Dr Oliver Mathematics
AQA Further Maths Level 2
June 2022 Paper 2
1 hour 45 minutes

The total number of marks available is 80.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.

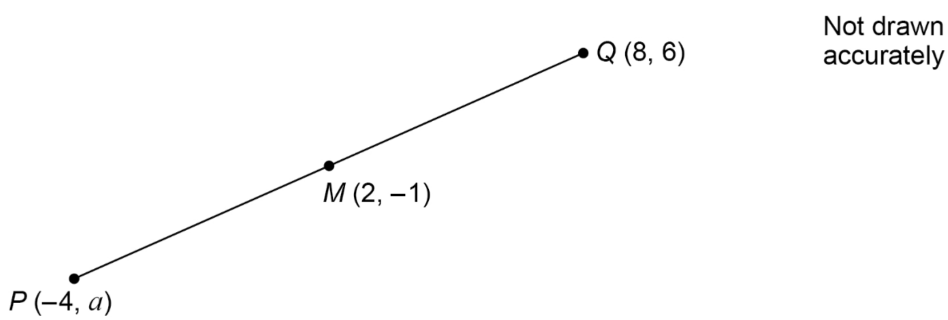
1. Factorise fully (2)

$$12w + 18w^2.$$

Solution

$$12w + 18w^2 = \underline{\underline{6w(2 + 3w)}}.$$

2. M is the midpoint of PQ. (2)



Work out the value of a.

Solution

Well,

$$\begin{aligned}\overrightarrow{OP} &= \overrightarrow{OQ} + \overrightarrow{QP} \\ &= \overrightarrow{OQ} + 2\overrightarrow{QM} \\ &= \begin{pmatrix} 8 \\ 6 \end{pmatrix} + 2\begin{pmatrix} 2-8 \\ -1-6 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ 6 \end{pmatrix} + 2\begin{pmatrix} -6 \\ -7 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ 6 \end{pmatrix} + \begin{pmatrix} -12 \\ -14 \end{pmatrix} \\ &= \begin{pmatrix} -4 \\ -8 \end{pmatrix};\end{aligned}$$

hence, $\underline{\underline{a = -8}}$.

3. (a) Work out (3)

$$3\begin{pmatrix} 4 & 2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ -1 & 5 \end{pmatrix}.$$

Give your answer as a single matrix.

Solution

$$\begin{aligned}3\begin{pmatrix} 4 & 2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ -1 & 5 \end{pmatrix} &= 3\begin{pmatrix} 6 & 10 \\ 2 & 0 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} 18 & 30 \\ 6 & 0 \end{pmatrix}}}.\end{aligned}$$

(b) (3)

$$\begin{pmatrix} 7 & a^2 \\ b & -5 \end{pmatrix}\begin{pmatrix} 2 \\ a \end{pmatrix} = \begin{pmatrix} 78 \\ 12 \end{pmatrix}.$$

Work out the values of a and b.

Solution

$$\begin{pmatrix} 7 & a^2 \\ b & -5 \end{pmatrix}\begin{pmatrix} 2 \\ a \end{pmatrix} = \begin{pmatrix} 78 \\ 12 \end{pmatrix}$$
$$\Rightarrow \quad \begin{pmatrix} 14 + a^3 \\ 2b - 5a \end{pmatrix} = \begin{pmatrix} 78 \\ 12 \end{pmatrix}.$$

Now,

$$14 + a^3 = 78 \Rightarrow a^3 = 64$$
$$\Rightarrow \underline{\underline{a = 4}}$$

and

$$2b - 5(4) = 12 \Rightarrow 2b - 20 = 12$$
$$\Rightarrow 2b = 32$$
$$\Rightarrow \underline{\underline{b = 16}}.$$

4. Line A has equation (4)

$$y + 4x = 6.$$

Line B is parallel to line A and passes through the point $(2, 1)$.

The point $(d, 2d)$ lies on line B.

Work out the value of d.

Solution

Well, the equation of line B is

$$y + 4x = c,$$

for some constant c. Now,

$$(1) + 4(2) = c \Rightarrow c = 9;$$

so,

$$y + 4x = 9.$$

Finally,

$$x = d, y = 2d \Rightarrow 2d + 4(d) = 9$$
$$\Rightarrow 6d = 9$$
$$\Rightarrow \underline{\underline{d = 1\tfrac{1}{2}}}.$$

5. Work out all the **negative** integer values of x for which (3)

$$3x^2 < 48.$$

Solution

Now,

$$3x^2 < 48 \Rightarrow 3x^2 - 48 < 0$$
$$\Rightarrow 3(x^2 - 16) < 0$$

$$\left.\begin{array}{ll}\text{add to:} & 0 \\ \text{multiply to:} & -16\end{array}\right\} -4, +4$$

$$\Rightarrow 3(x + 4)(x - 4) < 0.$$

We need a 'table of signs':

	$x < -4$	$x = -4$	$-4 < x < 4$	$x = 4$	$x > 4$
$x + 4$	$-$	0	$+$	$+$	$+$
$x - 4$	$-$	$-$	$-$	0	$+$
$(x + 4)(x - 4)$	$+$	0	$-$	0	$+$

As we interested in all the negative integer values of x, we can say

$$\underline{\underline{x = -3,\ x = -2,\ \text{or } x = -1}}.$$

6. Prove algebraically that, when n is an integer, (3)

$$\frac{(2n + 1)^2 - (2n + 1)^2}{4}$$

is always even.

Solution

Well,

$\times$	$2n$	± 1
$2n$	$4n^2$	$\pm 2n$
± 1	$\pm 2n$	$+1$

so

$$\begin{aligned}\frac{(2n+1)^2-(2n+1)^2}{4} &= \frac{(4n^2+4n+1)-(4n^2-4n+1)}{4}\\ &= \frac{8n}{4}\\ &= 2n;\end{aligned}$$

hence, when n is an integer the expression is always even.

7. How many integers between 200 000 and 400 000 can be formed using only the digits (2)

$$1 \quad 2 \quad 3 \quad 5 \quad 8 \quad 9$$

with no repetition of any digit?

Solution

Clearly, there are two digits (2 or 3) for the first digit. Now, with no repetition of any digit, there are

$$2 \times 5! = \underline{\underline{240 \text{ ways}}}.$$

8. A curve has equation

$$y = x^3 - 5x^2.$$

At two points on the curve, the rate of change of y with respect to x is 4.

(a) Work out an equation, in terms of x, to represent this information. (2)

Give your answer in the form

$$ax^2 + bx + c = 0,$$

where a, b, and c are integers.

Solution

$$y = x^3 - 5x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 10x$$

and

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = 4 &\Rightarrow 3x^2 - 10x = 4 \\ &\Rightarrow \underline{\underline{3x^2 - 10x - 4 = 0}};\end{aligned}$$

hence, $\underline{\underline{a = 3}}$, $\underline{\underline{b = -10}}$, and $\underline{\underline{c = -4}}$.

(b) Hence, work out the two possible values of x. (2)
Give your answers to 3 significant figures.

Solution
Quadratic formula:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{10 \pm \sqrt{10^2 - 4 \times 3 \times (-4)}}{2 \times 3} \\ &= \frac{10 \pm \sqrt{148}}{6} \\ &= -0.360\,920\,843,\ 3,\ 3.694\,254\,177 \text{ (FCD)} \\ &= \underline{\underline{-0.361,\ 3.69 \text{ (3 sf)}}}.\end{aligned}$$

9. The first three terms of a linear sequence are

$$30 \quad 30 + 4k \quad 30 + 8k,$$

where k is a constant.

(a) Work out an expression, in terms of k, for the 4th term. (1)
Give your answer in its simplest form.

Solution
The common difference is

$$(30 + 4k) - 30 = 4k$$

and, hence, the 4th term is

$$(30 + 8k) + 4k = \underline{\underline{30 + 12k}}.$$

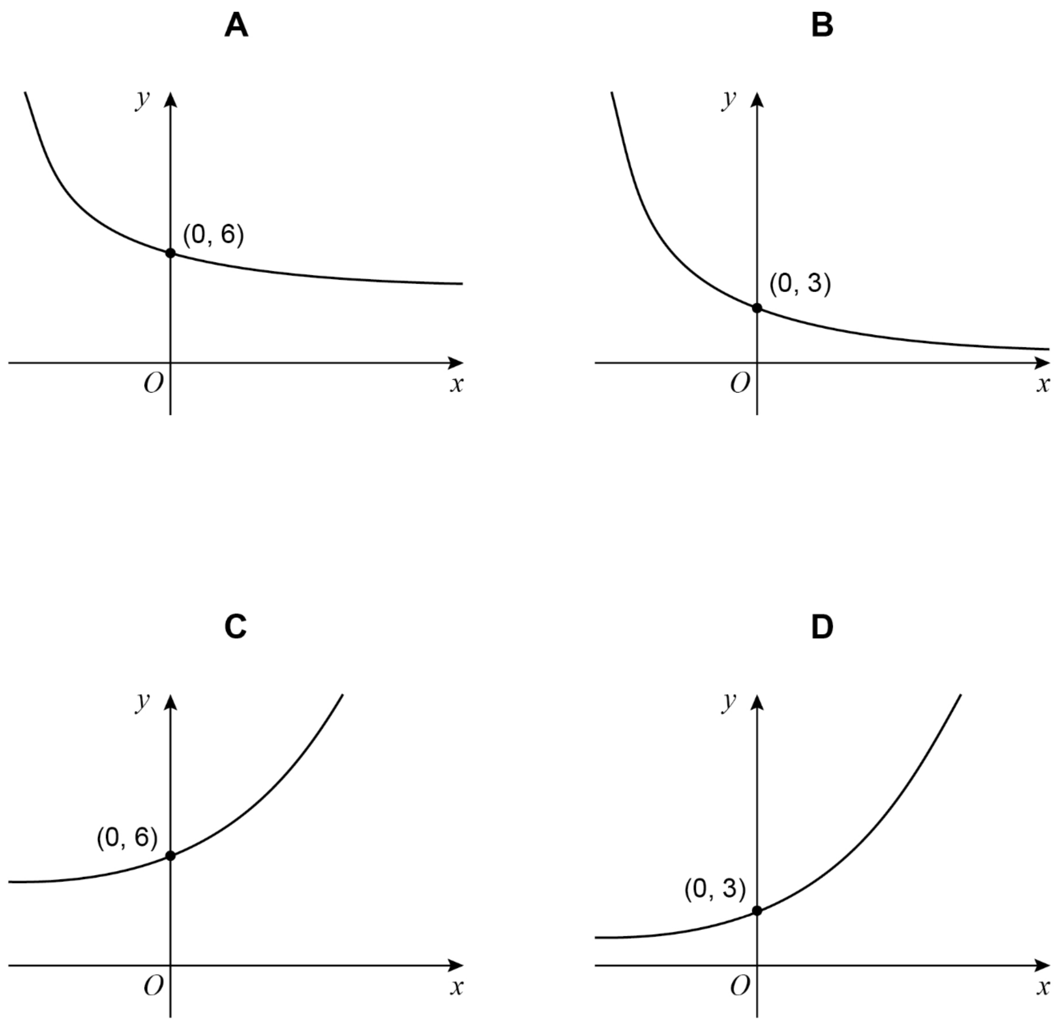
(b) The 100th term of the sequence is 525. (3)

Work out the value of k.

Solution

$$\begin{aligned} 30 + 4k(99) = 525 &\Rightarrow 30 + 396k = 525 \\ &\Rightarrow 396k = 495 \\ &\Rightarrow \underline{\underline{k = 1\tfrac{1}{4}}}. \end{aligned}$$

10. Here are four sketch graphs. (1)



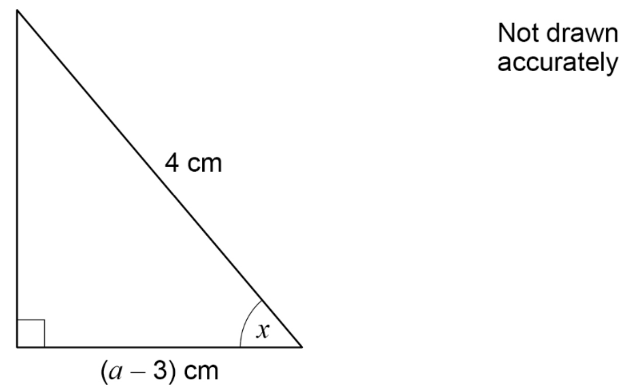
Circle the letter of the sketch graph that represents

$$y = 3 \times 2^x.$$

Solution

$\underline{\underline{D}}$.

11. Here is a right-angled triangle. (2)



You are given that $a > 5$

Use trigonometry to work out the range of values of x.

Solution

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos x = \frac{5-3}{4}$$
$$\Rightarrow \cos x = \tfrac{1}{2}$$
$$\Rightarrow x = 60;$$

as the triangle is right-angled,

$$\underline{\underline{0 < x < 60}}.$$

12. Work out the gradient of the curve (5)

$$y = \frac{12x^3 - 8x + 3}{4x^2}$$

at the point where $x = -1$.

You **must** show your working.

Solution

$$y = \frac{12x^3 - 8x + 3}{4x^2} \Rightarrow y = 3x - 2x^{-1} + \tfrac{3}{4}x^{-2}$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3 + 2x^{-2} - \tfrac{3}{2}x^{-3}.$$

Finally,

$$\begin{aligned} x = -1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 3 + 2 + \tfrac{3}{2} \\ \Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 6\tfrac{1}{2}}}. \end{aligned}$$

13. $A(-2, 5)$ and $B(4, 13)$ are points on a circle. (3)
AB is a diameter.

Work out the equation of the circle.
Give your answer in the form

$$(x - a)^2 + (y - b)^2 = c,$$

where a, b, and c are integers.

Solution

The circle's midpoint is at

$$\left(\frac{-2 + 4}{2}, \frac{5 + 13}{2}\right) = (1, 9);$$

we will call this point C. Now,

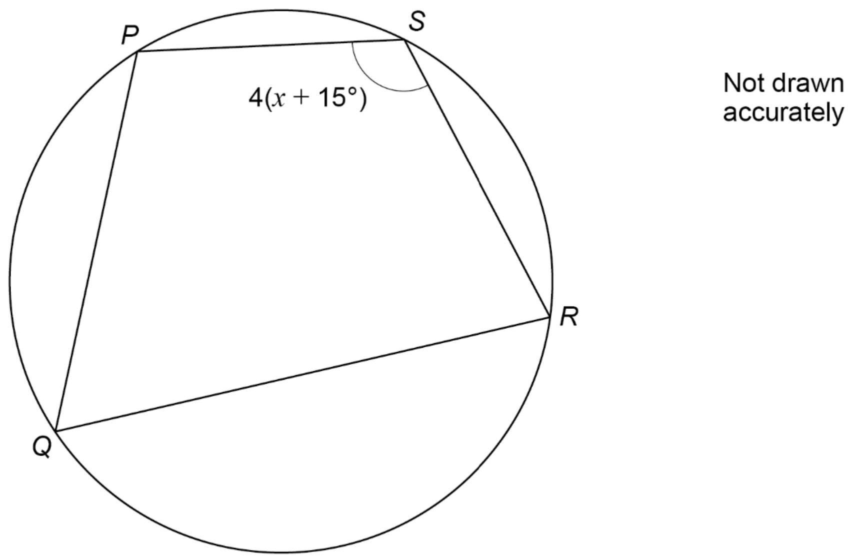
$$\begin{aligned} BC^2 &= (13 - 9)^2 + (4 - 1)^2 \\ &= 4^2 + 3^2 \\ &= 25. \end{aligned}$$

Finally, the equation of the circle is

$$\underline{\underline{(x - 1)^2 + (y - 9)^2 = 25}};$$

hence, $\underline{\underline{a = 1}}$, $\underline{\underline{b = 9}}$, and $\underline{\underline{c = 25}}$.

14. $PQRS$ is a cyclic quadrilateral. (3)



- Angle $PSR = 4(x+15)^\circ$.
- Angle PQR is 40° smaller than angle PSR.

Work out the value of x.

Solution

Opposite angles in a cyclic quadrilateral add up to 180°:

$$\begin{aligned} 4(x+15) + 4(x+15) - 40 = 180 &\Rightarrow 8(x+15) = 220 \\ &\Rightarrow x + 15 = 27\tfrac{1}{2} \\ &\Rightarrow \underline{\underline{x = 12\tfrac{1}{2}}}. \end{aligned}$$

15. Simplify fully (5)

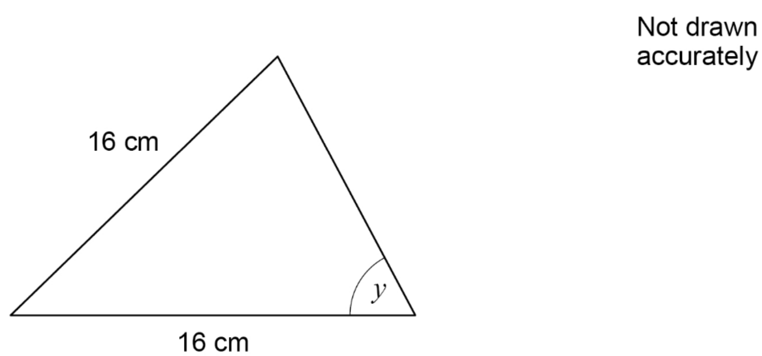
$$\left(\tfrac{1}{2}x + \tfrac{3}{5}x\right) \div \sqrt{\frac{x^6}{4}}.$$

Solution

Well,

$$\left(\tfrac{1}{2}x + \tfrac{3}{5}x\right) \div \sqrt{\frac{x^6}{4}} = \frac{\frac{11}{10}x}{\frac{1}{2}x^3} = \underline{\underline{\tfrac{11}{5}x^{-2}}}.$$

16. Here is an isosceles triangle. (4)
All the angles are acute.



The area of the triangle is 120 cm^2.

Work out the size of angle y.

Solution

Let the top angle be y (isosceles triangle) and then the left angle is $(180 - 2y)$.

Now,

$$\begin{aligned}\tfrac{1}{2} \times 16^2 \times \sin(180 - 2y) = 120 &\Rightarrow \sin(180 - 2y) = \tfrac{15}{16} \\ &\Rightarrow 180 - 2y = 69.635\,865\,19 \text{ (FCD)} \\ &\Rightarrow 2y = 110.364\,134\,8 \text{ (FCD)} \\ &\Rightarrow y = 55.182\,067\,4 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{y = 55.2 \text{ (3 sf)}}}.\end{aligned}$$

17. Solve the simultaneous equations (5)

$$\begin{aligned} a + 3b - 2c &= 4 \\ 4a - 3b + 5c &= -5 \\ 2a + b + 3c &= 9. \end{aligned}$$

Do **not** use trial and improvement.
You **must** show your working.

Solution

$$\begin{aligned} a + 3b - 2c &= 4 \quad (1) \\ 4a - 3b + 5c &= -5 \quad (2) \\ 2a + b + 3c &= 9 \quad (3) \end{aligned}$$

Do $4 \times (1)$ and $2 \times (3)$:

$$\begin{aligned} 4a + 12b - 8c &= 16 \quad (4) \\ 4a - 3b + 5c &= -5 \quad (2) \\ 4a + 2b + 6c &= 18 \quad (5) \end{aligned}$$

Do $(4) - (2)$ and $(5) - (2)$:

$$\begin{aligned} 15b - 13c &= 21 \quad (6) \\ 5b + c &= 23 \quad (7) \end{aligned}$$

Do $3 \times (7)$:

$$\begin{aligned} 15b - 13c &= 21 \quad (6) \\ 15b + 3c &= 69 \quad (8) \end{aligned}$$

Do $(8) - (7)$:

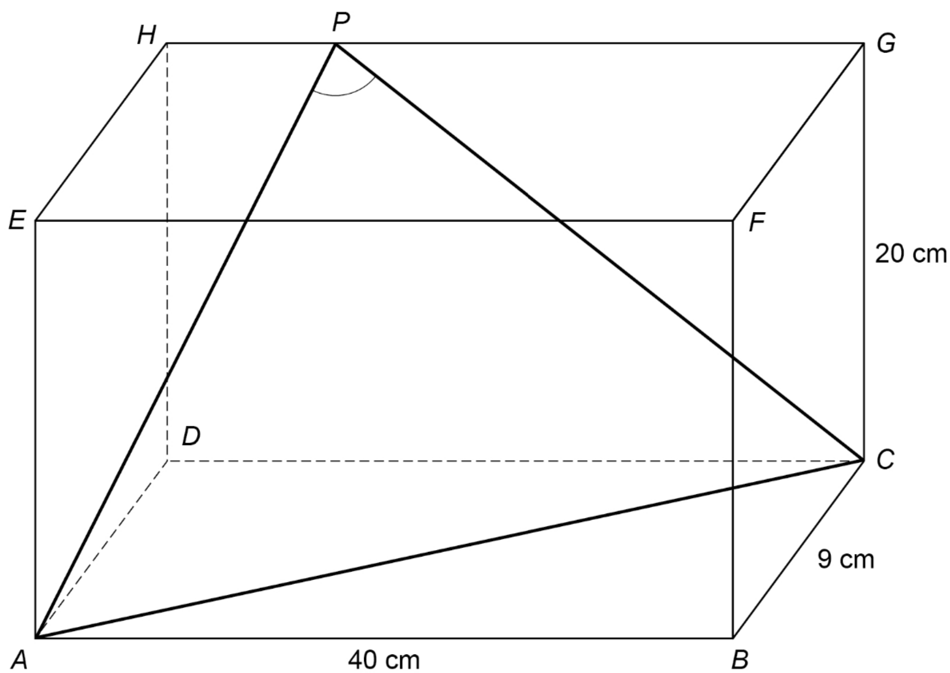
$$\begin{aligned} 16c = 48 &\Rightarrow \underline{\underline{c = 3}} \\ &\Rightarrow 5b + 3 = 23 \\ &\Rightarrow 5b = 20 \\ &\Rightarrow \underline{\underline{b = 4}} \\ &\Rightarrow a + 3(4) - 2(3) = 4 \\ &\Rightarrow \underline{\underline{a = -2}}. \end{aligned}$$

18. $ABCDEFGH$ is a cuboid. (5)

- $AB = 40$ cm.
- $BC = 9$ cm.
- $CG = 20$ cm.
- P is a point on HG such that

$$HP : PG = 3 : 7.$$

- $AP = 25$ cm.



Work out the size of angle APC.

Solution

Well,

$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{40^2 + 9^2} \\ &= 41. \end{aligned}$$

Next,

$$PG = \tfrac{7}{10} \times 40 = 28$$

and

$$\begin{aligned} CP &= \sqrt{CG^2 + PG^2} \\ &= \sqrt{20^2 + 28^2} \\ &= 4\sqrt{74}. \end{aligned}$$

Finally, we apply the cosine rule:

$$\begin{aligned} & AC^2 = AP^2 + CP^2 - 2 \times AP \times CP \times \cos APC \\ \Rightarrow\ & 41^2 = 25^2 + (4\sqrt{74})^2 - 2 \times 25 \times 4\sqrt{74} \times \cos APC \\ \Rightarrow\ & 200\sqrt{74}\cos APC = 128 \\ \Rightarrow\ & \cos APC = 0.074\,398\,488\,8 \text{ (FCD)} \\ \Rightarrow\ & \angle APC = 85.733\,338\,31 \text{ (FCD)} \\ \Rightarrow\ & \underline{\underline{\angle APC = 85.7^\circ \text{ (3 sf)}}}. \end{aligned}$$

19. Expand and simplify fully (3)

$$(3x + 4)(2x - 3)(5x - 2).$$

Solution

$\times$	$3x$	$+4$
$2x$	$6x^2$	$+8x$
-3	$-9x$	-12

So

$$(3x + 4)(2x - 3) = 6x^2 - x - 12.$$

$\times$	$6x^2$	$-x$	-12
$5x$	$30x^3$	$-5x^2$	$-60x$
-2	$-12x^2$	$+2x$	$+24$

Hence,

$$(3x+4)(2x-3)(5x-2) = \underline{\underline{30x^3 - 17x^2 - 58x + 24}}.$$

20.

$$\mathrm{f}(x) = 2x^3 + 11x^2 + 12x - 9.$$

(a) Use the factor theorem to show that $(2x-1)$ is a factor of $\mathrm{f}(x)$. (2)

Solution

Well,

$$\mathrm{f}(\tfrac{1}{2}) = \tfrac{1}{4} + \tfrac{11}{4} + 6 - 9 = 0;$$

so, $(2x-1)$ is a $\underline{\underline{\text{factor}}}$ of $\mathrm{f}(x)$

(b) Show that $\mathrm{f}(x) = 0$ has **exactly two** solutions. (4)

Solution

We use synthetic division:

$\frac{1}{2}$	2	11	12	−9
	↓	1	6	−9
	2	12	18	0

So

$$\begin{aligned}\mathrm{f}(x) &= (2x-1)(2x^2+12x+18)\\ &= 2(2x-1)(x^2+6x+9)\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & +6\\ \text{multiply to:} & +9\end{array}\right\} +3 \text{ (repeated)}$$

$$= 2(2x-1)(x+3)^2;$$

thus, $\mathrm{f}(x)$ has roots of

$$\underline{\underline{\tfrac{1}{2} \text{ and } -3 \text{ (repeated)}}}.$$

21. Work out the values of x between 0° and 360° for which (4)

$$2\tan^2 x = 3.$$

Give your answers to 1 decimal place.
You **must** show your working.

Solution

$$\begin{aligned} 2\tan^2 x = 3 &\Rightarrow \tan^2 x = \tfrac{3}{2} \\ &\Rightarrow \tan x = \pm\frac{\sqrt{6}}{2}. \end{aligned}$$

$\underline{\tan x = \frac{\sqrt{6}}{2}}$:

$$\begin{aligned} \tan x = \frac{\sqrt{6}}{2} &\Rightarrow x = 50.764\,479\,52,\ 230.764\,479\,52 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 50.8,\ 230.8 \text{ (3 sf)}}}. \end{aligned}$$

$\underline{\tan x = -\frac{\sqrt{6}}{2}}$:

$$\begin{aligned} \tan x = -\frac{\sqrt{6}}{2} &\Rightarrow x = 129.231\,520\,5,\ 309.231\,520\,5 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 129.2,\ 309.2 \text{ (3 sf)}}}. \end{aligned}$$

22. Using powers of 2 or otherwise, work out the non-zero value of x for which (4)

$$(16^x)^x = \frac{1}{2^{3x}}.$$

You **must** show your working.

Solution

Well,

$$\begin{aligned} (16^x)^x = \frac{1}{2^{3x}} &\Rightarrow [(2^4)^x)]^x = 2^{-3x} \\ &\Rightarrow 2^{4x^2} = 2^{-3x} \\ &\Rightarrow 4x^2 = -3x \\ &\Rightarrow 4x^2 + 3x = 0 \\ &\Rightarrow x(4x+3) = 0 \\ &\Rightarrow x = -\tfrac{3}{4} \text{ or } x = 0; \end{aligned}$$

as we are asked the non-zero value,

$$\underline{\underline{x = -\tfrac{3}{4}}}.$$