Dr Oliver Mathematics
GCSE Mathematics
2020 Paper 3H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Simplify (1)

$$n^3 \times n^5.$$

Solution

$$n^3 \times n^5 = \underline{\underline{n^8}}.$$

(b) Simplify (2)

$$\frac{c^3 d^4}{c^2 d}.$$

Solution

$$\frac{c^3 d^4}{c^2 d} = \frac{\cancel{c^3}\cancel{d^4}\,{}^3}{\cancel{c^2}\cancel{d}} = \underline{\underline{cd^3}}.$$

(c) Solve (2)

$$\frac{5x}{2} > 7.$$

Solution

$$\frac{5x}{2} > 7 \Rightarrow x > \tfrac{2}{5} \times 7 \Rightarrow \underline{\underline{x > 2\tfrac{4}{5}}}.$$

2. Andy cycles a distance of 30 km at an average speed of 24 km/h. (3)
He then runs a distance of 12 km at an average speed of 8 km/h.

Work out the total time Andy takes.
Give your answer in hours and minutes.

Solution

Cycle:

$$\frac{30}{24} = 1.25 \text{ hours.}$$

Runs:

$$\frac{12}{8} = 1.5 \text{ hours.}$$

He cycles/runs

$$\begin{aligned} 1.25 + 1.5 &= 2.75 \text{ hours} \\ &= \underline{\underline{2 \text{ hours } 45 \text{ minutes}}}. \end{aligned}$$

3. A number, m, is rounded to 1 decimal place. The result is 9.4. (2)

Complete the error interval for m.

Solution

$$\underline{\underline{9.35 \leqslant m < 9.45}}.$$

4. Maisie knows that she needs 3 kg of grass seed to make a rectangular lawn 5 m by 9 m.

Grass seed is sold in 2 kg boxes.

Maisie wants to make a rectangular lawn 10 m by 14 m.
She has 5 boxes of grass seed.

(a) Has Maisie got enough grass seed to make a lawn 10 m by 14 m? (4)
You must show all your working.

Solution

She needs

$$\frac{5 \times 9}{3} = 15 \text{ m}^2/\text{kg}$$

to 1 m^2. Well,

$$\frac{10 \times 14}{15} = 9\tfrac{1}{3}$$

and she has

$$5 \times 2 = 10$$

boxes of grass seed. Hence, Maisie has got enough grass seed.

Maisie opens the 5 boxes of grass seed.

She finds that 4 of the boxes contain 2 kg of grass seed.
The other box contains 1 kg of grass seed.

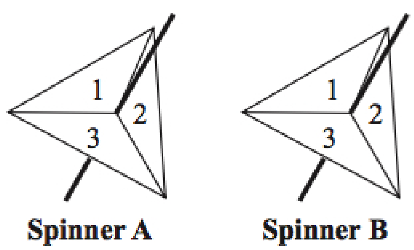
(b) Does this affect whether Maisie has enough grass seed to make her lawn? (1)
Give a reason for your answer.

Solution

$$4 \times 2 + 1 = 9;$$

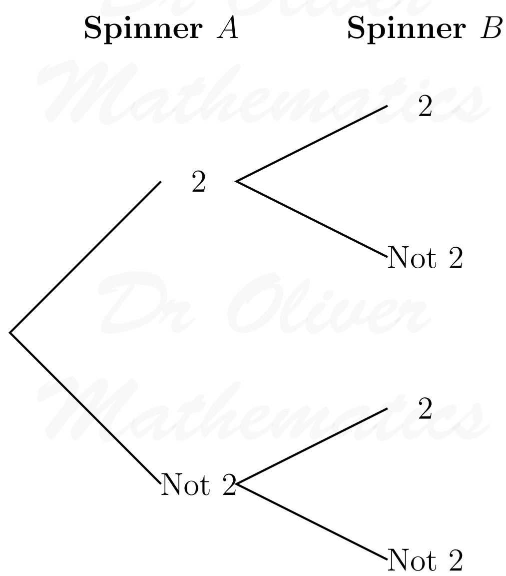
hence, Maisie does not have enough grass seed.

5. Amanda has two fair 3-sided spinners.

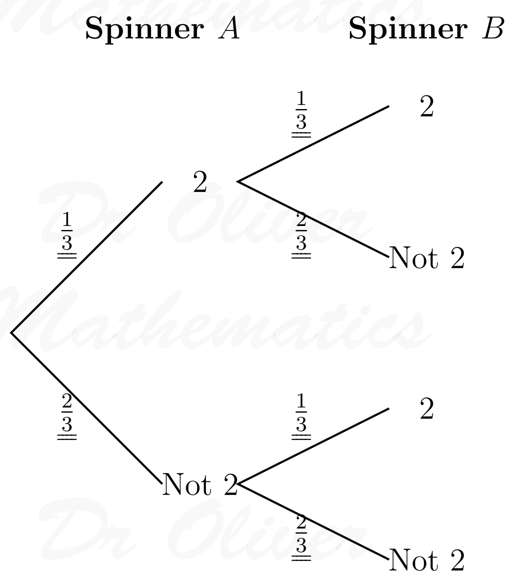


Amanda spins each spinner once.

(a) Complete the probability tree diagram. (2)



Solution

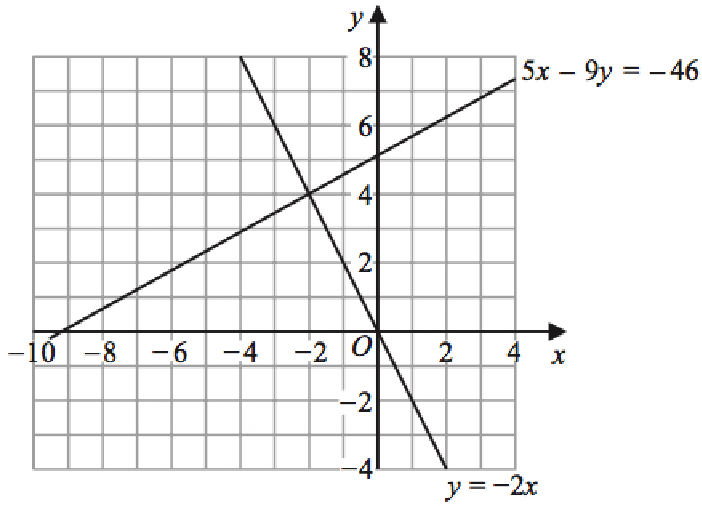


(b) Work out the probability that Spinner A lands on 2 and Spinner B does **not** land on 2. (2)

Solution

$$\begin{aligned} \mathrm{P}(2, \text{not } 2) &= \tfrac{1}{3} \times \tfrac{2}{3} \\ &= \underline{\underline{\tfrac{2}{9}}}. \end{aligned}$$

6. Here is a graph.



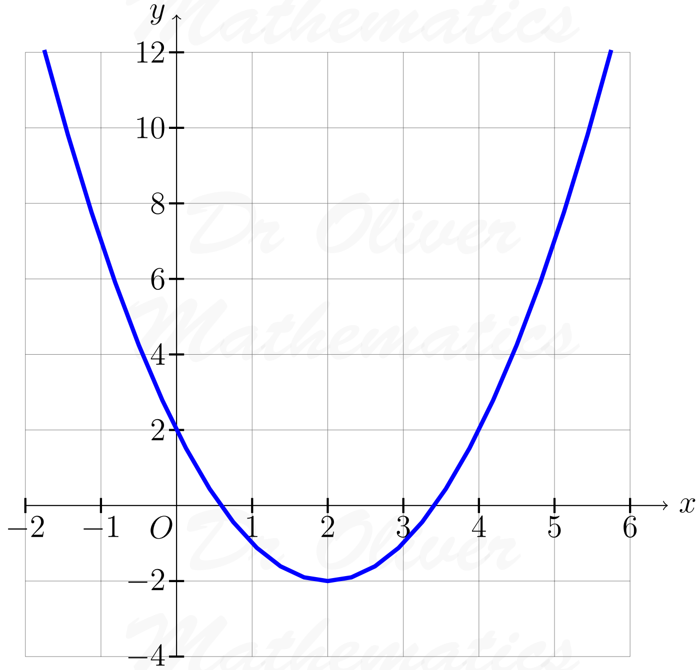
(a) Use these graphs to solve the simultaneous equations (1)

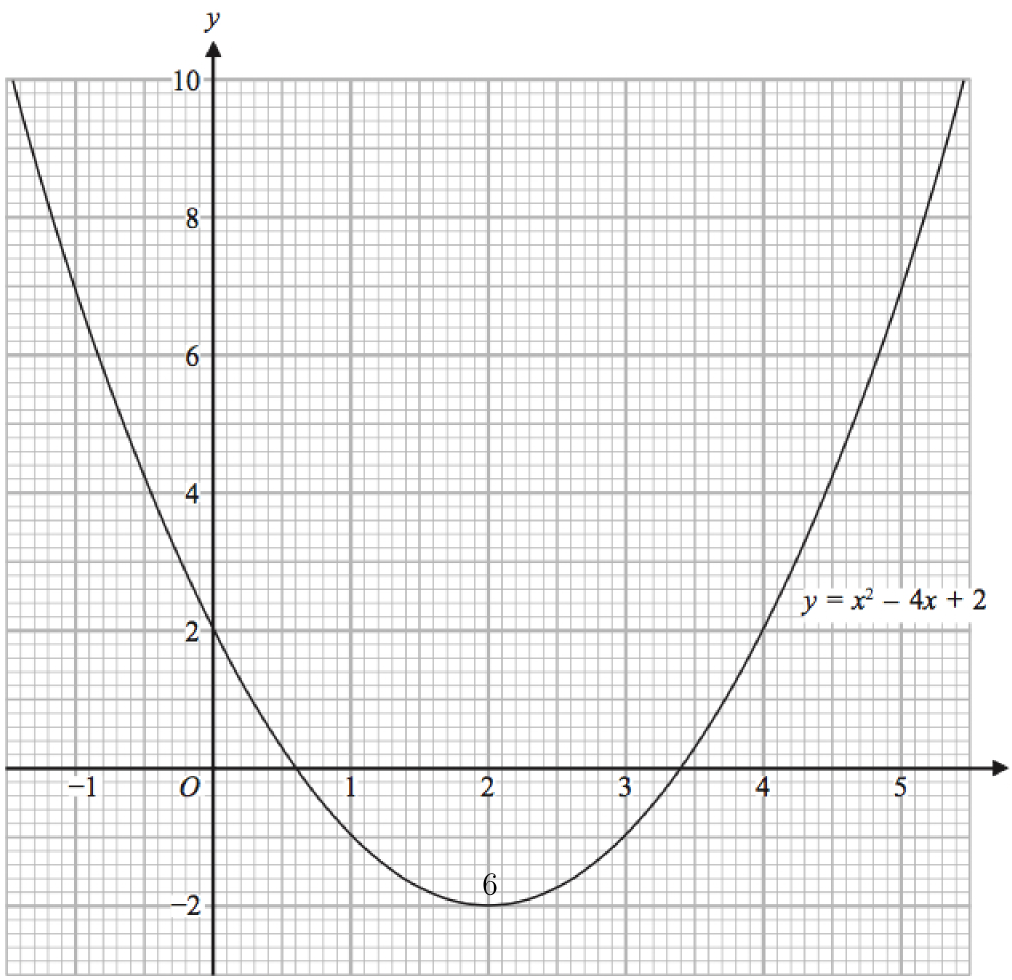
$$5x - 9y = -46y \qquad\qquad = -2x.$$

Solution

$\underline{\underline{x = -2,\ y = 4}}$.

Here is another graph.





(b) Use this graph to find estimates for the solutions of the quadratic equation (2)

$$x^2 - 4x + 2 = 0.$$

Solution

$$\underline{\underline{x = 0.6 \text{ or } x = 3.4}}.$$

7. There is a total of 45 boys and girls in a choir. (3)

The mean age of the 18 boys is 16.2 years.
The mean age of the 27 girls is 16.7 years.

Calculate the mean age of all 45 boys and girls.

Solution

The total ages of all the boys is

$$18 \times 16.2 = 291.6$$

and the total ages of all the girls is

$$27 \times 16.7 = 450.9.$$

Add them:

$$291.6 + 450.9 = 742.5$$

and divide by 45:

$$\frac{742.5}{45} = \underline{\underline{16.5 \text{ years}}}.$$

8. There are some counters in a bag. (3)
The counters are blue or green or red or yellow.

The table shows the probabilities that a counter taken at random from the bag will be blue or will be green.

Colour	Blue	Green	Red	Yellow
Probability	0.32	0.20		

The probability that a counter taken at random from the bag will be red is five times the probability that the counter will be yellow.

There are 300 counters in the bag.

Work out the number of yellow counters in the bag.

Solution

The probability that a counter is either red or yellow is

$$1 - 0.32 - 0.20 = 0.48.$$

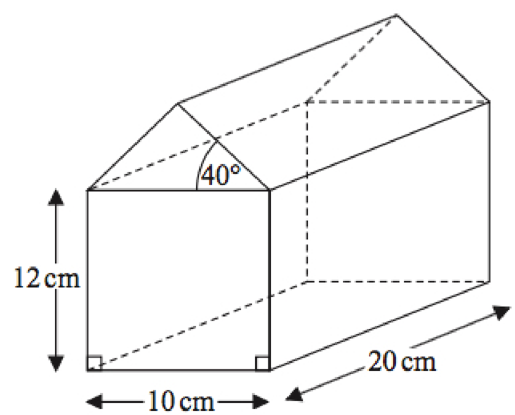
Now, the probability that the counter will be yellow is

$$\tfrac{1}{6} \times 0.48 = 0.08$$

and the number of yellow counters in the bag is

$$0.08 \times 300 = \underline{\underline{24}}.$$

9. The diagram shows a prism. (5)



The cross section of the prism has exactly one line of symmetry.

Work out the volume of the prism.
Give your answer correct to 3 significant figures.

Solution

Let x cm be the height of the triangle. Now,

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan 40^\circ = \frac{x}{5} \\ &\Rightarrow x = 5 \tan 40^\circ. \end{aligned}$$

Hence,

$$\begin{aligned} \text{volume} &= \text{cross-sectional area} \times \text{breadth} \\ &= [(10 \times 12) + (\tfrac{1}{2} \times 10 \times 5 \tan 40^\circ)] \times 20 \\ &= 20(120 + 25 \tan 40^\circ) \\ &= 2\,819\,549\,816 \text{ (FCD)} \\ &= \underline{\underline{2\,820 \text{ cm}^3 \text{ (3 sf)}}}. \end{aligned}$$

10. A person's heart beats approximately 10^5 times each day.
A person lives for approximately 81 years.

(a) Work out an estimate for the number of times a person's heart beats in their lifetime. Give your answer in standard form correct to 2 significant figures. (2)

Solution

$$\begin{aligned} 10^5 \times 365 \times 81 &= 2.956\,5 \times 10^9 \text{ (exact)} \\ &= \underline{\underline{3.0 \times 10^9 \text{ (2 sf)}}}. \end{aligned}$$

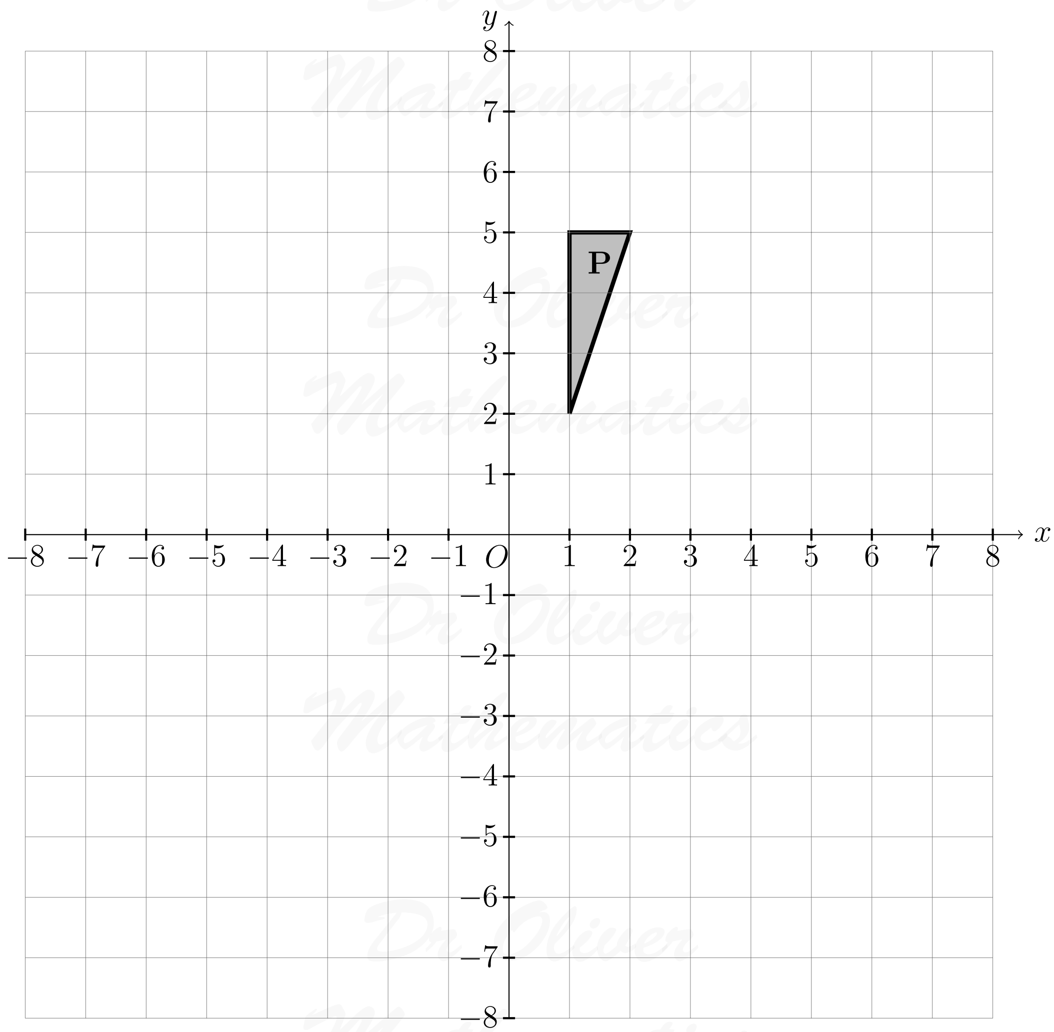
2×10^{12} red blood cells have a total mass of 90 grams.

(b) Work out the average mass of 1 red blood cell. Give your answer in standard form. (2)

Solution

$$\frac{90}{2 \times 10^{12}} = \underline{\underline{4.5 \times 10^{-11} \text{ g}}}.$$

11. The diagram shows a triangle **P** on a grid.



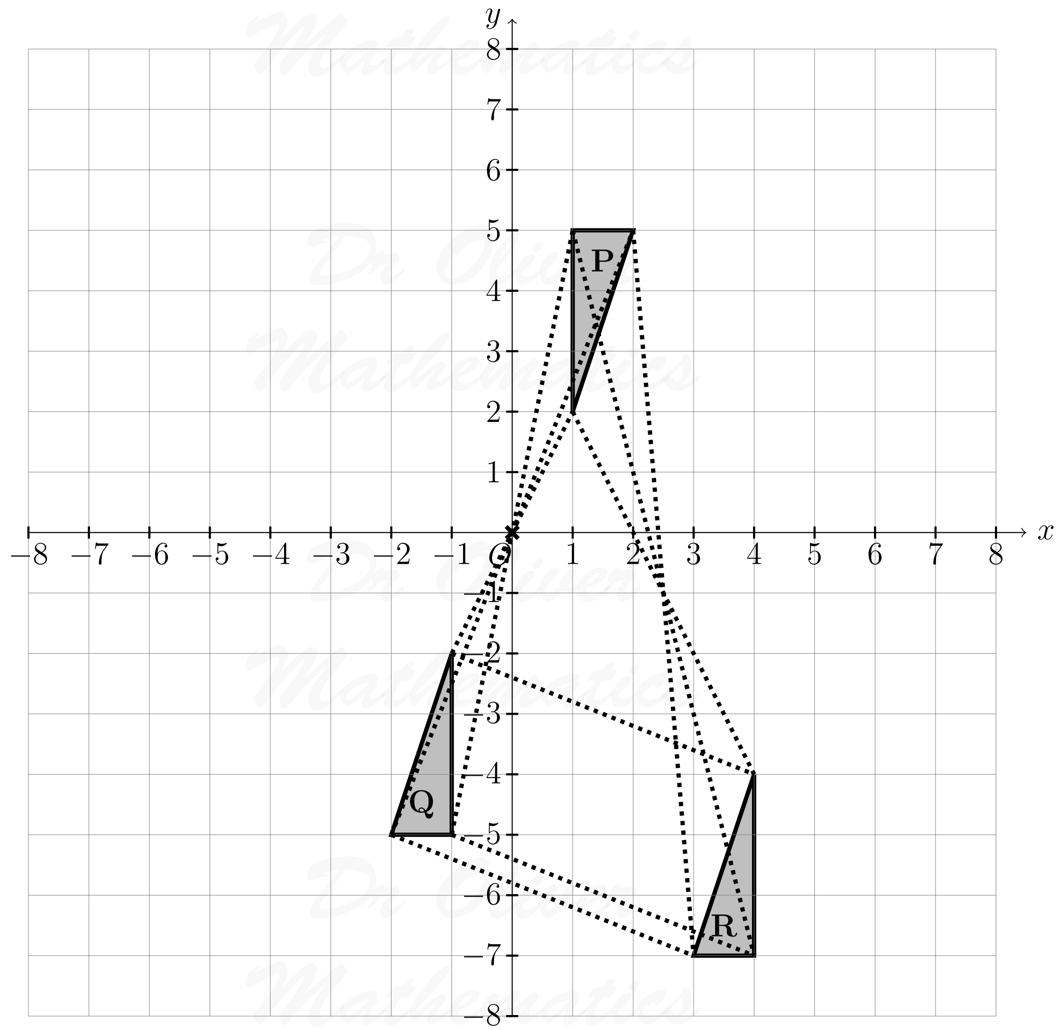
Triangle **P** is rotated 180° about $(0, 0)$ to give triangle **Q**.
Triangle **Q** is translated by

$$\begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

to give triangle **R**.

(a) Describe fully the single transformation that maps triangle **P** onto triangle **R**. (3)

Solution



$\underline{\underline{\text{Rotation}}}$, $\underline{\underline{180^\circ}}$ about $\underline{\underline{(2.5, -1)}}$.

Under the transformation that maps triangle **P** onto triangle **R**, the point A is invariant.

(b) Write down the coordinates of point A. (1)

Solution

$\underline{\underline{A(2.5, -1)}}$.

12. (a) Express (3)

$$\frac{x}{x+2} + \frac{2x}{x-4}$$

as a single fraction in its simplest form.

Solution

$$\begin{aligned}\frac{x}{x+2}+\frac{2x}{x-4} &= \frac{x(x-4)}{(x+2)(x-4)}+\frac{2x(x+2)}{(x+2)(x-4)}\\ &= \frac{x(x-4)+2x(x+2)}{(x+2)(x-4)}\\ &= \frac{x^2-4x+2x^2+4x}{(x+2)(x-4)}\\ &= \underline{\underline{\frac{3x^2}{(x+2)(x-4)}}}.\end{aligned}$$

(b) Expand and simplify (3)

$$(x-3)(2x+3)(4x+5).$$

Solution

$\times$	$2x$	$+3$
x	$2x^2$	$+3x$
-3	$-6x$	-9

So

$$(x-3)(2x+3) = 2x^2-3x-9.$$

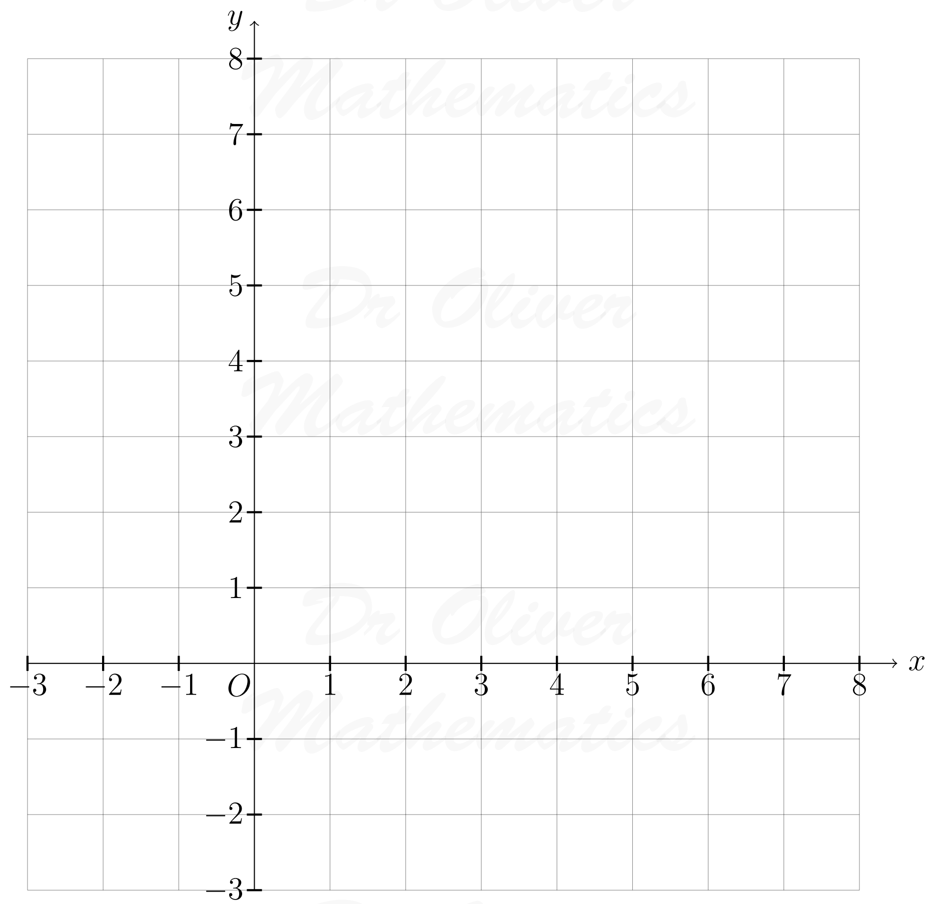
$\times$	$2x^2$	$-3x$	-9
$4x$	$8x^3$	$-12x^2$	$-36x$
$+5$	$+10x^2$	$-15x$	-45

Hence,

$$(x-3)(2x+3)(4x+5) = \underline{\underline{8x^3-2x^2-51x-45}}.$$

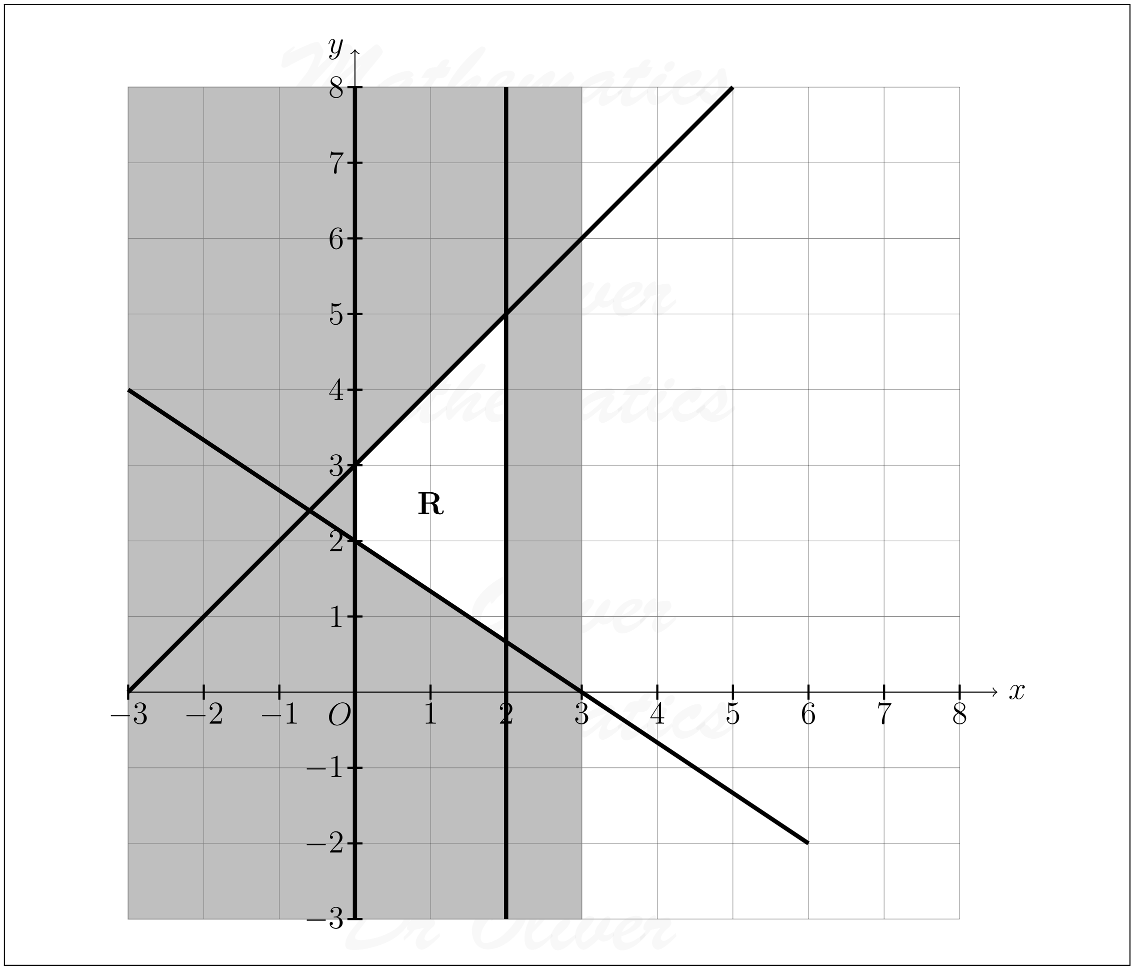
13. (a) On the grid show, by shading, the region that satisfies all these inequalities. (4)

$$x \geqslant 0 \qquad x \leqslant 2 \qquad y \leqslant x+3 \qquad 2x+3y \geqslant 6.$$



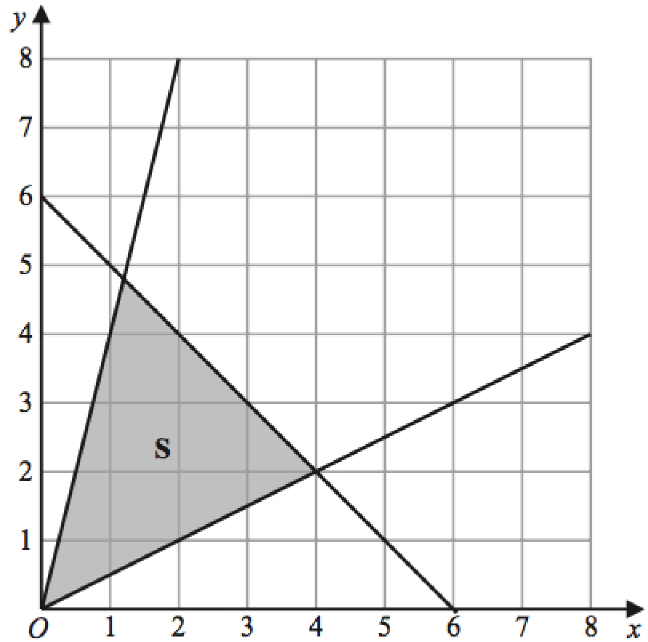
Label the region **R**.

Solution



(b) The diagram below shows the region **S** that satisfies the inequalities: (1)

$$y \leqslant 4x \qquad y \geqslant \tfrac{1}{2}x \qquad x + y \leqslant 6.$$

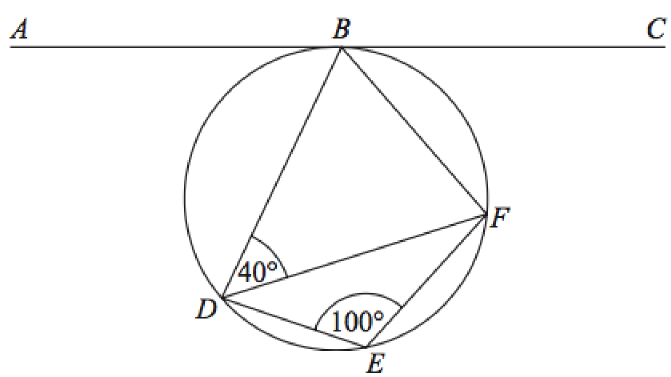


Geoffrey says that the point with coordinates $(2, 4)$ does not satisfy all the inequalities because it does not lie in the shaded region.

Is Geoffrey correct?
You must give a reason for your answer.

Solution
No: $(2, 4)$ satisfies $y \leqslant 4x$, $y \geqslant \frac{1}{2}x$, and $x + y \leqslant 6$ and lies on the edge of **S**.

14. Points B, D, E, and F lie on a circle. (4)
ABC is the tangent to the circle at B.



Find the size of angle ABD.
You must give a reason for each stage of your working.

Solution

$\angle DBF = 180 - 100 = 80^\circ$ (cyclic quadrilateral)
$\angle BFD = 180 - 40 - 80 = 60^\circ$ (angles in a triangle)
$\angle ABD = \underline{\underline{60^\circ}}$ (alternate segment theorem)

15. Prove algebraically that $0.7\dot{3}$ can be written as $\frac{11}{15}$. (2)

Solution

Let $x = 0.7\dot{3}$. Then

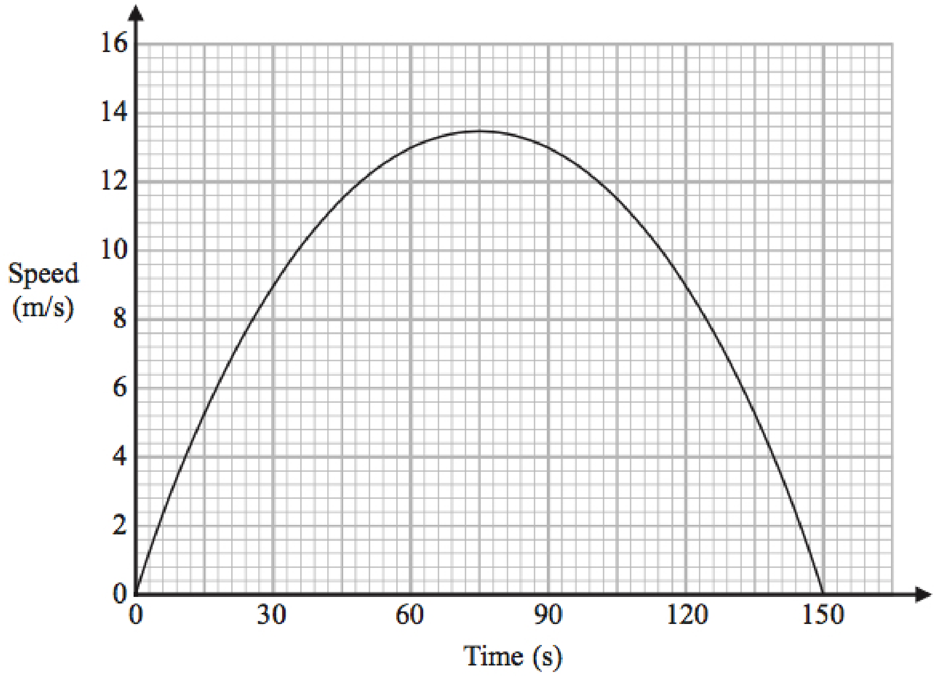
$$10x = 7.\dot{3} \quad (1)$$
$$100x = 73.\dot{3} \quad (2)$$

Do (2) − (1):

$$\begin{aligned} 90x = 66 &\Rightarrow x = \tfrac{66}{90} \\ &\Rightarrow x = \tfrac{6\times 11}{6\times 15} \\ &\Rightarrow x = \tfrac{\cancel{6}\times 11}{\cancel{6}\times 15} \\ &\Rightarrow \underline{\underline{x = \tfrac{11}{15}}}, \end{aligned}$$

as required.

16. Here is a speed-time graph for a car.



(a) Work out an estimate for the distance the car travelled in the first 30 seconds. (2)

Solution

$$\begin{aligned}\text{Distance} &= \text{area under the graph}\\ &\approx \text{area under the triangle}\\ &= \tfrac{1}{2} \times 30 \times 8.8\\ &= \underline{\underline{132\text{ m}}}.\end{aligned}$$

(b) Is your answer to part (a) an underestimate or an overestimate of the actual distance the car travelled in the first 30 seconds? (1)
Give a reason for your answer.

Solution

It is an underestimate as the triangle that we used does not quite exhaust the area under the graph.

Julian used the graph to answer this question.

Work out an estimate for the acceleration of the car at time 60 seconds.

Here is Julian's working.

$$\begin{aligned} \text{acceleration} &= \text{speed} \div \text{time} \\ &= 13 \div 60 \\ &= 0.21\dot{6} \text{ m/s}^2. \end{aligned}$$

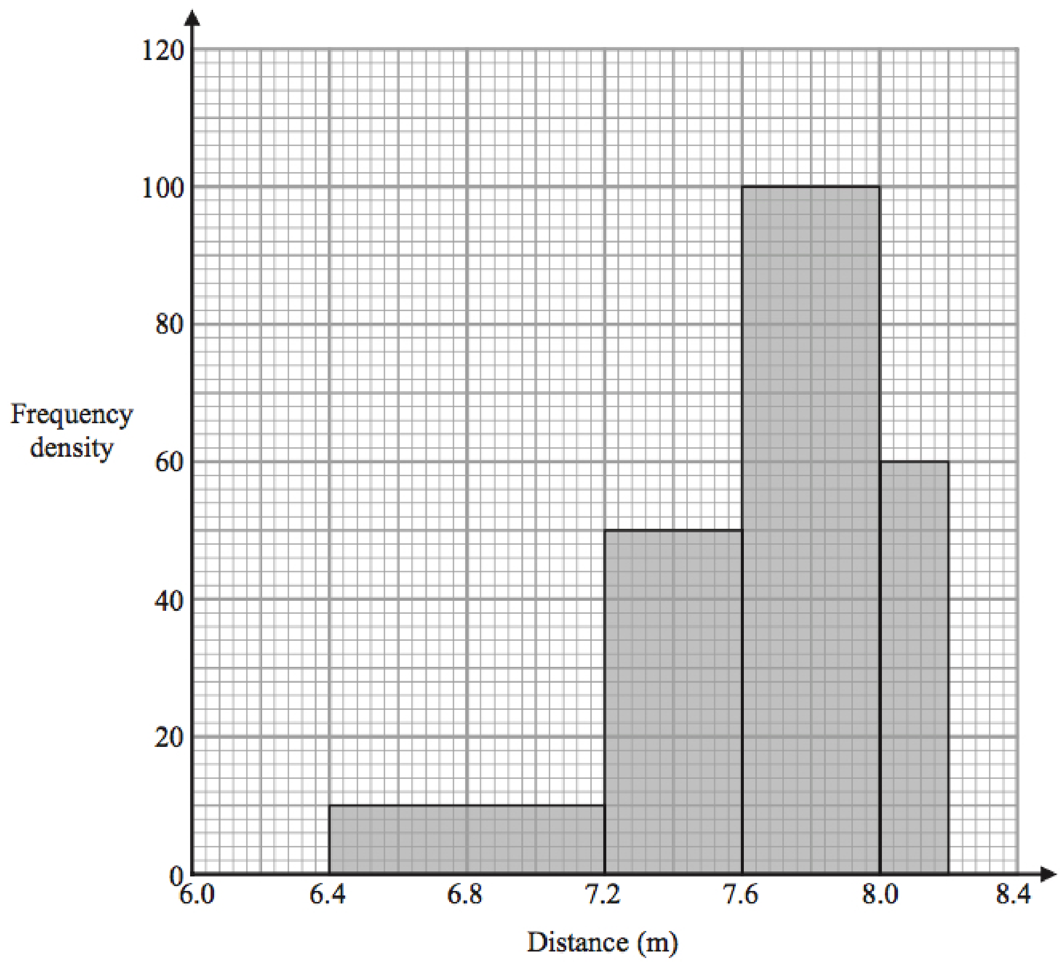
Julian's method does not give a good estimate of the acceleration at time 60 seconds.

(c) Explain why. (1)

Solution

E.g., he has not worked out the gradient at time 60 seconds.

17. The histogram gives information about the distances 80 competitors jumped in a long jump competition. (4)



Calculate an estimate for the mean distance.

Solution

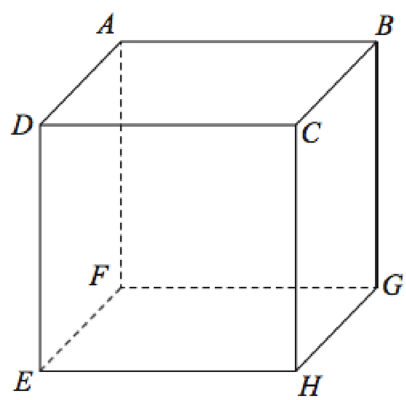
We need to work out the midpoints: 6.8, 7.4, 7.8, and 8.1.

Distance	Frequency Density	Width	Frequency
6.4 – 7.2	10	0.8	$0.8 \times 10 = 8$
7.2 – 7.6	50	0.4	$0.4 \times 50 = 20$
7.6 – 8.0	100	0.4	$0.4 \times 100 = 40$
8.0 – 8.2	60	0.2	$0.2 \times 60 = 12$

Hence,

$$\begin{aligned}\text{estimate for the mean distance} &\approx \frac{(6.8\times 8)+(7.4\times 20)+(7.8\times 40)+(8.1\times 12)}{80}\\ &= \frac{54.4+148+312+97.2}{80}\\ &= \frac{6.116}{80}\\ &= \underline{\underline{7.645\text{ m}}}.\end{aligned}$$

18. The diagram shows a cube. (4)



$AH = 11.3$ cm correct to the nearest mm.

Calculate the lower bound for the length of an edge of the cube.
You must show all your working.

Solution

Well, if $AH = a$, then

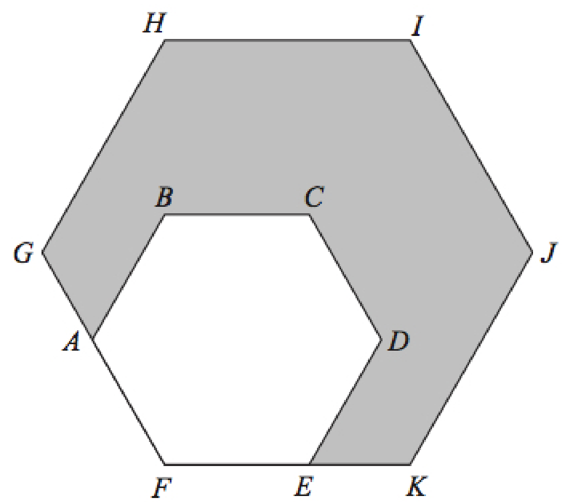
$$AH^2 = a^2 + a^2 + a^2 = 3a^2 \Rightarrow AH = a\sqrt{3}.$$

Now,

$$\begin{aligned} & 11.25 \leqslant AH\sqrt{3} < 11.35\\ \Rightarrow\ & \frac{11.25}{\sqrt{3}} \leqslant \text{length of an edge of the cube} < \frac{11.35}{\sqrt{3}}\\ \Rightarrow\ & 6.495\,190\,528 \leqslant \text{length of an edge of the cube} < 6.552\,925\,555 \text{ (FCD)}.\end{aligned}$$

Hence, the lower bound is $\underline{\underline{6.495\,190\,528 \text{ (FCD)}}}$.

19. $ABCDEF$ is a regular hexagon with sides of length x. (4)
This hexagon is enlarged, centre F, by scale factor p to give hexagon $FGHIJK$.



Show that the area of the shaded region in the diagram is given by

$$\tfrac{3\sqrt{3}}{2}(p^2 - 1)x^2.$$

Solution

Well,

$$\begin{aligned}\text{shape } ABCDEF &= 6 \times (\tfrac{1}{2} \times x \times x \times \sin 60^\circ) \\ &= \tfrac{3\sqrt{3}}{2}x^2\end{aligned}$$

and

$$\begin{aligned}\text{shape } FGHIJK &= p^2 \text{shape } ABCDEF \\ &= \tfrac{3\sqrt{3}}{2}p^2x^2.\end{aligned}$$

Hence,

$$\begin{aligned}\text{shaded region} &= \text{shape } FGHIJK - \text{shape } ABCDEF \\ &= \tfrac{3\sqrt{3}}{2}p^2x^2 - \tfrac{3\sqrt{3}}{2}x^2 \\ &= \underline{\underline{\tfrac{3\sqrt{3}}{2}(p^2-1)x^2}},\end{aligned}$$

as required.

20. Here is a list of five numbers. (1)

$$98^{53} \qquad 98^{64} \qquad 98^{73} \qquad 98^{88} \qquad 98^{91}.$$

Find the lowest common multiple of these five numbers.

Solution

$\underline{\underline{98^{91}}}$ as the other numbers all go into this:

$$\frac{98^{91}}{98^{53}} = 98^{38}, \; \frac{98^{91}}{98^{64}} = 98^{27}, \; \frac{98^{91}}{98^{73}} = 98^{18}, \text{ and } \frac{98^{91}}{98^{88}} = 98^{3}.$$

21. $5c + d = c + 4d$.

(a) Find the ratio $c : d$. (2)

Solution

$$\begin{aligned}5c + d = c + 4d &\Rightarrow 4c = 3d \\ &\Rightarrow \underline{\underline{c : d = 3 : 4}}.\end{aligned}$$

$$6x^2 = 7xy + 20y^2,$$

where $x > 0$ and $y > 0$.

(b) Find the ratio $x : y$. (3)

Solution

$$6x^2 = 7xy + 20y^2 \Rightarrow 6x^2 - 7xy - 20y^2 = 0$$

$$\left.\begin{array}{ll} \text{add to:} & \qquad\qquad -7 \\ \text{multiply to:} & (+6) \times (-20) = -120 \end{array}\right\} -15,\ +8$$

$$\begin{aligned}
&\Rightarrow 6x^2 - 15xy + 8xy - 20y^2 = 0 \\
&\Rightarrow 3x(2x - 5y) + 4y(2x - 5y) = 0 \\
&\Rightarrow (3x + 4y)(2x - 5y) = 0 \\
&\Rightarrow 3x = -4y \text{ or } 2x = 5y.
\end{aligned}$$

Well, we can rule out $3x = -4y$ as both $x, y > 0$. Hence,

$$2x = 5y \Rightarrow \underline{\underline{x : y = 5 : 2}}.$$