Dr Oliver Mathematics
Advance Level Further Mathematics
Further Mechanics 1: Calculator
1 hour 30 minutes

The total number of marks available is 75.
You must write down all the stages in your working.

1. Figure 1 represents the plan of part of a smooth horizontal floor, where W_1 and W_2 are two fixed parallel vertical walls. The walls are 3 metres apart.

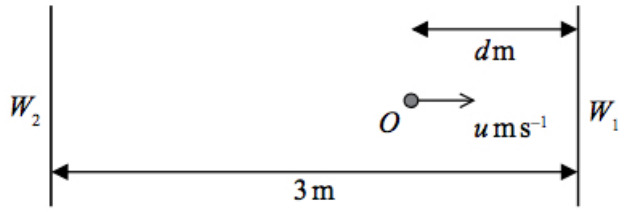

Figure 1: a smooth horizontal floor

A particle lies at rest at a point O on the floor between the two walls, where the point O is d metres, $0 < d \leqslant 3$, from W_1.

At time $t = 0$, the particle is projected from O towards W_1 with speed u ms^{-1} in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{2}{3}$. The particle returns to O at time $t = T$ seconds, having bounced off each wall once.

(a) Show that (6)

$$T = \frac{45 - 5d}{4u}.$$

The value of u is fixed, the particle still hits each wall once but the value of d can now vary.

(b) Find the least possible value of T, giving your answer in terms of u. (2)
You must give a reason for your answer.

2. Figure 2 represents the plan view of part of a horizontal floor, where AB and BC are fixed vertical walls with AB perpendicular to BC.

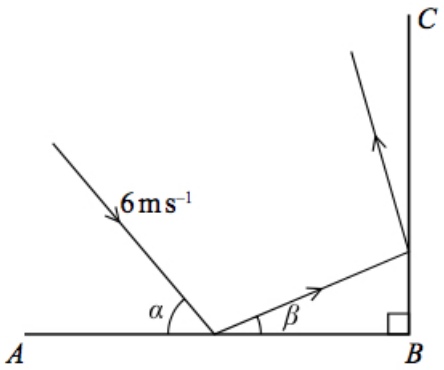

Figure 2: a horizontal floor

A small ball is projected along the floor towards AB with speed 6 ms^{-1} on a path that makes an angle α with AB, where $\tan\alpha = \frac{4}{3}$. The ball hits AB and then hits BC.

Immediately after hitting AB, the ball is moving at an angle β to AB, where $\tan\beta = \frac{1}{3}$.

The coefficient of restitution between the ball and AB is e.

The coefficient of restitution between the ball and BC is $\frac{1}{2}$.

By modelling the ball as a particle and the floor and walls as being smooth,

(a) show that the value of $e = \frac{1}{4}$, (5)

(b) find the speed of the ball immediately after it hits BC. (4)

(c) Suggest two ways in which the model could be refined to make it more realistic. (2)

3. A particle P, of mass 0.5 kg, is moving with velocity $(4\mathbf{i} + 4\mathbf{j})$ ms^{-1} when it receives an impulse $\mathbf{I}$ of magnitude 2.5 Ns. (9)

As a result of the impulse, the direction of motion of P is deflected through an angle of $45°$.

Given that $\mathbf{I} = (\lambda\mathbf{i} + \mu\mathbf{j})$ Ns, find all the possible pairs of values of λ and μ.

4. A car of mass 600 kg pulls a trailer of mass 150 kg along a straight horizontal road. The trailer is connected to the car by a light inextensible towbar, which is parallel to the direction of motion of the car. The resistance to the motion of the trailer is modelled as a constant force of magnitude 200 N. At the instant when the speed of the car is v ms^{-1}, the resistance to the motion of the car is modelled as a force of magnitude $(200 + \lambda v)$ N, where λ is a constant.

When the engine of the car is working at a constant rate of 15 kW, the car is moving at a constant speed of 25 ms^{-1}.

(a) Show that $\lambda = 8$. (4)

Later on, the car is pulling the trailer up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{15}$.

The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 200 N at all times. At the instant when the speed of the car is v ms^{-1}, the resistance to the motion of the car from non-gravitational forces is modelled as a force of magnitude $(200 + 8v)$ N.

The engine of the car is again working at a constant rate of 15 kW.

When $v = 10$, the towbar breaks. The trailer comes to instantaneous rest after moving a distance d metres up the road from the point where the towbar broke.

(b) Find the acceleration of the car immediately after the towbar breaks. (4)

(c) Use the work-energy principle to find the value of d. (4)

5. A particle P of mass $3m$ and a particle Q of mass $2m$ are moving along the same straight line on a smooth horizontal plane. The particles are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is u and the speed of Q is $2u$.

Immediately after the collision P and Q are moving in opposite directions.

The coefficient of restitution between P and Q is e.

(a) Find the range of possible values of e, justifying your answer. (8)

Given that Q loses 75% of its kinetic energy as a result of the collision,

(b) find the value of e. (3)

6. (In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.)
A smooth uniform sphere A has mass 0.2 kg and another smooth uniform sphere B, with the same radius as A, has mass 0.4 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision, the velocity of A is $(3\mathbf{i} + 2\mathbf{j})$ ms^{-1} and the velocity of B is $(-4\mathbf{i} - \mathbf{j})$ ms^{-1}.

At the instant of collision, the line joining the centres of the spheres is parallel to **i**.

The coefficient of restitution between the spheres is $\frac{3}{7}$.

(a) Find the velocity of A immediately after the collision. (7)

(b) Find the magnitude of the impulse received by A in the collision. (2)

(c) Find, to the nearest degree, the size of the angle through which the direction of motion of A is deflected as a result of the collision. (3)

7. A particle P, of mass m, is attached to one end of a light elastic spring of natural length a and modulus of elasticity kmg.

The other end of the spring is attached to a fixed point O on a ceiling.

The point A is vertically below O such that $OA = 3a$.

The point B is vertically below O such that $OB = \frac{1}{2}a$.

The particle is held at rest at A, then released and first comes to instantaneous rest at the point B.

(a) Show that $k = \frac{4}{3}$. (3)

(b) Find, in terms of g, the acceleration of P immediately after it is released from rest at A. (3)

(c) Find, in terms of g and a, the maximum speed attained by P as it moves from A to B. (6)