Dr Oliver Mathematics
GCSE Mathematics
2003 June Paper 5H: Non-Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. Using the information that

$$97 \times 123 = 11\,931$$

write down the value of

(a) 9.7×12.3, (1)

Solution

Divide the first term by 10, divide the second term by 10, which means the answer is divided by 100:

$$9.7 \times 12.3 = \underline{\underline{119.31}}.$$

(b) $0.97 \times 123\,000$ (1)

Solution

Divide the first term by 100, multiply the second term by 1 000, which means the answer is multiplied by 10:

$$0.97 \times 123\,000 = \underline{\underline{119\,310}}.$$

(c) $11.931 \div 9.7$. (1)

Solution

$$\frac{11\,931}{97} = 123;$$

divide the first term by 1 000, divide the second term by 10, which means the answer is divided by 100:

$$11.931 \div 9.7 = \underline{\underline{1.23}}.$$

2. Ben bought a car for £12 000. (3)
Each year the value of the car depreciated by 10%.
Work out the value of the car two years after he bought it.

Solution

$$12\,000 \times 0.9^2 = 12\,000 \times 0.81.$$

$\times$	10	2
0.8	8	+1.6
0.01	+0.1	+0.02

Finally,

$$\begin{aligned} 12\,000 \times 0.81 &= (8 + 1.6 + 0.1 + 0.02) \times 1\,000 \\ &= 9.72 \times 1\,000 \\ &= \underline{\underline{£9\,720}}. \end{aligned}$$

3. Solve (2)

$$7r + 2 = 5(r - 4).$$

Solution

$$\begin{aligned} 7r + 2 = 5(r - 4) &\Rightarrow 7r + 2 = 5r - 20 \\ &\Rightarrow 2r = -22 \\ &\Rightarrow \underline{\underline{r = -11}}. \end{aligned}$$

4. (a) $-2 < x \leqslant 1$. (2)
x is an integer.
Write down all the possible values of x.

Solution

$\underline{\underline{-1, 0, 1}}$.

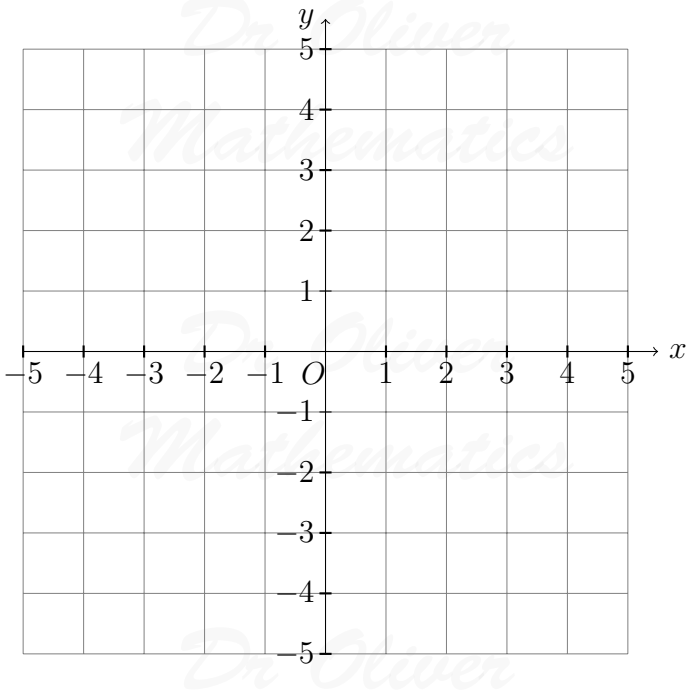
(b) $-2 < x \leqslant 1$. (3)

$y > -2$.

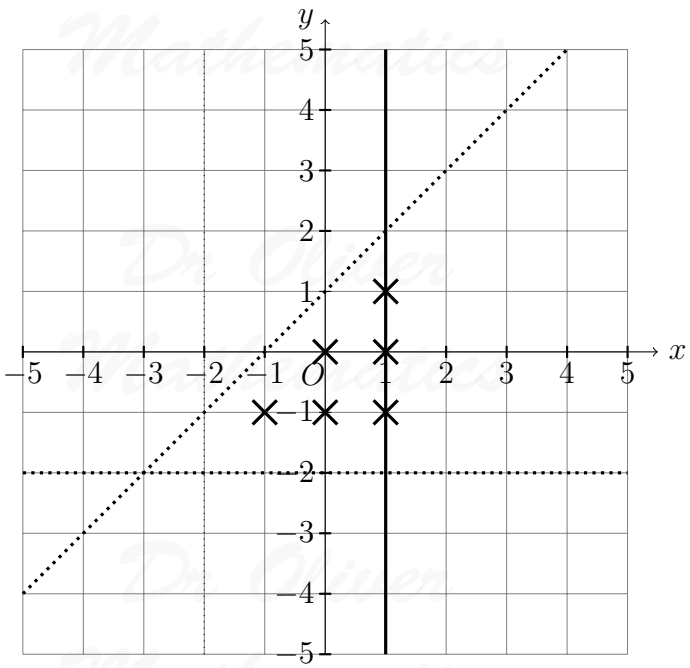
$y < x + 1$.

x and y are integers.

On the grid, mark with a cross ($\times$), each of the six points which satisfies all these three inequalities.



Solution



5. Here are the first five terms of an arithmetic sequence: (2)

$$6, 11, 16, 21, 26.$$

Find an expression, in terms of n, for the nth term of the sequence.

Solution

Let the

$$n\text{th term} = an + b.$$

$$\begin{array}{cccccccc} 6 & & 11 & & 16 & & 21 \\ & 5 & & 5 & & 5 & \\ a+b & & 2a+b & & 3a+b & & 4a+b \\ & a & & a & & a & \end{array}$$

We compare terms:

$$a = 5$$

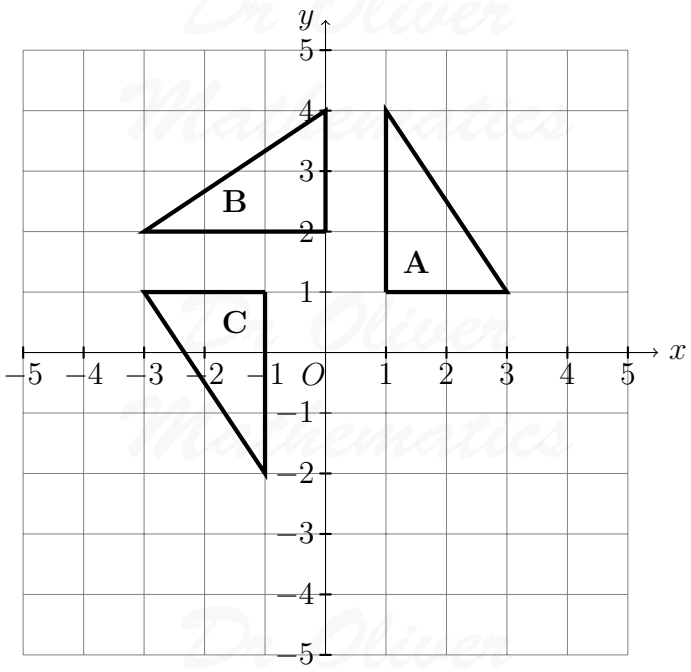
and

$$\begin{aligned} a + b = 6 &\Rightarrow 5 + b = 6 \\ &\Rightarrow b = 1. \end{aligned}$$

Hence,

$$n\text{th term} = \underline{\underline{5n + 1}}.$$

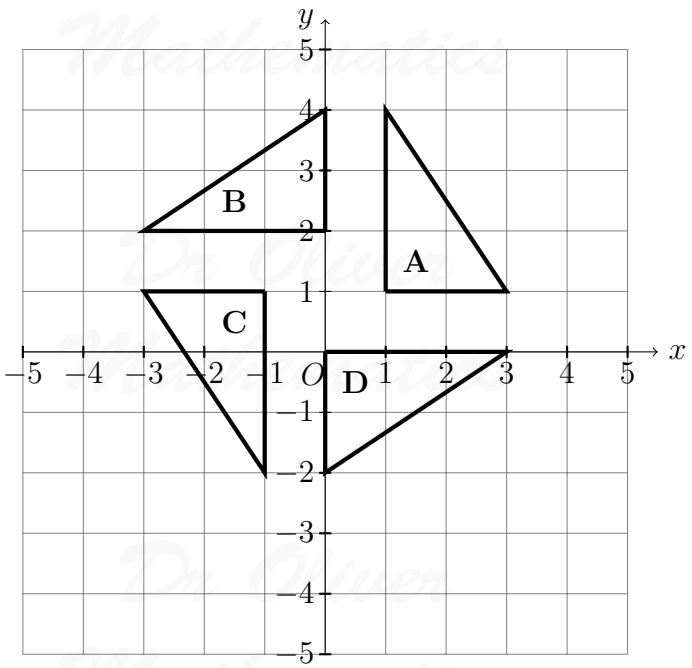
6. Here is a grid.



Shape **A** is rotated 90° anticlockwise, centre $(0, 1)$, to shape **B**.
Shape **B** is rotated 90° anticlockwise, centre $(0, 1)$, to shape **C**.
Shape **C** is rotated 90° anticlockwise, centre $(0, 1)$, to shape **D**.

(a) Mark the position of shape **D** (2)

Solution

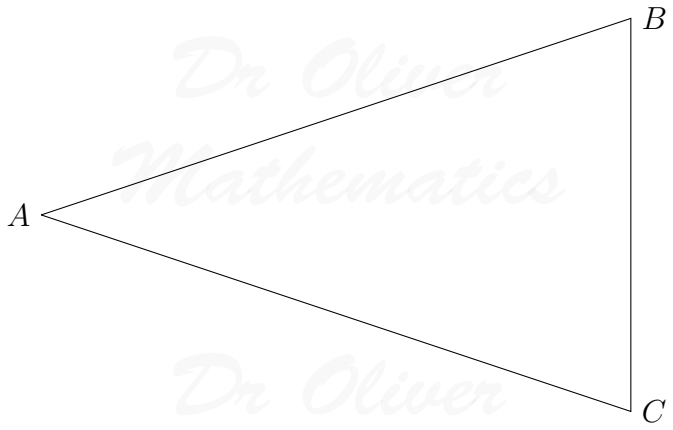


(b) Describe the single transformation that takes shape **C** to shape **A**. (2)

Solution

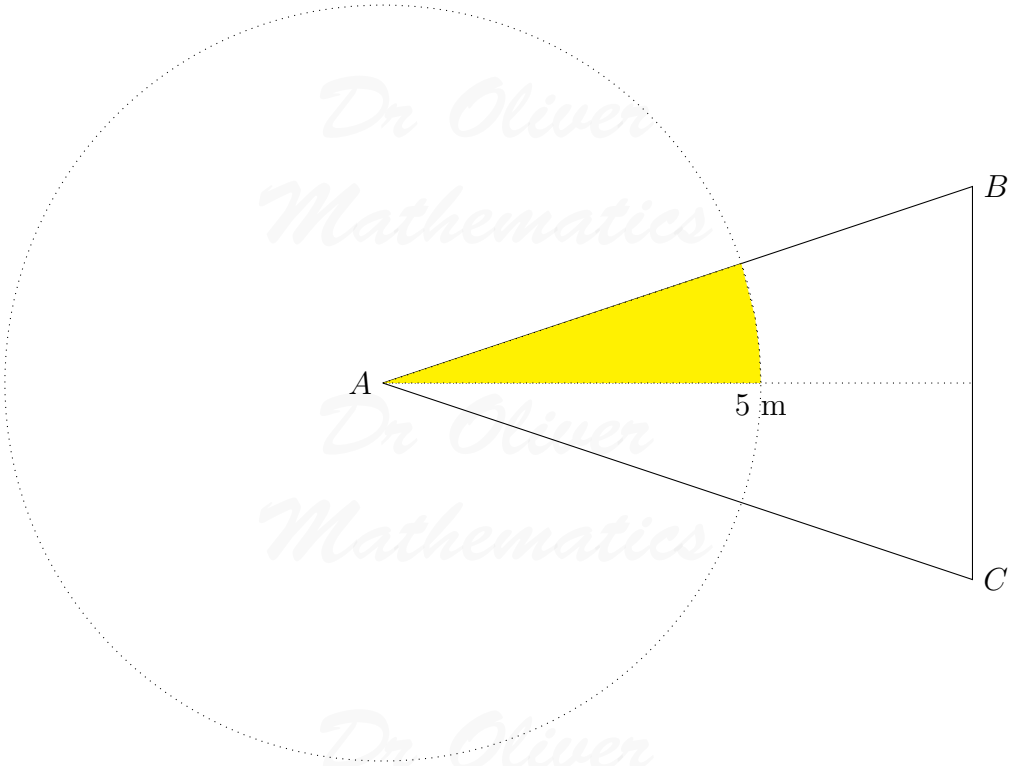
E.g., rotation, 180°, about (0, 1).

7. The diagram represents a triangular garden ABC. (3)
The scale of the diagram is 1 cm represents 1 m.
A tree is to be planted in the garden so that it is nearer to AB than to AC and within 5 m of point A.



On the diagram, shade the region where the tree may be planted.

Solution



8. This table shows some expressions. (3)
The letters x, y, and z represent lengths.
Place a tick in the appropriate column for each expression to show whether the expression can be used to represent a length, an area, a volume, or none of these.

Expression	Length	Area	Volume	None of these
$x + y + z$				
xyz				
$xy + yz + xz$				

Solution

Expression	Length	Area	Volume	None of these
$x + y + z$	✓			
xyz			✓	
$xy + yz + xz$		✓		

9. Mr Beeton is going to open a restaurant.
He wants to know what type of restaurant people like.
He designs a questionnaire.

(a) Design a suitable question he could use to find out what type of restaurant people like. (2)

Solution

E.g., Where do you eat out last week/last month/the last three months?
Never eat out, Pizza, Indian, Chinese, Other (please specify)

He asks his family "Do you agree that pizza is better than pasta?"
This is **not** a good way to find out what people who might use his restaurant like to eat.

(b) Write down **two** reasons why this is **not** a good way to find out what people who might use his restaurant like to eat. (2)

Solution

E.g., it is a biased question, restricted sample of people (his family), not specifying a range of foods, nothing to do with eating habits, etc.

10. A spaceship travelled for 6×10^2 hours at a speed of 8×10^4 km/h.

(a) Calculate the distance travelled by the spaceship. (3)
Give your answer in standard form.

Solution

$$(6 \times 10^2) \times (8 \times 10^4) = 48 \times 10^6$$
$$= \underline{\underline{4.8 \times 10^7 \text{ km}}}.$$

One month an aircraft travelled 2×10^5 km.
The next month the aircraft travelled 3×10^4 km.

(b) Calculate the total distance travelled by the aircraft in the two months. (2)
Give your answer as an ordinary number.

Solution

$$\begin{aligned} 2 \times 10^5 + 3 \times 10^4 &= 200\,000 + 30\,000 \\ &= \underline{\underline{230\,000 \text{ km}}}. \end{aligned}$$

11. (a) Expand and simplify $(x + y)^2$. (2)

Solution

$$(x + y)^2 = \underline{\underline{x^2 + 2xy + y^2}}.$$

(b) Hence, or otherwise, find the value of (2)

$$3.47^2 + 2 \times 3.47 \times 1.53 + 1.53^2.$$

Solution

$$\begin{aligned} 3.47^2 + 2 \times 3.47 \times 1.53 + 1.53^2 &= (3.47 + 1.53)^2 \\ &= 5^2 \\ &= \underline{\underline{25}}. \end{aligned}$$

12. In the diagram, A, B, and C are points on the circle, centre O.

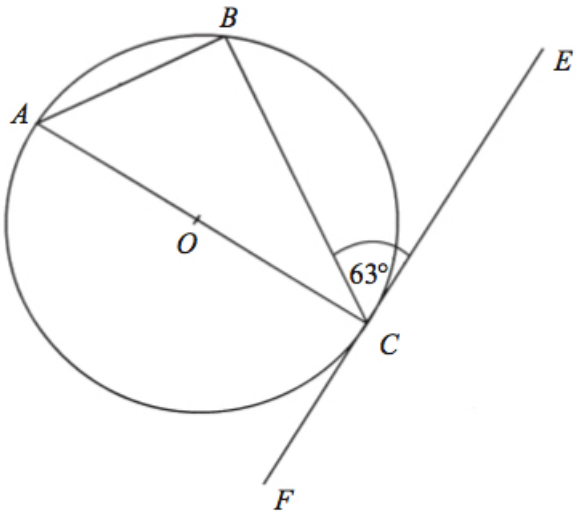


Diagram **NOT** accurately drawn

Angle $BCE = 63°$.
FE is a tangent to the circle at point C.

(a) Calculate the size of angle ACB. (2)
Give reasons for your answer.

Solution
Angle $ACB = 90 - 63 = \underline{\underline{27°}}$ (complementary angle).

(b) Calculate the size of angle BAC. (2)
Give reasons for your answer.

Solution
Angle $BAC = \underline{\underline{63°}}$ (alternating segment theorem).

13. Simplify fully

(a) $(p^3)^3$, (1)

Solution
$$(p^3)^3 = \underline{\underline{p^9}}.$$

(b) $\dfrac{3q^4 \times 2q^5}{q^3}$. (2)

Solution

$$\frac{3q^4 \times 2q^5}{q^3} = \frac{6q^9}{q^3}$$
$$= \underline{\underline{6q^6}}.$$

14. Mary recorded the heights, in centimetres, of the girls in her class.
She put the heights in order.

132	144	150	152	160	162	162	167
167	170	172	177	181	182	182	

(a) Find (2)

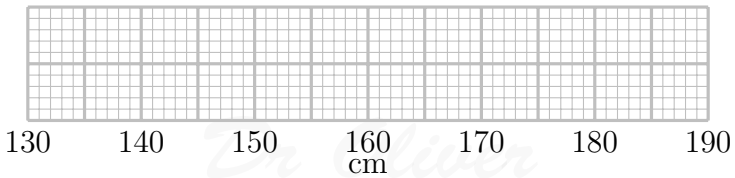
(i) the lower quartile,

Solution
$\underline{\underline{152 \text{ cm}}}$.

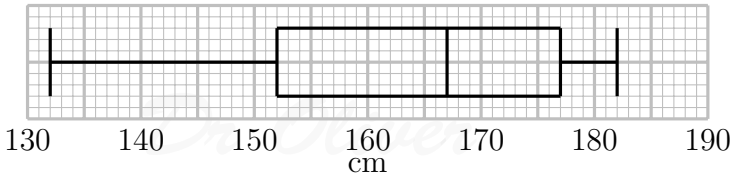
(ii) the upper quartile.

Solution
$\underline{\underline{177 \text{ cm}}}$.

(b) On the grid, draw a box plot for this data. (3)



Solution



15. The diagram shows a sector of a circle, centre O. (4)

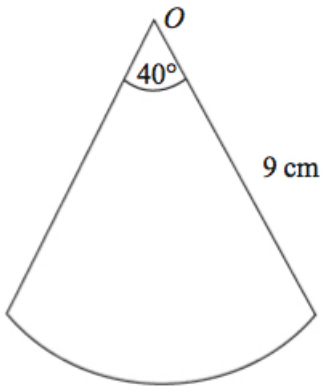


Diagram **NOT** accurately drawn

The radius of the circle is 9 cm.
The angle at the centre of the circle is 40°.
Find the perimeter of the sector.
Leave your answer in terms of π.

Solution

$$\begin{aligned}\text{Perimeter} &= 9 + 9 + \frac{40}{360} \times \pi \times 9^2 \\ &= 18 + \frac{1}{9}\pi \times 81 \\ &= \underline{\underline{(18 + 9\pi) \text{ cm}}}.\end{aligned}$$

16. Work out

(a) 4^0, (1)

Solution

$$4^0 = \underline{\underline{1}}.$$

(b) 4^{-2}, (1)

Solution

$$4^{-2} = \frac{1}{4^2} = \underline{\underline{\tfrac{1}{16}}}.$$

(c) $16^{\frac{3}{2}}$. (1)

Solution

$$16^{\frac{3}{2}} = (\sqrt{16})^3 = 4^3 = \underline{\underline{64}}.$$

17. The force, F, between two magnets is inversely proportional to the square of the distance, x, between them.
When $x = 3$, $F = 4$.

(a) Find an expression for F in terms of x. (3)

Solution

$$F = \frac{k}{x^2}$$

for some constant k. Now,

$$4 = \frac{k}{3^2} \Rightarrow k = 4 \times 3^2$$
$$\Rightarrow k = 36,$$

and so

$$\underline{\underline{F = \frac{36}{x^2}}}.$$

(b) Calculate F when $x = 2$. (1)

Solution

$$F = \frac{36}{2^2} = \frac{36}{4} = \underline{\underline{9}}.$$

(c) Calculate x when $F = 64$. (2)

Solution

$$64 = \frac{36}{x^2} \Rightarrow x^2 = \tfrac{36}{64}$$
$$\Rightarrow x = \tfrac{6}{8} \text{ (as } x > 0)$$
$$\Rightarrow \underline{\underline{x = \tfrac{3}{4}}}.$$

18. Work out (3)

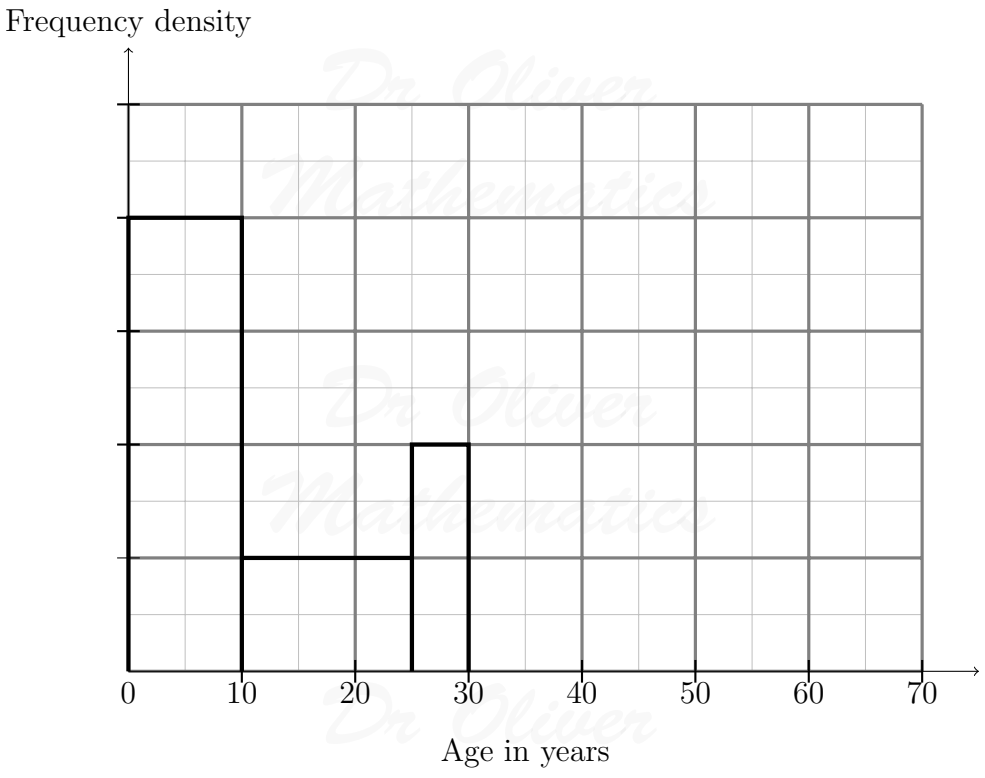
$$\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}.$$

Give your answer in its simplest form.

Solution

$$\begin{aligned}\left[\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}\right. &= \frac{5^2-(\sqrt{3})^2}{\sqrt{22}}\\ &= \frac{25-3}{\sqrt{22}}\\ &= \frac{22}{\sqrt{22}}\\ &= \underline{\underline{\sqrt{22}}}.\end{aligned}$$

19. The incomplete table and histogram give some information about the ages of the people who live in a village.



(a) Use the information in the histogram to complete the frequency table below. (2)

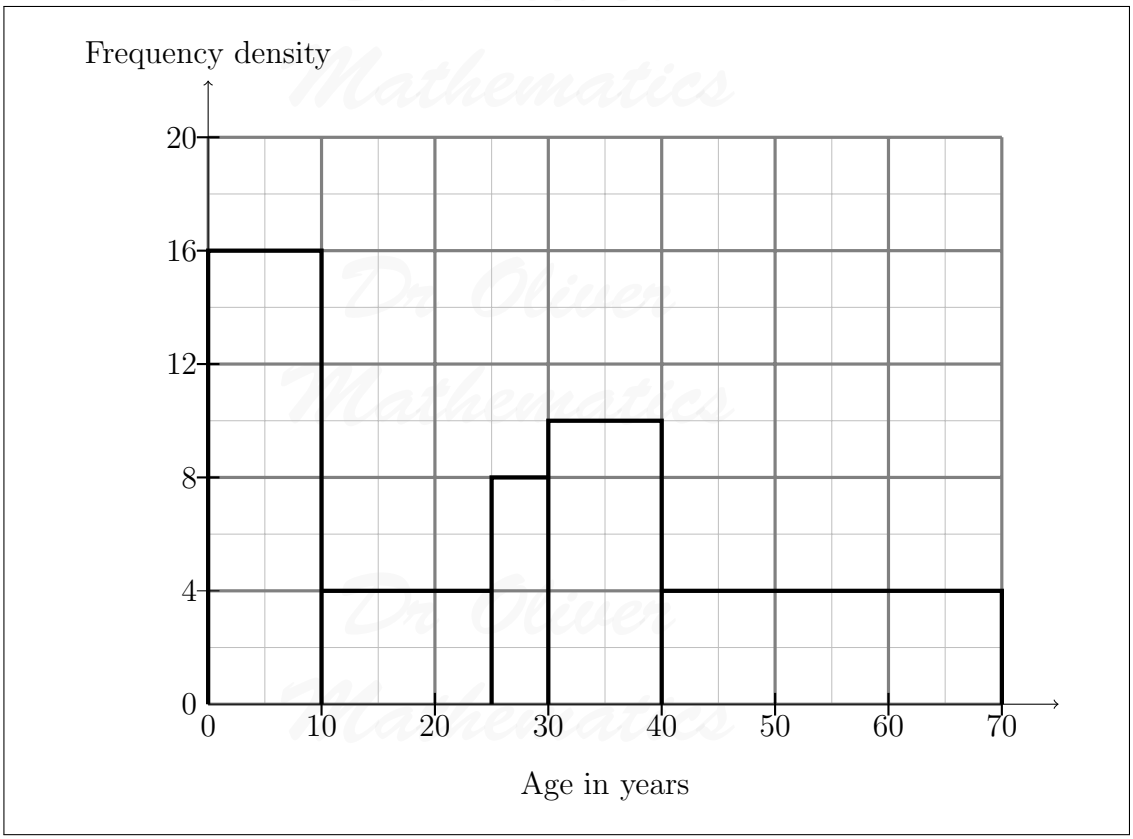
Age (x) in years	Frequency
$0 < x \leqslant 10$	160
$10 < x \leqslant 25$	
$25 < x \leqslant 30$	
$30 < x \leqslant 40$	100
$40 < x \leqslant 70$	120

Solution

Age (x) in years	Frequency	Width	FD
$0 < x \leqslant 10$	160	10	$\frac{160}{10} = 16$
$10 < x \leqslant 25$	$\underline{\underline{60}}$	15	$\frac{60}{15} = 4$
$25 < x \leqslant 30$	$\underline{\underline{40}}$	5	$\frac{40}{5} = 8$
$30 < x \leqslant 40$	100	10	$\frac{100}{10} = 10$
$40 < x \leqslant 70$	120	30	$\frac{120}{30} = 4$

(b) Complete the histogram. (2)

Solution



20. Simplify fully

(a) $2(3x+4)-3(4x-5)$, (2)

Solution

$$2(3x+4)-3(4x-5) = 6x+8-12x+15$$
$$= \underline{\underline{-6x+23}}.$$

(b) $(2xy^3)^5$, (2)

Solution

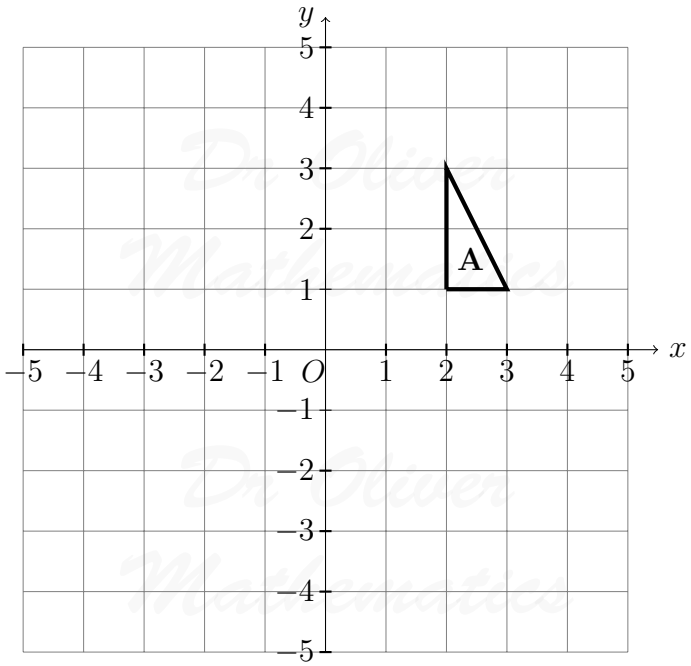
$$(2xy^3)^5 = \underline{\underline{32x^5y^{15}}}.$$

(c) $\dfrac{n^2-1}{n+1} \times \dfrac{2}{n-2}$. (3)

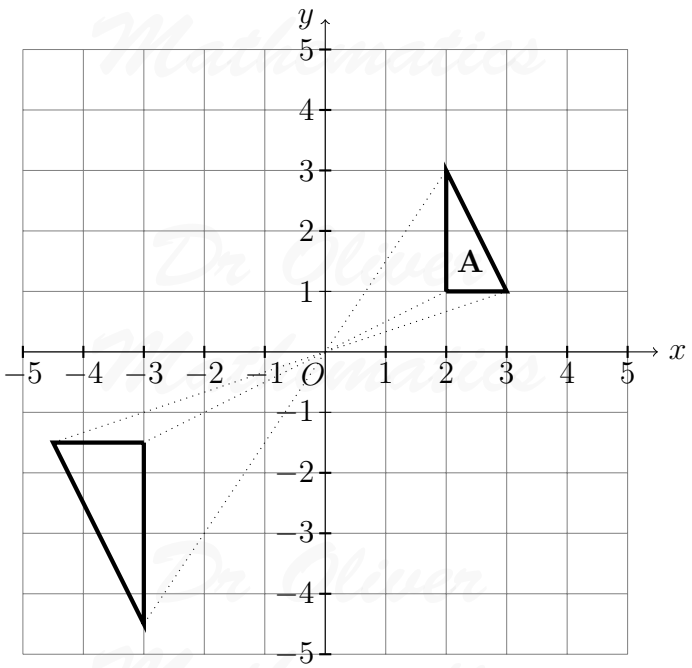
Solution

$$\frac{n^2-1}{n+1} \times \frac{2}{n-2} = \frac{(n-1)(n+1)}{n+1} \times \frac{2}{n-2}$$
$$= \underline{\underline{\frac{2(n-2)}{n-2}}}.$$

21. Enlarge triangle **A** by scale factor $-1\frac{1}{2}$, centre O. (3)



Solution



22. A bag contains 3 black beads, 5 red beads, and 2 green beads. (5)
Gianna takes a bead at random from the bag, records its colour, and replaces it.
She does this two more times.
Work out the probability that, of the three beads Gianna takes, exactly two are the same colour.

Solution

$$
\begin{aligned}
&\text{P(exactly two are the same colour)}\\
=&\,\text{P}(2B) + \text{P}(2R) + \text{P}(2G)\\
=&(3 \times \tfrac{3}{10} \times \tfrac{3}{10} \times \tfrac{7}{10}) + (3 \times \tfrac{5}{10} \times \tfrac{5}{10} \times \tfrac{5}{10}) + (3 \times \tfrac{2}{10} \times \tfrac{2}{10} \times \tfrac{8}{10})\\
=&3(\tfrac{63}{1\,000} + \tfrac{125}{1\,000} + \tfrac{32}{1\,000})\\
=&3 \times \tfrac{220}{1\,000}\\
=&\underline{\underline{\tfrac{660}{1\,000} \text{ or } 0.66}}.
\end{aligned}
$$

23. The diagram shows a regular hexagon $ABCDEF$ with centre O.

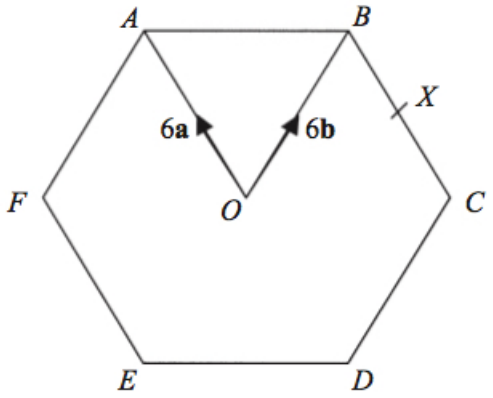


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 6\mathbf{a}$.
$\overrightarrow{OB} = 6\mathbf{b}$.

(a) Express in terms of **a** and/or **b** (2)

(i) $\overrightarrow{AB}$,

Solution

$$\overrightarrow{AB} = \overrightarrow{AO} - \overrightarrow{OB} = \underline{\underline{6\mathbf{b} - 6\mathbf{a}}}.$$

(ii) $\overrightarrow{EF}$.

Solution

$$\overrightarrow{EF} = \overrightarrow{OA} = \underline{\underline{6\mathbf{a}}}.$$

X is the midpoint of BC.

(b) Express $\overrightarrow{EX}$ in terms of **a** and/or **b** (2)

Solution

$$\begin{aligned}\overrightarrow{EX} &= \overrightarrow{EO} + \overrightarrow{OC} + \overrightarrow{CX} \\ &= 6\mathbf{b} + (6\mathbf{b} - 6\mathbf{a}) + 3\mathbf{a} \\ &= \underline{\underline{12\mathbf{b} - 3\mathbf{a}}}.\end{aligned}$$

Y is the point on AB extended, such that $AB : BY = 3 : 2$.

(c) Prove that E, X, and Y lie on the same straight line. (3)

Solution

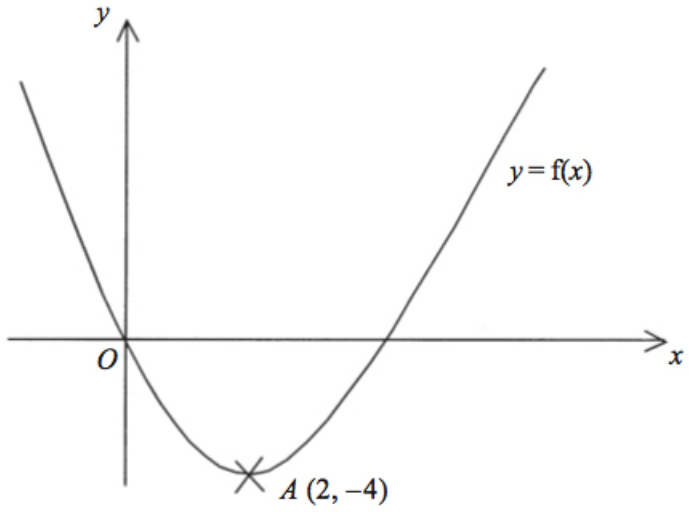
$$\overrightarrow{AY} = \tfrac{5}{3}\overrightarrow{AB}$$

and

$$\begin{aligned}\overrightarrow{EY} &= \overrightarrow{EO} + \overrightarrow{OA} + \overrightarrow{AY}\\ &= 6\mathbf{b} + 6\mathbf{a} + \tfrac{5}{3}\overrightarrow{AB}\\ &= 6\mathbf{b} + 6\mathbf{a} + \tfrac{5}{3}(6\mathbf{b} - 6\mathbf{a})\\ &= 6\mathbf{b} + 6\mathbf{a} + (10\mathbf{b} - 10\mathbf{a})\\ &= 16\mathbf{b} - 4\mathbf{a}\\ &= \tfrac{4}{3}(12\mathbf{b} - 3\mathbf{a})\\ &= \tfrac{4}{3}\overrightarrow{EX};\end{aligned}$$

hence, E, X, and Y lie on the same straight line.

24. This is a sketch of the curve with equation $y = \mathrm{f}(x)$.



It passes through the origin O.
The only vertex of the curve is at $A(2, -4)$.

(a) Write down the coordinates of the vertex of the curve with equation (4)

(i) $y = \mathrm{f}(x - 3)$,

Solution
$(5, -4)$.

(ii) $y = \mathrm{f}(x) - 5$,

Solution
$\underline{\underline{(2, -9)}}$.

(iii) $y = -\mathrm{f}(x)$,

Solution
$\underline{\underline{(2, 4)}}$.

(iv) $y = \mathrm{f}(2x)$.

Solution
$\underline{\underline{(1, -4)}}$.

The curve with equation $y = x^2$ has been translated to give the curve $y = \mathrm{f}(x)$.

(b) Find $\mathrm{f}(x)$ in terms of x. (4)

Solution

$$y = (x-2)^2 - 4 \Rightarrow y = (x^2 - 4x + 4) - 4$$
$$\Rightarrow \underline{\underline{y = x^2 - 4x}}.$$