Dr Oliver Mathematics
Mathematics
Differentiation Part 2
Past Examination Questions

This booklet consists of 25 questions across a variety of examination topics.
The total number of marks available is 229.

1. Figure 1 shows the plan of a stage in the shape of a rectangle joined to a semicircle.

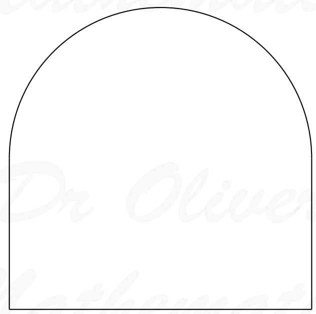

Figure 1: the stage

The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 metres.

(a) Show the the area, $A\text{m}^2$, of the stage is given by (4)

$$A = 80x - \left(2 + \frac{\pi}{2}\right) x^2.$$

(b) Use calculus to find the value of x at which A has a stationary value. (4)

(c) Prove that the value of x you found in (b) gives the maximum value of A. (2)

(d) Calculate, to the nearest m^2, the maximum area of the stage. (2)

2. Find the coordinates of the stationary point on the curve with equation $y = 2x^2 - 12x$.

3. The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(a) Find $\dfrac{\text{d}y}{\text{d}x}$. (2)

(b) Using the result from part (a), find the coordinates of the turning points of C. (4)

(c) Find $\dfrac{\text{d}^2y}{\text{d}x^2}$. (2)

(d) Hence, or otherwise, determine the nature of the turning points of C. (2)

4. Figure 2 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$.

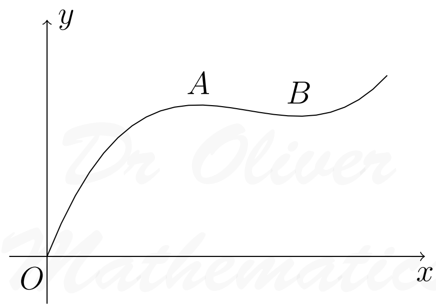

Figure 2: $y = x^3 - 8x^2 + 20x$

The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B. (4)

5. (3)

$$f(x) = x^3 + 3x^2 + 5.$$

Find $f''(x)$.

6. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum. (5)

(b) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v. (2)

(c) Calculate the minimum total cost of the journey. (2)

7. A solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by (4)

$$V = 200x - \tfrac{4}{3}x^3.$$

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3. (5)

(c) Justify that the value of V you have found is a maximum. (2)

8. Figure 3 shows an open-topped water tank, in the shape of a cuboid, which is made from sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

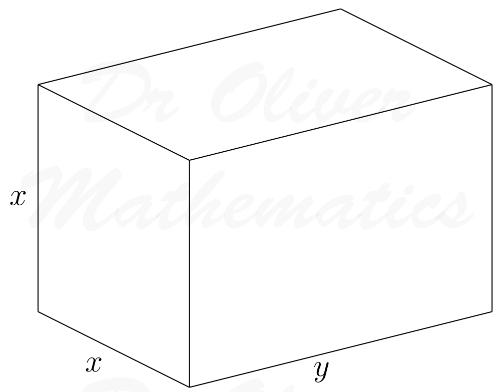

Figure 3: open-topped water tank

The capacity of the tank is 100 m^3.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by (4)

$$A = \frac{300}{x} + 2x^2.$$

(b) Use calculus to find the value of x for which A is stationary. (4)

(c) Prove that this value of x gives a minimum value for A. (2)

(d) Calculate the minimum area of sheet metal needed to make the tank. (2)

9. Figure 4 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$. (3)

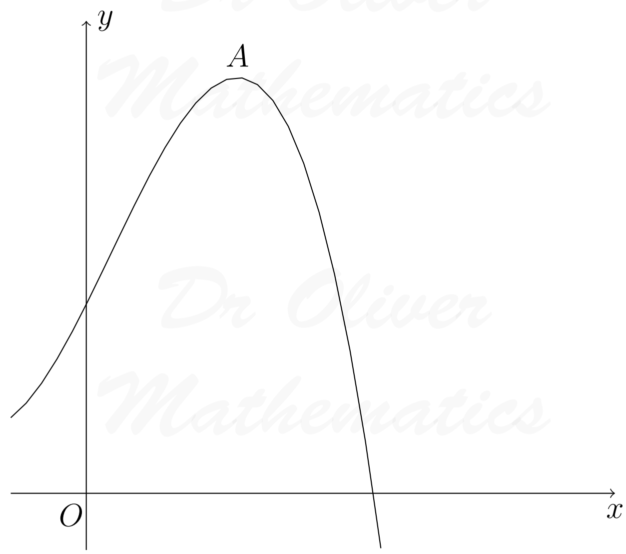

Figure 4: $y = 10 + 8x + x^2 - x^3$

The curve has a maximum turning point A.

Using calculus, show that the x-coordinate of A is 2.

10. A solid right circular cylinder has radius r cm and height h cm.

    The total surface area of the cylinder is 800 cm^2.

    (a) Show that the volume, V cm^3, of the cylinder is given by (4)

    $$V = 400r - \pi r^3.$$

    Given that r varies,

    (b) use calculus to find the maximum value of V, to the nearest cm^3. (6)

    (c) Justify that the value of V you have found is a maximum. (2)

11. Figure 5 shows a closed box used by a shop for packing pieces of cake.

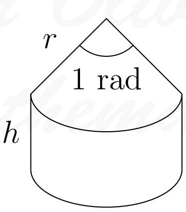

Figure 5: packing pieces of cake

The box is a right prism of height h cm. The cross-section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(a) Show that the surface area of the box, S cm^2, is given by (5)

$$S = r^2 + \frac{1800}{r^2}.$$

(b) Use calculus to find the value of r for which S is stationary. (4)

(c) Prove that this value of r gives a minimum value for S. (2)

(d) Find, to the nearest cm^2, this minimum value for S. (2)

12. The curve C has equation $y = 12\sqrt{x} - x^{\frac{3}{2}} - 10$, $x > 0$.

(a) Use calculus to find the coordinates of the turning point on C. (7)

(b) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. (2)

(c) State the nature of the turning point. (1)

13.

$$y = x^2 - k\sqrt{x}, \text{ where } k \text{ is a constant.}$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. (2)

(b) Given that y is decreasing at $x = 4$, find the set of possible value of k. (2)

14. The volume V cm^3 of a box, of height h cm, is given by

$$V = 4x(5 - x)^2, \; 0 < x < 5.$$

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}x}$. (4)

(b) Hence find the maximum volume of the box. (4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum. (2)

15. A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm.

The volume of the cuboid 81 cm^3.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by (3)

$$L = 12x + \frac{162}{x^2}.$$

(b) Use calculus to find the minimum value of L. (6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum. (2)

16. Figure 6 shows a flowerbed.

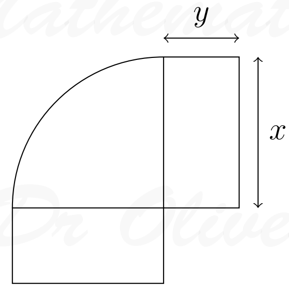

Figure 6: the flowerbed

Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to$y$ metres.

Given that the area of the flowerbed is 4 m^2,

(a) show that (3)

$$y = \frac{16 - \pi x^2}{8x}.$$

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation (3)

$$P = \frac{8}{x} + 2x.$$

(c) Use calculus to find the minimum value for P. (5)

(d) Find the width of each rectangle when the perimeter is a minimum. Give your answer to the nearest centimetre. (2)

17. A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and h mm.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x, (1)

(b) show that the surface area, A mm^2, of a tablet is given by (3)

$$A = 2\pi x^2 + \frac{120}{x}.$$

The manufacturer needs to minimise the surface area, A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum. (5)

(d) Calculate the minimum value of A, giving your answer to the nearest integer. (2)

(e) Show that this value of A is a minimum. (2)

18. The curve C has equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$.

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$. (4)

(b) Find the x-coordinate of the other turning point Q on the curve. (1)

(c) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. (1)

(d) Hence, or otherwise, state with justification, the nature of each of these turning points P and Q. (3)

19. The curve with equation

$$y = x^2 - 32\sqrt{x} + 20,\ x > 0,$$

has a stationary point P.

Use calculus

(a) to find the coordinates of P, (6)

(b) to determine the nature of the stationary point P. (3)

20. Using calculus, find the coordinates of the stationary point on the curve with equation (6)

$$y = 2x + 3 + \frac{8}{x^2},\ x > 0.$$

21. Figure 7 shows a closed letter box $ABFEHGCD$, which is made to be attached to a wall of a house.

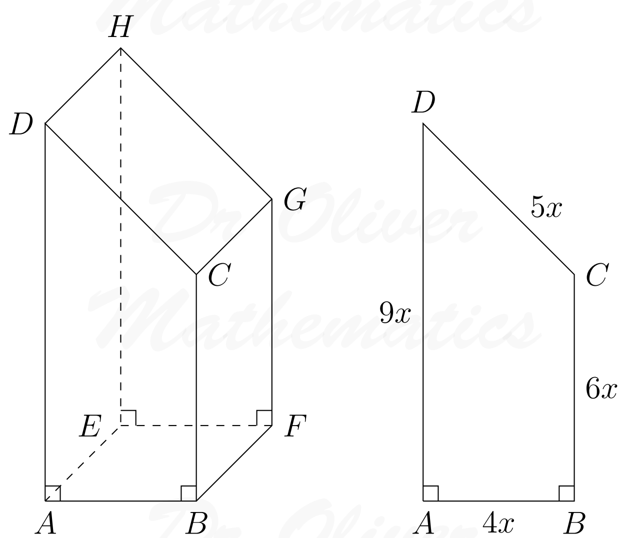

Figure 7: a letter box

The letter box is a right prism of length y cm. The base $ABFE$ of the prism is a rectangle. The total surface area of the six faces is S cm^2.

The cross-sectional $ABCD$ of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm, and $CD = 5x$ cm

The angle $DAB = 90°$ and the angle $ABC = 90°$.

The volume of the letter box is 9000 cm^3.

(a) Show that (2)

$$y = \frac{320}{x^2}.$$

(b) Hence show that the surface area of the letter box, S cm^2, is given by (4)

$$S = 60x^2 + \frac{7680}{x}.$$

(c) Use calculus to find the minimum value of S. (6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum. (2)

22. Figure 8 shows the plan of a pool.

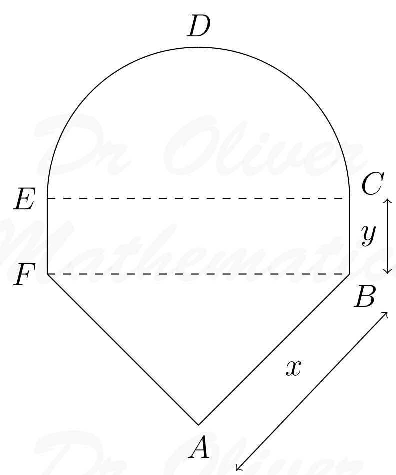

Figure 8: the pool

The shape of the pool $ABCDEFA$ consists of a rectangle $BCEF$ joined to an equilateral triangle BFA and a semi-circle CDE, as shown in the figure.

Given that $AB = x$ metres, $EF = y$ metres, and that the area of the pool is 50 m^2,

(a) show that (3)

$$y = \frac{50}{x} - \frac{x}{8}(\pi + 2\sqrt{3}).$$

(b) Hence show that the perimeter, P metres, of the pool is given by (3)

$$P = \frac{100}{x} + \frac{x}{4}(\pi + 8 - 2\sqrt{3}).$$

(c) Use calculus to find the minimum value of P, giving your answer to 3 significant figures. (5)

(d) Justify, by further differentiation, that the value of P you have found is a minimum. (2)

23. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.

The cost of polishing the surface area of this grass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by (4)

$$C = 6\pi r^2 + \frac{300\pi}{r}.$$

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound. (5)

(c) Justify that the answer that you have obtained in part (b) is a minimum. (1)

24. Figure 9 shows a plan of a sheep enclosure.

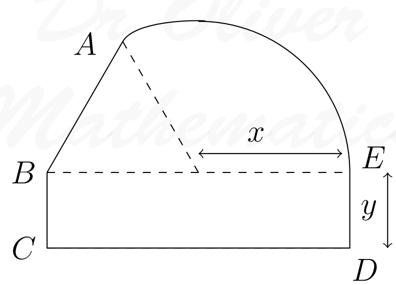

Figure 9: the sheep enclosure

The enclosure $ABCDEA$, as shown in the figure, consists of a rectangle $BCDE$ joined to an equilateral triangle BFA and a sector FEA of a circle with radius x metres and centre F.

The points B, F, and E lie on a straight line with $FE = x$ metres and $10 \leqslant x \leqslant 25$.

(a) Find, in m^2, the exact area of the sector FEA, giving your answer in terms of x, in its simplest form. (2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is 1000 m^2,

(b) show that (3)

$$y = \frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3}).$$

(c) Hence show that the perimeter P metres of the enclosure is given by (3)

$$P = \frac{1000}{x} + \frac{x}{12}(4\pi + 36 - 3\sqrt{3}).$$

(d) Use calculus to find the minimum value for P, giving your answer to the nearest metre. (5)

(e) Justify, by further differentiation, that the value of P you have found is a minimum. (2)

25. Figure 10 shows a sketch of part of the curve with equation (3)

$$y = 4x^3 + 9x^2 - 30x - 8, \ -0.5 \leqslant x \leqslant 2.2.$$

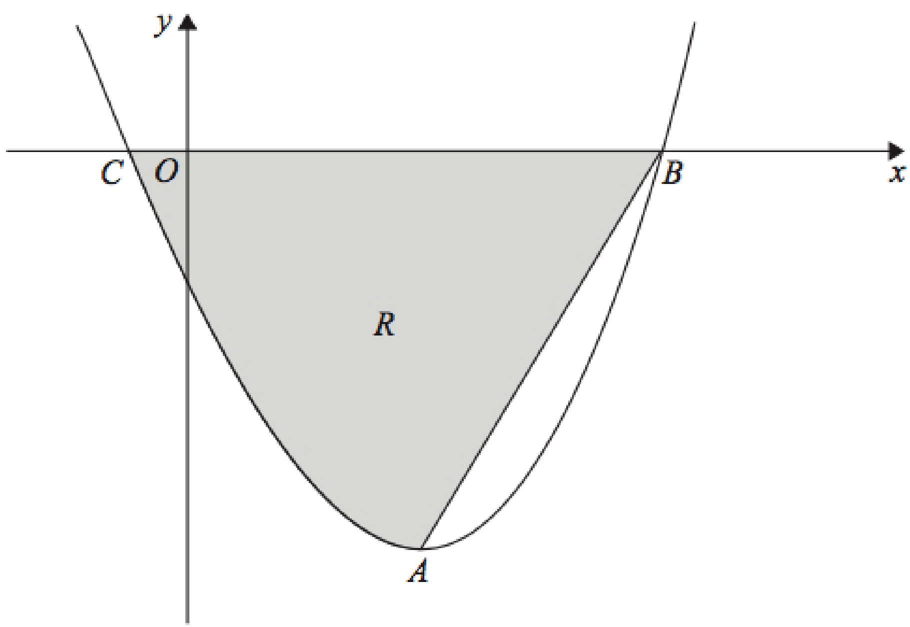

Figure 10: $y = 4x^3 + 9x^2 - 30x - 8$

The curve has a turning point at the point A. Using calculus, show that the x-coordinate of A is 1.