correct? (2)

You must show how you get your answer.

Solution

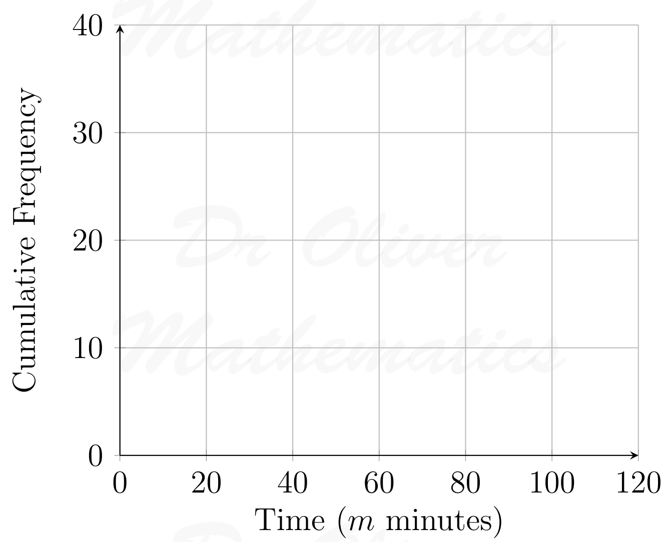
$$\begin{aligned} 20 \text{ m/s} &= 60 \times 20 \text{ m/min} \\ &= 1\,200 \text{ m/min} \\ &= 1.2 \text{ km/min} \\ &= 1.2 \times 60 \text{ km/h} \\ &= 72 \text{ km/h}. \end{aligned}$$

<u>No</u>: they are equal!

10. The cumulative frequency table shows information about the times, in minutes, taken by 40 people to complete a puzzle.

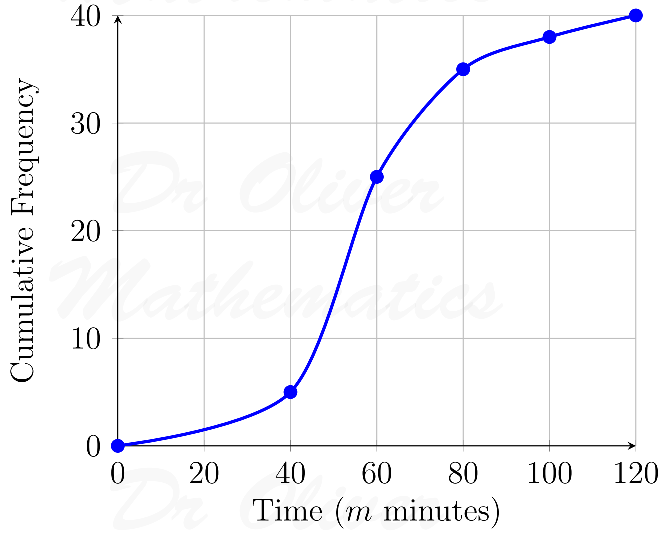
Time (m minutes)	Cumulative Frequency
$20 < m \leqslant 40$	5
$20 < m \leqslant 60$	25
$20 < m \leqslant 80$	35
$20 < m \leqslant 100$	38
$20 < m \leqslant 120$	40

(a) On the grid below, draw a cumulative frequency graph for this information. (2)



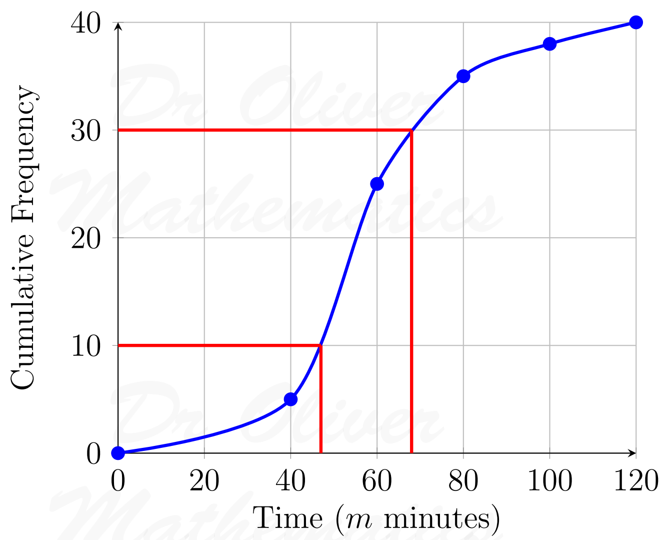
Dr Oliver Mathematics

Solution



(b) Use your graph to find an estimate for the interquartile range. (2)

Solution



Hence,

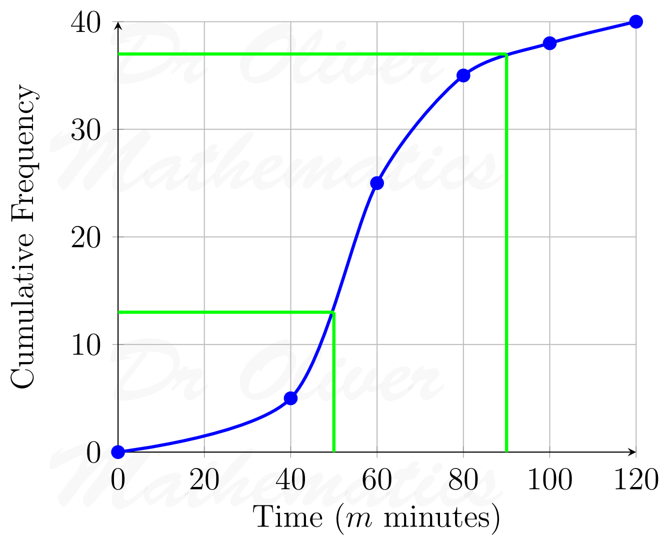
$$\begin{aligned} \text{IQR} &= \text{UQ} - \text{LQ} \\ &= 68 - 47 \\ &= \underline{\underline{21 \text{ minutes}}}. \end{aligned}$$

Dr Oliver Mathematics

One of the 40 people is chosen at random.

(c) Use your graph to find an estimate for the probability that this person took between 50 minutes and 90 minutes to complete the puzzle. (2)

Solution



Hence,

$$\text{Probability} = \frac{37 - 13}{40} = \frac{24}{40} = \underline{\underline{\tfrac{3}{5}}}.$$

11. There are p counters in a bag. (2)
12 of the counters are yellow.

Shafiq takes at random 30 counters from the bag.
5 of these 30 counters are yellow.

Work out an estimate for the value of p.

Solution

$$\frac{5}{30} = \frac{12}{p} \Rightarrow 5p = 360$$
$$= \underline{\underline{p = 72}}.$$

12. (1)

$$T = \tfrac{1}{2}q + 5.$$

Here is Spencer's method to make q the subject of the formula.

$$2 \times T = q + 5$$
$$q = 2T - 5.$$

What mistake did Spencer make in the first line of his method?

Solution

He forgot to multiply the five: it should be

$$2 \times T = q + 2 \times 5.$$

13. (a) Write (2)

$$\frac{5}{x+1} + \frac{2}{3x}$$

as a single fraction in its simplest form.

Solution

$$\begin{aligned} \frac{5}{x+1} + \frac{2}{3x} &= \frac{5(3x)}{3x(x+1)} + \frac{2(x+1)}{3x(x+1)} \\ &= \frac{5(3x) + 2(x+1)}{3x(x+1)} \\ &= \frac{15x + 2x + 2}{3x(x+1)} \\ &= \underline{\underline{\frac{17x+2}{3x(x+1)}}}. \end{aligned}$$

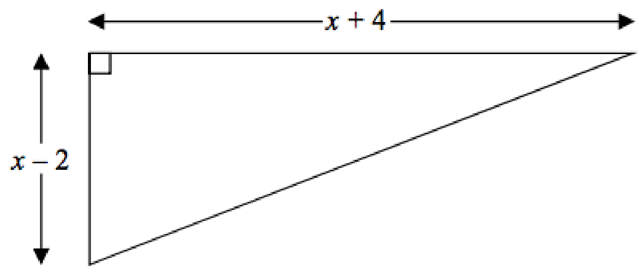
(b) Factorise (1)

$$(x+y)^2 + 3(x+y).$$

Solution

$$\begin{aligned}(x+y)^2 + 3(x+y) &= (x+y)[(x+y)+3] \\ &= \underline{\underline{(x+y)(x+y+3)}}.\end{aligned}$$

14. The diagram shows a right-angled triangle. (4)



All the measurements are in centimetres.

The area of the triangle is 27.5 cm^2.

Work out the length of the shortest side of the triangle.
You must show all your working.

Solution

$$\begin{aligned}\text{Area} = 27.5 &\Rightarrow \tfrac{1}{2} \times (x-2) \times (x+4) = 27.5 \\ &\Rightarrow (x-2)(x+4) = 55\end{aligned}$$

Dr Oliver Mathematics

$\times$	x	-2
x	x^2	$-2x$
$+4$	$+4x$	-8

$$\Rightarrow x^2 + 2x - 8 = 55$$
$$\Rightarrow x^2 + 2x - 63 = 0$$

$$\left.\begin{array}{ll} \text{add to:} & +2 \\ \text{multiply to:} & -63 \end{array}\right\} -7, +9$$

$$\Rightarrow (x - 7)(x + 9) = 0$$
$$\Rightarrow x = 7 \text{ or } x = -9.$$

$x \neq -9$ as it is one of the sides of a triangle. So,

$$x = 7 \Rightarrow x - 2 = \underline{\underline{5 \text{ cm}}}.$$

15. Express (3)

$$0.4\dot{1}\dot{8}$$

as a fraction.
You must show all your working.

Solution

Let $x = 0.4\dot{1}\dot{8}$. Then

$$10x = 4.\dot{1}\dot{8} \quad (1)$$
$$1\,000x = 418.\dot{1}\dot{8} \quad (2)$$

Do (2) − (1):

$$990x = 414 \Rightarrow 18 \times 55x = 18 \times 23$$
$$\Rightarrow 55x = 23$$
$$\Rightarrow \underline{\underline{x = \tfrac{23}{55}}}.$$

Dr Oliver Mathematics

16. (a) Rationalise the denominator of (2)

$$\frac{22}{\sqrt{11}}.$$

Give your answer in its simplest form.

Solution

$$\frac{22}{\sqrt{11}} = \frac{22}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}}$$
$$= \frac{22\sqrt{11}}{11}$$
$$= \underline{\underline{2\sqrt{11}}}.$$

(b) Show that (3)

$$\frac{\sqrt{3}}{2\sqrt{3}-1}$$

can be written in the form

$$\frac{a+\sqrt{3}}{b},$$

where a and b are integers.

Solution

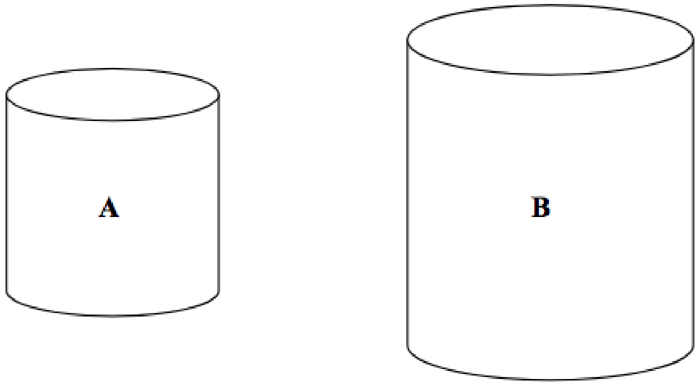
$$\frac{\sqrt{3}}{2\sqrt{3}-1} = \frac{\sqrt{3}}{2\sqrt{3}-1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1}$$

$\times$	$2\sqrt{3}$	-1
$2\sqrt{3}$	12	$-2\sqrt{3}$
$+1$	$+2\sqrt{3}$	-1

$$= \frac{\sqrt{3}(2\sqrt{3}+1)}{12-1}$$
$$= \underline{\underline{\frac{6+\sqrt{3}}{11}}};$$

hence, $\underline{\underline{a = 6}}$ and $\underline{\underline{b = 11}}$.

17. **A** and **B** are two similar cylindrical containers. (4)



The surface area of container **A** : the surface area of container **B** = 4 : 9.

Tyler fills container **A** with water.
She then pours all the water into container **B**.
Tyler repeats this and stops when container **B** is full of water.

Work out the number of times that Tyler fills container **A** with water.
You must show all your working.

Solution

Well, the ASF (area scale factor) of

$$4 : 9 = 2^2 : 3^2$$

which means that the LSF (length scale factor) is

$$2 : 3$$

and the VSF (volume scale factor) is

$$2^3 : 3^3 = 8 : 27.$$

Tyler fills container **A** with water is

$$\frac{27}{8} = 3\tfrac{3}{8};$$

hence, she fills **A** 4 times.

Dr Oliver Mathematics

18. The function f is given by

$$f(x) = 2x^3 - 4.$$

(a) Show that **(2)**

$$f^{-1}(50) = 3.$$

Solution

$$\begin{aligned} 2x^3 - 4 = 50 &\Rightarrow 2x^3 = 54 \\ &\Rightarrow x^3 = 27 \\ &\Rightarrow \underline{\underline{x = 3}}, \end{aligned}$$

as required.

The functions g and h are given by

$$g(x) = x + 2 \text{ and } h(x) = x^2.$$

(b) Find the values of x for which **(4)**

$$h\,g(x) = 3x^2 + x - 1.$$

Solution

$$\begin{aligned} h\,g(x) &= h(g(x)) \\ &= h(x + 2) \\ &= (x + 2)^2 \\ &= x^2 + 4x + 4. \end{aligned}$$

Now,

$$\begin{aligned} h\,g(x) = 3x^2 + x - 1 &\Rightarrow x^2 + 4x + 4 = 3x^2 + x - 1 \\ &\Rightarrow 2x^2 - 3x - 5 = 0 \end{aligned}$$

Dr Oliver Mathematics

$$\left.\begin{array}{ll}\text{add to:} & -3 \\ \text{multiply to:} & (+2)\times(-5) = -10\end{array}\right\} -5,\ +2$$

$$\begin{aligned}
&\Rightarrow 2x^2 - 5x + 2x - 5 = 0 \\
&\Rightarrow x(2x-5) + 1(2x-5) = 0 \\
&\Rightarrow (x+1)(2x-5) = 0 \\
&\Rightarrow x+1 = 0 \text{ or } 2x-5 = 0 \\
&\Rightarrow \underline{\underline{x = -1 \text{ or } x = 2\tfrac{1}{2}}}.
\end{aligned}$$

19. Given that (3)

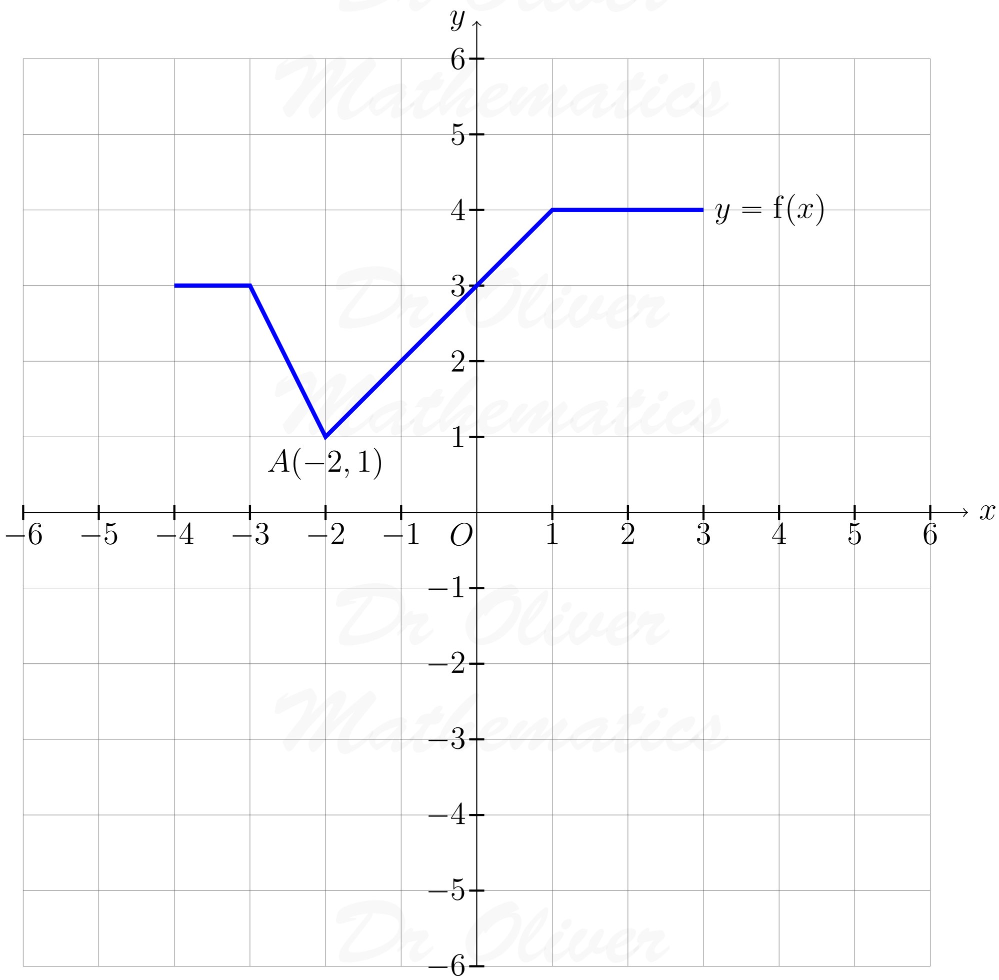
$$9^{-\frac{1}{2}} = 27^{\frac{1}{4}} \div 3^{x+1},$$

find the exact value of x.

Solution

$$\begin{aligned}
9^{-\frac{1}{2}} = 27^{\frac{1}{4}} \div 3^{x+1} &\Rightarrow 3^{x+1} = 27^{\frac{1}{4}} \div 9^{-\frac{1}{2}} \\
&\Rightarrow 3^{x+1} = (3^3)^{\frac{1}{4}} \div (3^2)^{-\frac{1}{2}} \\
&\Rightarrow 3^{x+1} = 3^{\frac{3}{4}} \div 3^{-1} \\
&\Rightarrow 3^{x+1} = 3^{\frac{3}{4}} \times 3^{1} \\
&\Rightarrow 3^{x+1} = 3^{\frac{7}{4}} \\
&\Rightarrow x+1 = \tfrac{7}{4} \\
&\Rightarrow \underline{\underline{x = \tfrac{3}{4}}}.
\end{aligned}$$

20. The graph of $y = \mathrm{f}(x)$ is shown on the grid.

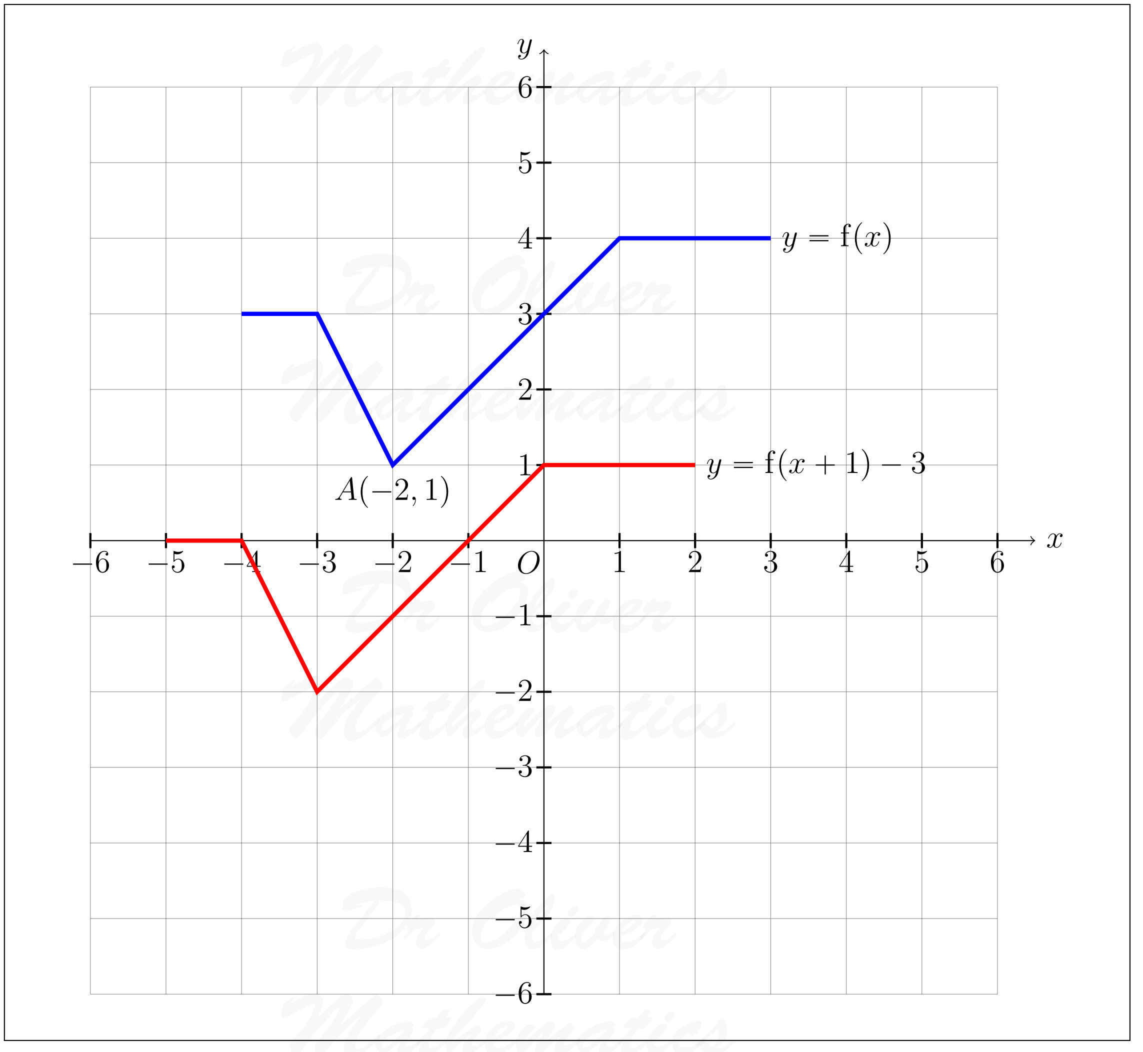


(a) On the grid, draw the graph with equation $y = \mathrm{f}(x + 1) - 3$. (2)

Solution

It is a translation by the vector

$$\begin{pmatrix} -1 \\ -3 \end{pmatrix}.$$



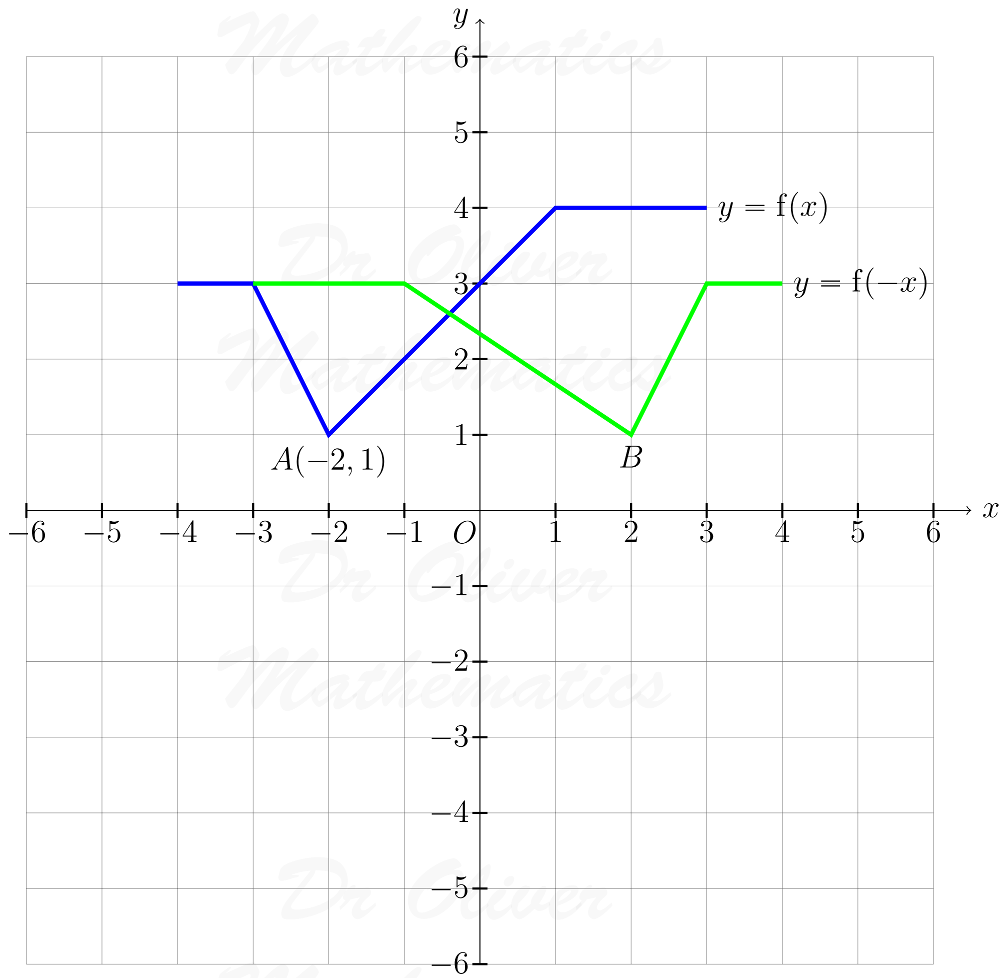
Point $A(-2, 1)$ lies on the graph of $y = \mathrm{f}(x)$.

When the graph of $y = \mathrm{f}(x)$ is transformed to the graph with equation $y = \mathrm{f}(-x)$, point A is mapped to point B.

(b) Write down the coordinates of point B. (1)

Solution

It is a reflection in the y-axis:



$\underline{\underline{B(2, 1)}}$.

21. Sketch the graph of (5)

$$y = 2x^2 - 8x - 5,$$

showing the coordinates of the turning point and the exact coordinates of any interceptswith the coordinate axes.

Solution

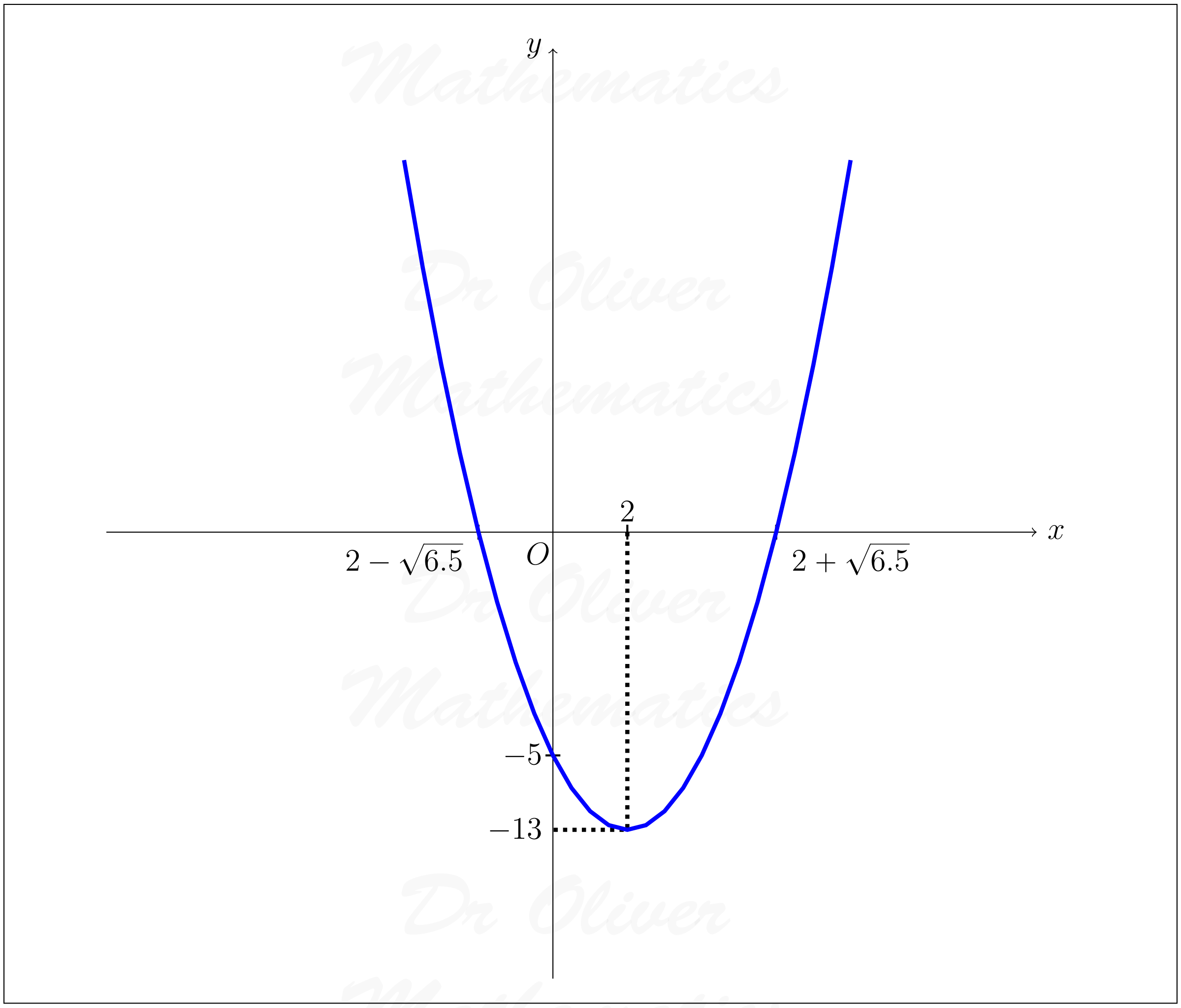
Well,

$$\begin{aligned}2x^2 - 8x - 5 &= 2[x^2 - 4x] - 5\\&= 2[(x^2 - 4x + 4) - 4] - 5\\&= 2[(x-2)^2 - 4] - 5\\&= 2(x-2)^2 - 8 - 5\\&= 2(x-2)^2 - 13\end{aligned}$$

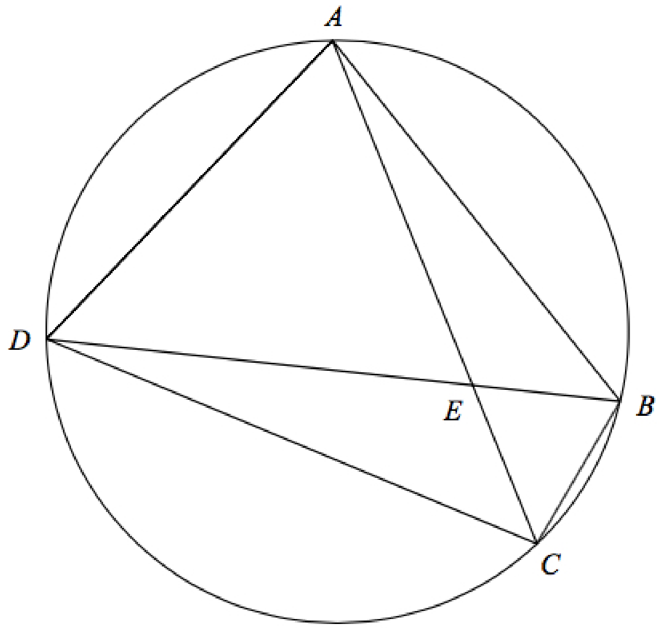
so the turning point is $\underline{\underline{(2, -13)}}$. Now,

$$\begin{aligned}2(x-2)^2 - 13 = 0 &\Rightarrow 2(x-2)^2 = 13\\&\Rightarrow (x-2)^2 = 6.5\\&\Rightarrow x - 2 = \pm\sqrt{6.5}\\&\Rightarrow x = 2 \pm \sqrt{6.5}\end{aligned}$$

so the exact coordinates of any intercepts with the coordinate axes are $\underline{\underline{(2-\sqrt{6.5}, 0)}}$ and $\underline{\underline{(2+\sqrt{6.5}, 0)}}$.



22. A, B, C, and D are four points on a circle. (4)



AEC and DEB are straight lines.
Triangle AED is an equilateral triangle.

Prove that triangle ABC is congruent to triangle DCB.

Solution

BC is common to both triangles.
$\angle CAB = \angle CDB$ (angles in the same segment are equal)
$\angle ACB = \angle ADB$ (angles in the same segment are equal)
So, the triangles ABC and DCB are congruent (AAS).

Dr Oliver Mathematics