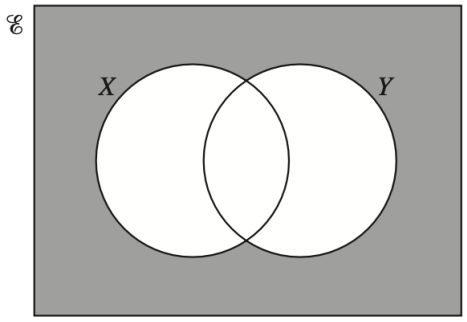
# Dr Oliver Mathematics
# Cambridge O Level Additional Mathematics
# 2008 June Paper 2: Calculator
# 2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. (a) Express, in set notation, the set represented by the shaded region. (1)




(b) In a class of 30 students, 17 are studying politics, 14 are studying economics, and 10 are studying both of these subjects.

(i) Illustrate this information using a Venn diagram. (1)

Find the number of students studying

(ii) neither of these subjects, (1)

(iii) exactly one of these subjects. (1)

2. Given that (4)

$$\mathbf{A} = \begin{pmatrix} 7 & 6 \\ 3 & 4 \end{pmatrix},$$

find $\mathbf{A}^{-1}$ and hence solve the simultaneous equations

$$7x + 6y = 17$$
$$3x + 4y = 3.$$

3. Sketch the graph of (4)

$$y = |x^2 - 8x + 12|.$$

4. Find the coefficient of $x^4$ in the expansion of

   (a) $(1 + 2x)^6$, (2)

   (b) $(1 - \frac{1}{4}x)(1 + 2x)^6$. (3)

5. Two variables, $x$ and $y$, are related by the equation

$$y = 6x^2 + \frac{32}{x^3}.$$

   (a) Obtain an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. (2)

   (b) Use your expression to find the approximate change in the value of $y$ when $x$ increases from 2 to 2.04. (3)

6. The function f is defined by

$$\mathrm{f}(x) = 2 + \sqrt{x - 3}, \text{ for } x \geqslant 3.$$

   Find

   (a) the range of f, (1)

   (b) an expression for $\mathrm{f}^{-1}(x)$. (2)

   The function g is defined by

$$\mathrm{g}(x) = \frac{12}{x} + 2, \text{ for } x > 0.$$

   Find

   (c) $\mathrm{g}\,\mathrm{f}(12)$. (2)

7. Given that

$$\log_p X = 9 \text{ and } \log_p Y = 6,$$

   find

   (a) $\log_p \sqrt{X}$, (1)

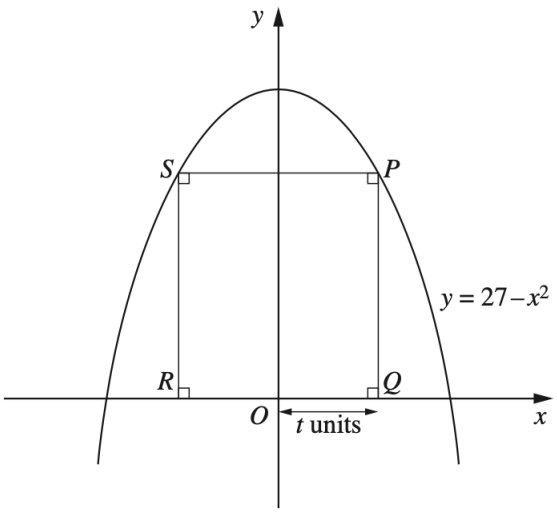
   (b) $\log_p \left(\dfrac{1}{X}\right)$, (1)

   (c) $\log_p(XY)$, (2)

   (d) $\log_Y X$ (2)

8. The diagram shows part of the curve

$$y = 27 - x^2.$$




- The points $P$ and $S$ lie on this curve.
- The points $Q$ and $R$ lie on the $x$-axis and $PQRS$ is a rectangle.
- The length of $OQ$ is $t$ units.

(a) Find the length of $PQ$ in terms of $t$ and hence show that the area, $A$ square units, of $PQRS$ is given by (2)

$$A = 54t - 2t^3.$$

(b) Given that $t$ can vary, find the value of $t$ for which $A$ has a stationary value. (3)

(c) Find this stationary value of $A$ and determine its nature. (3)

9. A musician has to play 4 pieces from a list of 9.
Of these 9 pieces,

- 4 were written by Beethoven,
- 3 by Handel, and
- 2 by Sibelius.

Calculate the number of ways the 4 pieces can be chosen if

(a) there are no restrictions, (2)

(b) there must be 2 pieces by Beethoven, 1 by Handel, and 1 by Sibelius, (3)

(c) there must be at least one piece by each composer. (4)

10. The line (9)

$$2x + y = 12$$

intersects the curve

$$x^2 + 3xy + y^2 = 176$$

at the points $A$ and $B$.

Find the equation of the perpendicular bisector of $AB$.

11. (a) Find all the angles between $0^\circ$ and $360^\circ$ which satisfy

(i) $2\sin x - 3\cos x = 0$, (3)

(ii) $2\sin^2 y - 3\cos y = 0$. (5)

(b) Given that $0 \leqslant z \leqslant 3$ radians, find, correct to 2 decimal places, all the values of $z$ for which (3)

$$\sin(2z + 1) = 0.9.$$

**EITHER**

12. The point $P(0, 5)$ lies on the curve for which

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{\frac{1}{2}x}.$$

The point $Q$, with $x$-coordinate 2, also lies on the curve.

(a) Find, in terms of e, the $y$-coordinate of $Q$. (5)

The tangents to the curve at the points $P$ and $Q$ intersect at the point $R$.

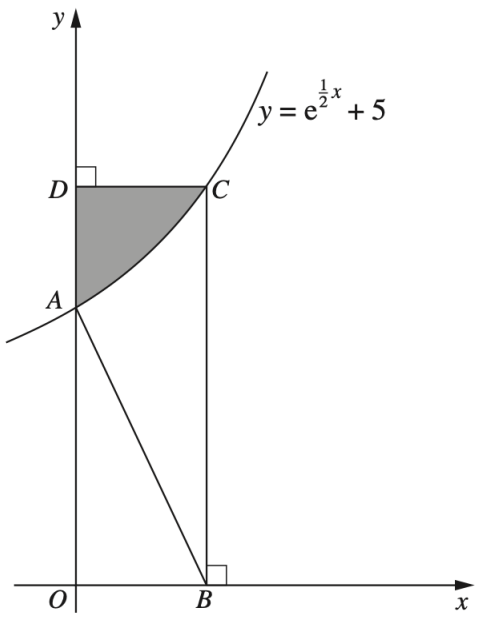
(b) Find, in terms of e, the $x$-coordinate of $R$. (5)

**OR**

13. The diagram shows part of the curve

$$y = \mathrm{e}^{\frac{1}{2}x} + 5$$

crossing the $y$-axis at $A$.




The normal to the curve at $A$ meets the $x$-axis at $B$.

(a) Find the coordinates of $B$. (4)

The line through $B$, parallel to the $y$-axis, meets the curve at $C$.

The line through $C$, parallel to the $x$-axis, meets the $y$-axis at $D$.

(b) Find the area of the shaded region. (6)