Dr Oliver Mathematics
Mathematics
Coordinates Part 1
Past Examination Questions

This booklet consists of 24 questions across a variety of examination topics. The total number of marks available is 213. Calculators may not be used.

1. The points $A(1, 7)$, $B(20, 7)$ and $C(p, q)$ form the vertices of a triangle ABC, as shown in Figure 1.

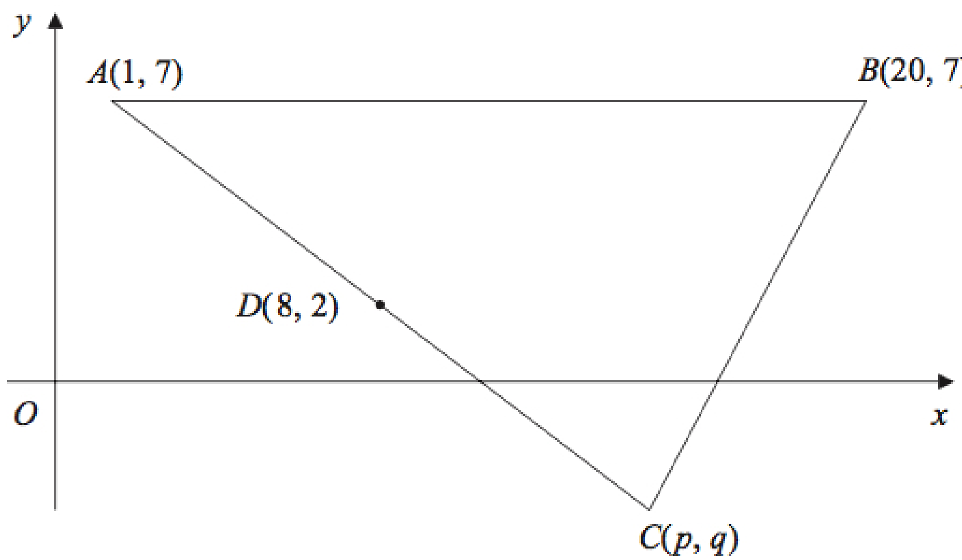

Figure 1: ABC

The point $D(8, 2)$ is the mid-point of AC.

(a) Find the value of p and the value of q. (2)

Solution

$\overrightarrow{AD} = \overrightarrow{DC}$; hence, $\underline{\underline{p = 15}}$ and $\underline{\underline{q = -3}}$.

The line l, which passes through D and is perpendicular to AC, intersects AB at E.

(b) Find an equation for l, in the form $ax + by + c = 0$, where a, b, and c are integers. (5)

Solution

$$\begin{aligned}\text{Gradient of } AC &= \frac{2-7}{8-1} \\ &= -\frac{5}{7}\end{aligned}$$

and hence

$$\text{gradient of } l = \frac{7}{5}.$$

Now,

$$\begin{aligned}y - 2 = \frac{7}{5}(x-8) &\Rightarrow 5(y-2) = 7(x-8) \\ &\Rightarrow 5y - 10 = 7x - 56 \\ &\Rightarrow \underline{\underline{7x - 5y - 46 = 0}}.\end{aligned}$$

(c) Find the exact x-coordinate of E. (2)

Solution

We make $y = 7$:

$$\begin{aligned}7x - 5y - 46 = 0 &\Rightarrow 7x = 81 \\ &\Rightarrow \underline{\underline{x = 11\tfrac{4}{7}}}.\end{aligned}$$

2. The line l_1 passes through the point $(9, -4)$ and has gradient $\frac{1}{3}$.

(a) Find an equation for l_1, in the form $ax + by + c = 0$, where a, b, and c are integers. (3)

Solution

$$\begin{aligned}y + 4 = \tfrac{1}{3}(x-9) &\Rightarrow 3(y+4) = x - 9 \\ &\Rightarrow 3y + 12 = x - 9 \\ &\Rightarrow \underline{\underline{x - 3y - 21 = 0}}.\end{aligned}$$

The line l_2 passes through the origin O and has gradient -2. The lines l_1 and l_2 intersect at the point P.

(b) Calculate the coordinates of P. (4)

Solution

l_2 has equation $y = -2x$. Now,

$$\begin{aligned} x - 3(-2x) - 21 = 0 &\Rightarrow 7x = 21 \\ &\Rightarrow x = 3 \\ &\Rightarrow y = -6; \end{aligned}$$

hence, $\underline{\underline{P(3, -6)}}$.

Given that l_1 crosses the y-axis at the point C,

(c) calculate the exact area of $\triangle OCP$. (3)

Solution

$$x = 0 \Rightarrow -3y - 21 = 0 \Rightarrow y = -7$$

and hence $C(0, -7)$. Finally,

$$\begin{aligned} \text{area} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times 7 \times 3 \\ &= \underline{\underline{10\tfrac{1}{2} \text{ units}^2}}. \end{aligned}$$

3. The line L has equation $y = 5 - 2x$.

(a) Show that the point $P(3, -1)$ lies on L. (1)

Solution

$$x = 3 \Rightarrow y = 5 - 2 \times 3 = -1,$$

and hence the point $\underline{\underline{P(3, -1)}}$ lies on L.

(b) Find an equation of the line perpendicular to L, which passes through P. Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Solution

$$\text{Gradient of the line} = \tfrac{1}{2}$$

and

$$y + 1 = -\tfrac{1}{2}(x-3) \Rightarrow 2(y+1) = x-3$$
$$\Rightarrow 2y + 2 = x - 3$$
$$\Rightarrow \underline{\underline{x + 2y - 5 = 0}}.$$

4. The line l_1 passes through the points $P(-1, 2)$ and $Q(11, 8)$.

   (a) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants. (4)

Solution

$$\text{Gradient} = \frac{8-2}{11-(-1)} = \frac{1}{2}$$

and

$$y - 2 = \tfrac{1}{2}(x+1) \Rightarrow \underline{\underline{y = \tfrac{1}{2}x + \tfrac{5}{2}}}.$$

The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S.

   (b) Calculate the coordinates of S. (5)

Solution

$$\text{Gradient of } l_2 = -2.$$

The equation of l_2 is

$$y - 0 = -2(x-10) \Rightarrow y = -2x + 20.$$

Next,

$$\tfrac{1}{2}x + \tfrac{5}{2} = -2x + 20 \Rightarrow \tfrac{5}{2}x = \tfrac{35}{2}$$
$$\Rightarrow x = 7$$
$$\Rightarrow y = 6;$$

hence, $\underline{\underline{S(7, 6)}}$.

   (c) Show that the length of RS is $3\sqrt{5}$. (2)

Solution

$$\begin{aligned}\text{Length} &= \sqrt{(10-7)^2+(0-6)^2}\\ &= \sqrt{45}\\ &= \underline{\underline{3\sqrt{5}}},\end{aligned}$$

as required.

(d) Hence, or otherwise, find the exact area of triangle PQR. (4)

Solution

$$\begin{aligned}PQ &= \sqrt{(-1-11)^2+(2-8)^2}\\ &= \sqrt{180}\\ &= 6\sqrt{5},\end{aligned}$$

and

$$\begin{aligned}\text{area} &= \tfrac{1}{2}\times PQ\times RS\\ &= \tfrac{1}{2}\times 6\sqrt{5}\times 3\sqrt{5}\\ &= \tfrac{1}{2}\times 90\\ &= \underline{\underline{45 \text{ units}^2}}.\end{aligned}$$

5. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

(a) Show that the point $P(4, 8)$ lies on C. (1)

Solution

$$x = 4 \Rightarrow y = 4\times 4 + 3\times 4^{\frac{3}{2}} - 2\times 4^2 = 16 + 24 - 32 = 8;$$

hence, $\underline{\underline{P(4,8) \text{ lies on } C}}$.

You are given, when $x = 4$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -3$.

(b) Show that an equation of the normal to C at the point P is $3y = x + 20$. (4)

Solution

$$\text{Gradient of the normal} = \tfrac{1}{3}$$

and

$$\begin{aligned} y - 8 = \tfrac{1}{3}(x-4) &\Rightarrow 3(y-8) = x - 4 \\ &\Rightarrow 3y - 24 = x - 4 \\ &\Rightarrow \underline{\underline{3y = x + 20}}, \end{aligned}$$

as required.

The normal to C at P cuts the x-axis at the point Q.

(c) Find the length PQ, giving your answer in a simplified surd form. (3)

Solution

$$y = 0 \Rightarrow x + 20 = 0 \Rightarrow x = -20.$$

Finally,

$$\begin{aligned} \text{length} &= \sqrt{(-20-4)^2 + (0-8)^2} \\ &= \sqrt{640} \\ &= \underline{\underline{8\sqrt{10}}}. \end{aligned}$$

6. The curve C has equation

$$y = x^2(x-6) + \frac{4}{x},\ x > 0.$$

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$. (4)

Solution

$$\begin{aligned} x = 1 &\Rightarrow y = -5 + 4 = -1, \\ x = 2 &\Rightarrow y = -16 + 2 = -14. \end{aligned}$$

Finally,

$$\begin{aligned} \text{length} &= \sqrt{(2-1)^2 + (-14+1)^2} \\ &= \sqrt{1 + 169} \\ &= \underline{\underline{\sqrt{170}}}. \end{aligned}$$

(b) Show that the tangents to C at P and Q are parallel. (5)

Solution

$$y = x^2(x-6) + \frac{4}{x} \Rightarrow y = x^3 - 6x^2 + 4x^{-1}$$
$$\Rightarrow \frac{dy}{dx} = 3x^2 - 12x - 4x^{-2}.$$

$$x = 1 \Rightarrow \frac{dy}{dx} = 3 - 12 - 4 = -13,$$
$$x = 2 \Rightarrow \frac{dy}{dx} = 12 - 24 - 1 = -13.$$

Hence, the tangents to C at P and Q are parallel.

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Solution

$$\text{Gradient of the normal} = \tfrac{1}{13}$$

and

$$y + 1 = \tfrac{1}{13}(x-1) \Rightarrow 13(y+1) = x - 1$$
$$\Rightarrow 13y + 13 = x - 1$$
$$\Rightarrow x - 13y - 14 = 0.$$

7. The point $A(-6,4)$ and the point $B(8,-3)$ lie on the line L.

(a) Find an equation for L in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Solution

$$\text{Gradient} = \frac{4-(-3)}{-6-8} = -\frac{1}{2}$$

and the equation is

$$y - 4 = -\tfrac{1}{2}(x+6) \Rightarrow 2(y-4) = -(x+6)$$
$$\Rightarrow 2y - 8 = -x - 6$$
$$\Rightarrow x + 2y - 2 = 0.$$

(b) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer. (3)

Solution

$$\begin{aligned}\text{Distance} &= \sqrt{(-6-8)^2 + (4-(-3))^2}\\ &= \sqrt{196+49}\\ &= \sqrt{245}\\ &= \underline{\underline{7\sqrt{5}}}.\end{aligned}$$

8. The points $Q(1, 3)$ and $R(7, 0)$ lie on the line l_1, as shown in Figure 2.

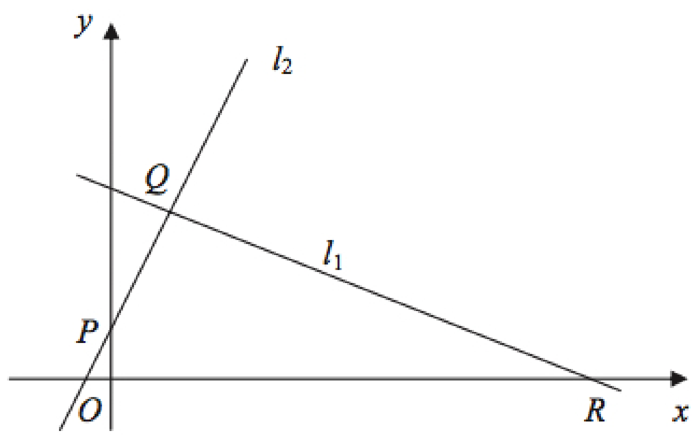

Figure 2: PQR

The length of QR is $a\sqrt{5}$.

(a) Find the value of a. (3)

Solution

$$\begin{aligned}QR &= \sqrt{(7-1)^2 + (3-0)^2}\\ &= \sqrt{45}\\ &= 3\sqrt{5};\end{aligned}$$

hence, $\underline{\underline{a = 3}}$.

The line l_2 is perpendicular to l_1, passes through Q and crosses the y-axis at the point P, as shown in Figure 2.

Find

(b) an equation for l_2, (5)

Solution

$$\text{Gradient of the line} = \frac{3-0}{1-7} = -\frac{1}{2}$$

and

$$\text{gradient of the normal} = 2.$$

The equation is

$$y - 3 = 2(x-1) \Rightarrow \underline{\underline{y = 2x + 1}}.$$

(c) the coordinates of P, (1)

Solution

$\underline{\underline{P(0, 1)}}$.

(d) the area of $\triangle PQR$. (4)

Solution

$$\begin{aligned} QP &= \sqrt{(0-1)^2 + (1-3)^2} \\ &= \sqrt{5}, \end{aligned}$$

and hence

$$\begin{aligned} \text{area} &= \tfrac{1}{2} \times QP \times QR \\ &= \tfrac{1}{2} \times \sqrt{5} \times 3\sqrt{5} \\ &= \tfrac{1}{2} \times 15 \\ &= \underline{\underline{7\tfrac{1}{2}\text{ units}^2}}. \end{aligned}$$

9. The line l_1 passes through the point $A(2, 5)$ and has gradient $-\frac{1}{2}$.

(a) Find an equation of l_1, giving your answer in the form $y = mx + c$. (3)

Solution

$$y - 5 = -\tfrac{1}{2}(x-2) \Rightarrow y - 5 = -\tfrac{1}{2}x + 1$$
$$\Rightarrow \underline{\underline{y = -\tfrac{1}{2}x + 6}}.$$

The point B has coordinates $(-2, 7)$.

(b) Show that B lies on l_1. (1)

Solution

$$x = -2 \Rightarrow y = -\tfrac{1}{2} \times (-2) + 6 = 7;$$

hence, $\underline{\underline{B \text{ lies on } (-2, 7)}}$.

(c) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer. (3)

Solution

$$\begin{aligned}\text{Length} &= \sqrt{(2-(2))^2 + (5-7)^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \\ &= \underline{\underline{2\sqrt{5}}}.\end{aligned}$$

The point C lies on l_1 and has x-coordinate equal to p. The length of AC is 5 units.

(d) Show that p satisfies (4)

$$p^2 - 4p - 16 = 0.$$

Solution

C is the point $(p, -\frac{1}{2}p + 6)$. Now,

$$\begin{aligned}AC^2 = 25 &\Rightarrow (p-2)^2 + (-\tfrac{1}{2}p + 6 - 5)^2 = 25 \\ &\Rightarrow (p-2)^2 + (-\tfrac{1}{2}p + 1)^2 = 25 \\ &\Rightarrow (p^2 - 4p + 4) + (\tfrac{1}{4}p^2 - p + 1) = 25 \\ &\Rightarrow \tfrac{5}{4}p^2 - 5p - 20 = 0 \\ &\Rightarrow \underline{\underline{p^2 - 4p - 16 = 0}},\end{aligned}$$

as required.

10. The points A and B have coordinates $(6, 7)$ and $(8, 2)$ respectively. The line l passes through the point A and is perpendicular to the line AB, as shown in Figure 3.

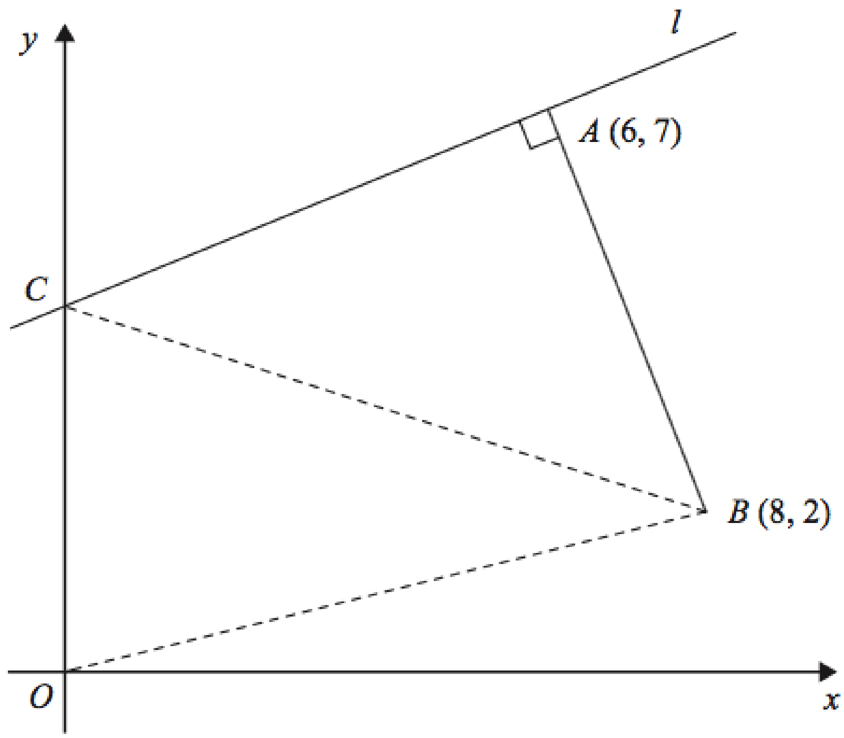

Figure 3: $\triangle OCB$

(a) Find an equation for l in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Solution

$$\text{Gradient of } AB = \frac{7-2}{6-8} = -\frac{5}{2}$$

and

$$\text{gradient of the perpendicular} = \frac{2}{5}.$$

Now,

$$\begin{aligned} y - 7 = \tfrac{2}{5}(x-6) &\Rightarrow 5(y-7) = 2(x-6) \\ &\Rightarrow 5y - 35 = 2x - 12 \\ &\Rightarrow 2x - 5y + 23 = 0. \end{aligned}$$

Given that l intersects the y-axis at the point C, find

(b) the coordinates of C, (2)

Solution

$$x = 0 \Rightarrow -5y + 23 = 0 \Rightarrow y = 4\tfrac{3}{5};$$

hence, $\underline{\underline{C(0, 4\tfrac{3}{5})}}$.

(c) the area of $\triangle OCB$, where O is the origin. (2)

Solution

$$\begin{aligned} \text{Area} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times \tfrac{23}{5} \times 8 \\ &= \tfrac{184}{10} \\ &= \underline{\underline{18\tfrac{2}{5} \text{ units}^2}}. \end{aligned}$$

11. The line l_1 has equation $3x + 5y - 2 = 0$.

(a) Find the gradient of l_1. (2)

Solution

$$\begin{aligned} 3x + 5y - 2 = 0 &\Rightarrow 5y = -3x + 2 \\ &\Rightarrow y = -\tfrac{3}{5}x + \tfrac{2}{5}; \end{aligned}$$

hence, $\underline{\underline{\text{the gradient} = -\tfrac{3}{5}}}$.

The line l_2 is perpendicular to l_1 and passes through the point $(3, 1)$.

(b) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants. (3)

Solution

$$\text{Gradient of the perpendicular } l_2 = \frac{5}{3}.$$

Hence,

$$\begin{aligned} y - 1 = \tfrac{5}{3}(x - 3) &\Rightarrow y - 1 = \tfrac{5}{3}x - 5 \\ &\Rightarrow \underline{\underline{y = \tfrac{5}{3}x - 4}}. \end{aligned}$$

12. (a) Find an equation of the line joining $A(7,4)$ and $B(2,0)$, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (3)

Solution

$$\text{Gradient} = \frac{4-0}{7-2} = \frac{4}{5}.$$

Now,

$$y - 0 = \frac{4}{5}(x-2) \Rightarrow 5y = 4x - 8$$
$$\Rightarrow \underline{\underline{4x - 5y - 8 = 0}}.$$

(b) Find the length of AB, leaving your answer in surd form. (2)

Solution

$$\begin{aligned}\text{Length} &= \sqrt{(7-2)^2 + (4-0)^2} \\ &= \sqrt{25+16} \\ &= \underline{\underline{\sqrt{41}}}.\end{aligned}$$

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(c) Find the value of t. (1)

Solution

$$t = 2 \times 4 = \underline{\underline{8}}.$$

(d) Find the area of triangle ABC. (2)

Solution

$$\begin{aligned}\text{Area} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times 5 \times 8 \\ &= \underline{\underline{20 \text{ units}^2}}.\end{aligned}$$

13. The line L_1 has equation $2y - 3x - k = 0$, where k is a constant. Given that the point $A(1,4)$ lies on L_1, find

(a) the value of k, (1)

Solution

$$x = 1, y = 4 \Rightarrow 8 - 3 - k = 0 \Rightarrow \underline{\underline{k = 5}}.$$

(b) the gradient of L_1. (2)

Solution

$$\begin{aligned} 2y - 3x - 5 = 0 &\Rightarrow 2y = 3x + 5 \\ &\Rightarrow y = \tfrac{3}{2}x + \tfrac{5}{2}; \end{aligned}$$

hence, the gradient is $\underline{\underline{\tfrac{3}{2}}}$.

The line L_2 passes through A and is perpendicular to L_1.

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Solution

$$\text{Gradient of the perpendicular } L_2 = -\frac{2}{3}$$

and the equation of the line is

$$\begin{aligned} y - 4 = -\frac{2}{3}(x - 1) &\Rightarrow 3(y - 4) = -2(x - 1) \\ &\Rightarrow 3y - 12 = -2x + 2 \\ &\Rightarrow \underline{\underline{2x + 3y - 14 = 0}}. \end{aligned}$$

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B. (2)

Solution

$$y = 0 \Rightarrow 2x = 14 \Rightarrow x = 7;$$

hence, L_2 crosses the x-axis at the point $\underline{\underline{B(7, 0)}}$

(e) Find the exact length of AB. (2)

Solution

$$\begin{aligned}\text{Length} &= \sqrt{(1-7)^2+(4-0)^2}\\ &= \sqrt{36+16}\\ &= \sqrt{52}\\ &= \underline{\underline{2\sqrt{13}}}.\end{aligned}$$

14. The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively. The line l is perpendicular to PQ and passes through the mid-point of PQ. Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (5)

Solution

$$\text{Gradient of } PQ = \frac{6-0}{-1-9} = -\frac{3}{5}$$

and

$$\text{gradient of the perpendicular} = \frac{5}{3}.$$

Mid-point of PQ is $(4, 3)$ and hence the equation is

$$\begin{aligned}y-3 = \frac{5}{3}(x-4) &\Rightarrow 3(y-3) = 5(x-4)\\ &\Rightarrow 3y-9 = 5x-20\\ &\Rightarrow \underline{\underline{5x-3y-11=0}}.\end{aligned}$$

15. The line l_1 has equation $2x - 3y + 12 = 0$.

(a) Find the gradient of l_1. (1)

Solution

$$\begin{aligned}2x-3y+12=0 &\Rightarrow 3y = 2x+12\\ &\Rightarrow y = \tfrac{2}{3}x+4;\end{aligned}$$

hence, the gradient is $\underline{\underline{\tfrac{2}{3}}}$.

The line l_1 crosses the x-axis at the point A and the y-axis at the point B, as shown in Figure 4.

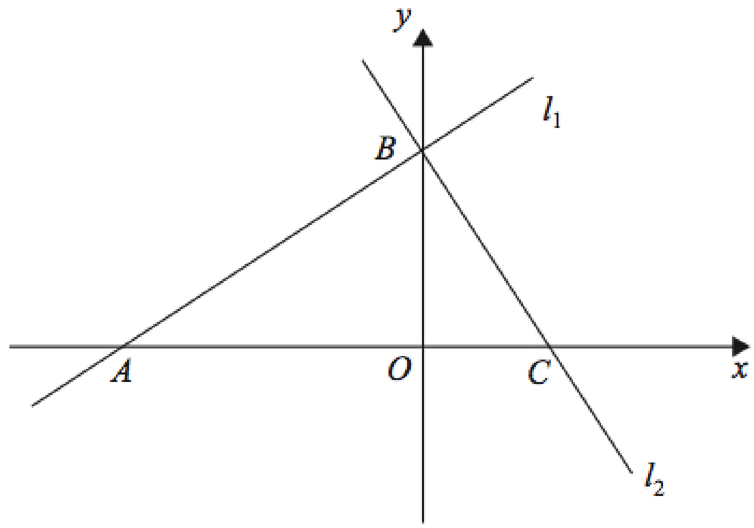

Figure 4: $\triangle ABC$

The line l_2 is perpendicular to l_1 and passes through B.

(b) Find an equation of l_2. (3)

Solution

$B(0, 4)$. Now,

$$\text{gradient of } l_1 = \frac{2}{3}$$

and

$$\text{gradient of the perpendicular} = -\frac{3}{2}.$$

Finally,

$$y - 4 = -\tfrac{3}{2}(x - 0) \Rightarrow \underline{\underline{y = -\tfrac{3}{2}x + 4}}.$$

The line l_2 crosses the x-axis at the point C.

(c) Find the area of triangle ABC. (4)

Solution

$A(-6, 0)$ and $C(\frac{8}{3}, 0)$. Finally,

$$\begin{aligned} \text{Area} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times (6 + \tfrac{8}{3}) \times 4 \\ &= \tfrac{1}{2} \times \tfrac{26}{3} \times 4 \\ &= \tfrac{104}{6} \\ &= \underline{\underline{17\tfrac{1}{3} \text{ units}^2}}. \end{aligned}$$

16. The line L_1 has equation $4y + 3 = 2x$. The point $A(p, 4)$ lies on L_1.

(a) Find the value of the constant p. (1)

Solution

$$16 + 3 = 2p \Rightarrow \underline{\underline{p = 9\tfrac{1}{2}}}.$$

The line L_2 passes through the point $C(2, 4)$ and is perpendicular to L_1.

(b) Find an equation for L_2 giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (5)

Solution

$$\begin{aligned} 4y + 3 = 2x \Rightarrow 4y &= 2x - 3 \\ \Rightarrow y &= \tfrac{1}{2}x - \tfrac{3}{4}. \end{aligned}$$

Now,

$$\text{gradient of } L_1 = \tfrac{1}{2}$$

and

$$\text{gradient of } L_2 = -2.$$

Finally,

$$\begin{aligned} y - 4 = -2(x - 2) &\Rightarrow y - 4 = -2x + 4 \\ &\Rightarrow \underline{\underline{2x + y - 8 = 0}}. \end{aligned}$$

The line L_1 and the line L_2 intersect at the point D.

(c) Find the coordinates of the point D. (3)

Solution
Now,

$$(4y+3)+y-8=0 \Rightarrow 5y=5$$
$$\Rightarrow y=1$$
$$\Rightarrow x=\tfrac{7}{2};$$

hence, $\underline{\underline{D(\tfrac{7}{2},1)}}$.

(d) Show that the length of CD is $\frac{3}{2}\sqrt{5}$. (3)

Solution

$$\begin{aligned}\text{Length} &= \sqrt{(2-\tfrac{7}{2})^2+(4-1)^2}\\ &= \sqrt{\tfrac{9}{4}+9}\\ &= \sqrt{\tfrac{45}{4}}\\ &= \sqrt{\tfrac{9}{4}\times 5}\\ &= \underline{\underline{\tfrac{3}{2}\sqrt{5}}}.\end{aligned}$$

A point B lies on L_1 and the length of $AB = \sqrt{80}$. The point E lies on L_2 such that the length of the line $CDE = 3$ times the length of CD.

(e) Find the area of the quadrilateral $ACBE$. (3)

Solution
Finally,

$$\begin{aligned}\text{Area} &= \text{area of } ABC + \text{area of } ABE\\ &= (\tfrac{1}{2}\times\tfrac{3}{2}\sqrt{5}\times\sqrt{80}) + (\tfrac{1}{2}\times 3\sqrt{5}\times\sqrt{80})\\ &= (\tfrac{1}{2}\times\tfrac{3}{2}\sqrt{400}) + (\tfrac{1}{2}\times 3\sqrt{400})\\ &= 10(\tfrac{3}{2}+3)\\ &= \underline{\underline{45 \text{ units}^2}}.\end{aligned}$$

17. The line l_1 has equation $y = -2x + 3$. The line l_2 is perpendicular to l_1 and passes through the point $(5, 6)$.

(a) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b, and c are integers. (3)

Solution

$$\text{Gradient of } L_1 = -2$$

and

$$\text{gradient of } L_2 = \tfrac{1}{2}.$$

Now,

$$\begin{aligned} y - 6 = \tfrac{1}{2}(x-5) &\Rightarrow 2(y-6) = x - 5 \\ &\Rightarrow 2y - 12 = x - 5 \\ &\Rightarrow \underline{\underline{x - 2y + 7 = 0}}. \end{aligned}$$

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(b) Find the x-coordinate of A and the y-coordinate of B. (2)

Solution

$\underline{\underline{A(-7, 0)}}$ and $\underline{\underline{B(0, \tfrac{7}{2})}}$.

Given that O is the origin,

(c) find the area of the triangle OAB. (2)

Solution

$$\begin{aligned} \text{Area} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times 7 \times \tfrac{7}{2} \\ &= \tfrac{49}{4} \\ &= \underline{\underline{12\tfrac{1}{4} \text{ units}^2}}. \end{aligned}$$

18. The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(a) Find an equation for L_1 in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Solution

$$\text{Gradient} = \frac{12-3}{11-(-1)} = \frac{3}{4}$$

and the equation is

$$\begin{aligned} y - 3 = \tfrac{3}{4}(x+1) &\Rightarrow 4(y-3) = 3(x+1) \\ &\Rightarrow 4y - 12 = 3x + 3 \\ &\Rightarrow \underline{\underline{3x - 4y + 15 = 0}}. \end{aligned}$$

The line L_2 has equation $3y + 4x - 30 = 0$.

(b) Find the coordinates of the point of intersection of L_1 and L_2. (3)

Solution

L_1 has equation

$$y - 3 = \tfrac{3}{4}(x+1) \Rightarrow y = \tfrac{3}{4}x + \tfrac{15}{4}$$

and

$$\begin{aligned} 3(\tfrac{3}{4}x + \tfrac{15}{4}) + 4x - 30 = 0 &\Rightarrow \tfrac{9}{4}x + \tfrac{45}{4} + 4x - 30 = 0 \\ &\Rightarrow \tfrac{25}{4}x = \tfrac{75}{4} \\ &\Rightarrow x = 3 \\ &\Rightarrow y = 6; \end{aligned}$$

hence, the point of intersection is $\underline{\underline{(3, 6)}}$.

19. The line L_1 has equation $4x + 2y - 3 = 0$.

(a) Find an gradient of L_1 (2)

Solution

$$\begin{aligned} 4x + 2y - 3 = 0 &\Rightarrow 2y = -4x + 3 \\ &\Rightarrow y = -2x + \tfrac{3}{2}; \end{aligned}$$

hence, the gradient is $\underline{\underline{-2}}$.

The line L_2 is perpendicular to L_1 and passes through the point $(2, 5)$.

(b) Find the equation of L_2 in the form $y = mx + c$, where m and c are constants. (3)

Solution

$$\text{Gradient of } L_2 = \tfrac{1}{2}$$

and

$$y - 5 = \tfrac{1}{2}(x - 2) \Rightarrow y - 5 = \tfrac{1}{2}x - 1$$
$$\Rightarrow \underline{\underline{y = \tfrac{1}{2}x + 4}}.$$

20. The line l_1, shown in Figure 5 has equation $2x + 3y = 26$.

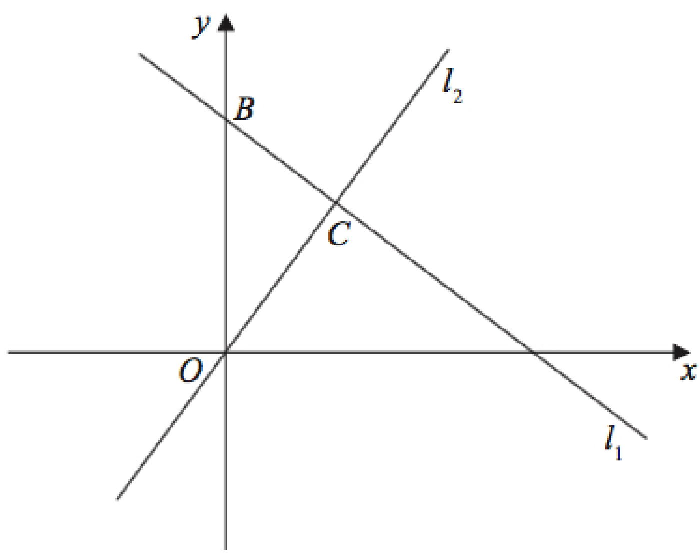

Figure 5: $\triangle OBC$

The line l_2 passes through the origin O and is perpendicular to l_1.

(a) Find an equation for the line l_2. (4)

Solution

$$2x + 3y = 26 \Rightarrow 3y = -2x + 26$$
$$\Rightarrow y = -\tfrac{2}{3}x + \tfrac{26}{3}.$$

Now,

$$\text{gradient of } l_1 = -\tfrac{2}{3}$$

and

$$\text{gradient of } l_2 = \tfrac{3}{2}.$$

Finally,

$$y - 0 = \tfrac{3}{2}(x - 0) \Rightarrow \underline{\underline{y = \tfrac{3}{2}x}}.$$

The line l_2 intersects the line l_1 at the point C. Line l_1 crosses the y-axis at the point B, as shown in Figure 5.

(b) Find the area of triangle OBC. Give your answer in the form $\frac{a}{b}$, where a and b are integers to be determined. (6)

Solution

$$\begin{aligned} 2x + 3y = 26 &\Rightarrow 2x + 3(\tfrac{3}{2}x) = 26 \\ &\Rightarrow \tfrac{13}{2}x = 26 \\ &\Rightarrow x = 4 \\ &\Rightarrow y = 6. \end{aligned}$$

Now, $B(0, \frac{26}{3})$ and $C(4, 6)$. Finally,

$$\begin{aligned} \text{Area} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times \tfrac{26}{3} \times 4 \\ &= \tfrac{104}{6} \\ &= \underline{\underline{\tfrac{52}{3} \text{ units}^2}}. \end{aligned}$$

21. Figure 6 shows a right angled triangle LMN.

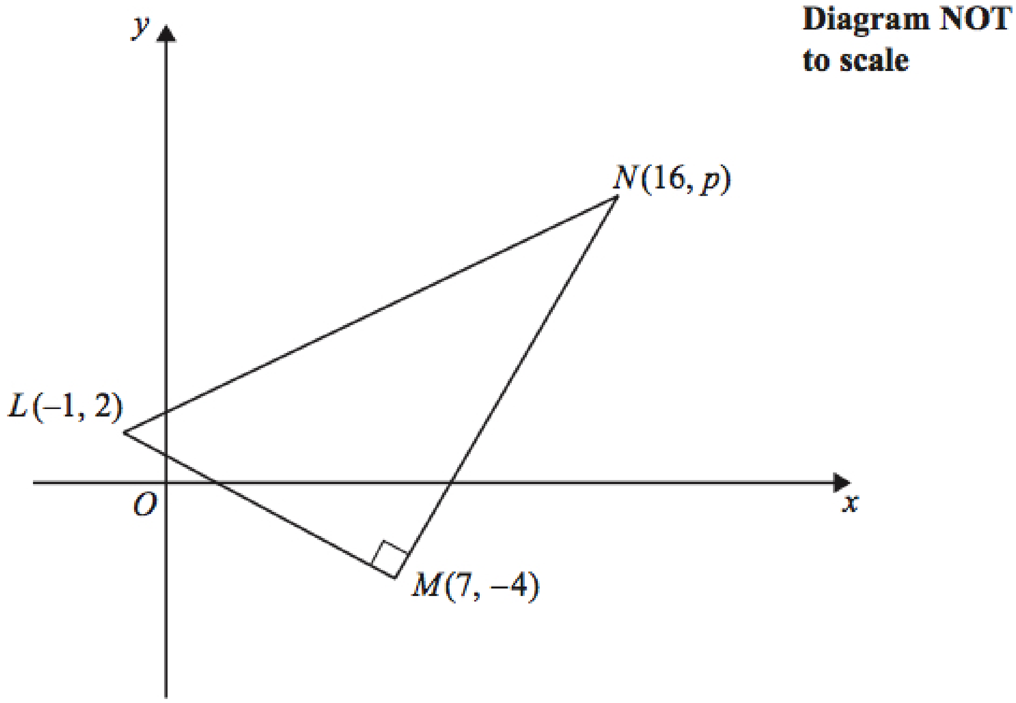

Figure 6: $\triangle LMN$

The points L and M have coordinates $(-1, 2)$ and $(7, -4)$ respectively.

(a) Find an equation for the straight line passing through the points L and M. Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Solution

$$\text{Gradient} = \frac{2 - (-4)}{-1 - 7} = -\frac{3}{4}$$

and the equation is

$$\begin{aligned} y - 2 = -\tfrac{3}{4}(x+1) &\Rightarrow 4(y-2) = -3(x+1) \\ &\Rightarrow 4y - 8 = -3x - 3 \\ &\Rightarrow \underline{\underline{3x + 4y - 5 = 0}}. \end{aligned}$$

Given that the coordinates of point N are $(16, p)$, where p is a constant, and angle $LMN = 90°$,

(b) find the value of p. (3)

Solution

$$\text{Gradient of } MN = \frac{4}{3}$$

and the equation is

$$\begin{aligned} y + 4 = \tfrac{4}{3}(x-7) &\Rightarrow 3(y+4) = 4(x-7) \\ &\Rightarrow 3y + 12 = 4x - 28 \\ &\Rightarrow 3y = 4x - 40. \end{aligned}$$

Now,

$$3y = 4 \times 16 - 40 = 24 \Rightarrow \underline{\underline{y = 8}}.$$

Given that there is a point K such that the points L, M, N, and K form a rectangle,

(c) find the y-coordinate of K. (2)

Solution

$$\begin{pmatrix} 16 \\ 8 \end{pmatrix} + \begin{pmatrix} -8 \\ 6 \end{pmatrix} = \begin{pmatrix} 8 \\ 14 \end{pmatrix};$$

hence, $\underline{\underline{y = 14}}$.

22. A curve C is given by (4)

$$y = 9x - 4x^3.$$

The points A and B lie on C and have x-coordinates of -2 and 1 respectively. Show that the length of AB is $k\sqrt{10}$ where k is a constant to be found.

Solution

$$\begin{aligned} x = 1 &\Rightarrow y = 9 \times 1 - 4 \times 1^3 = 5, \\ x = -2 &\Rightarrow y = 9 \times (-2) - 4 \times (-2)^3 = 14. \end{aligned}$$

Finally,

$$\begin{aligned} \text{length} &= \sqrt{(-2-1)^2 + (14-5)^2} \\ &= \sqrt{9 + 81} \\ &= \sqrt{90} \\ &= \underline{\underline{3\sqrt{10}}}. \end{aligned}$$

23. The points $P(0, 2)$ and $Q(3, 7)$ lie on the line l_1, as shown in Figure 7.

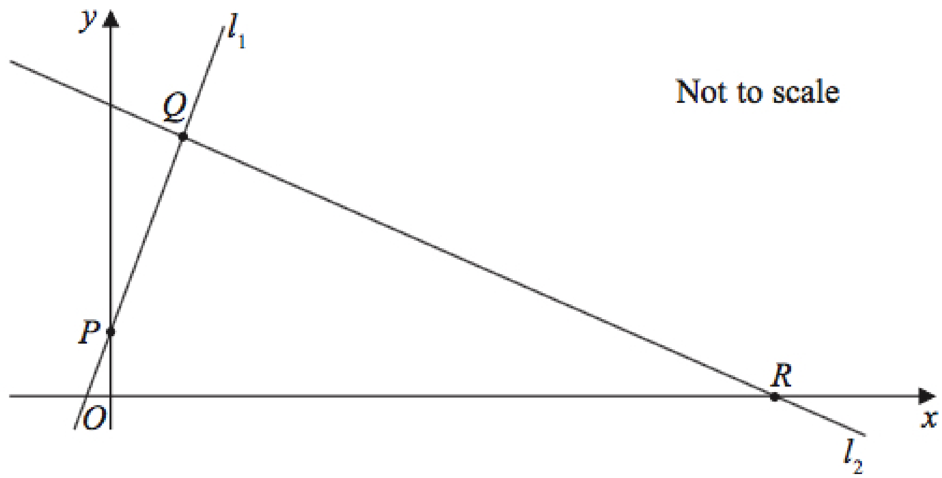

Figure 7: P, Q, and R

The line l_2 is perpendicular to l_1, passes through Q and crosses the x-axis at the point R, as shown in Figure 7. Find

(a) an equation for l_2, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers, (5)

Solution

$$\text{Gradient of } l_1 = \frac{7-2}{3-0} = \frac{5}{3}$$

and

$$\text{gradient of } l_2 = -\tfrac{3}{5}.$$

Finally,

$$\begin{aligned} y - 7 = -\tfrac{3}{5}(x-3) &\Rightarrow 5(y-7) = -3(x-3) \\ &\Rightarrow 5y - 35 = -3x + 9 \\ &\Rightarrow \underline{\underline{3x + 5y - 44 = 0}}. \end{aligned}$$

(b) the exact coordinates of R, (2)

Solution

$$y = 0 \Rightarrow 3x - 44 = 0 \Rightarrow x = \tfrac{44}{3};$$

hence, $\underline{\underline{R(\frac{44}{3}, 0)}}$.

(c) the exact area of the quadrilateral $ORQP$, where O is the origin. (5)

Solution

$$\begin{aligned}\text{Length of } PQ &= \sqrt{(3-0)^2 + (7-2)^2} \\ &= \sqrt{9+34} \\ &= \sqrt{34}\end{aligned}$$

and

$$\begin{aligned}\text{length of } QR &= \sqrt{(\tfrac{44}{3} - 3)^2 + (7-0)^2} \\ &= \sqrt{(\tfrac{35}{3})^2 + 49} \\ &= \sqrt{\tfrac{1225}{9} + 49} \\ &= \sqrt{136\tfrac{1}{9} + 49} \\ &= \sqrt{185\tfrac{1}{9}} \\ &= \sqrt{\tfrac{1666}{9}} \\ &= \sqrt{\tfrac{49}{9} \times 34} \\ &= \tfrac{7}{3}\sqrt{34}.\end{aligned}$$

Finally,

$$\begin{aligned}\text{Area} &= \text{area of } OPR + \text{area of } PQR \\ &= (\tfrac{1}{2} \times \tfrac{44}{3} \times 2) + (\tfrac{1}{2} \times \tfrac{7}{3}\sqrt{34} \times \sqrt{34}) \\ &= \tfrac{88}{6} + \tfrac{238}{6} \\ &= \tfrac{326}{6} \\ &= \underline{\underline{54\tfrac{1}{3} \text{ units}^2}}.\end{aligned}$$

24. The straight line l_1, shown in Figure 8, has equation $5y = 4x + 10$.

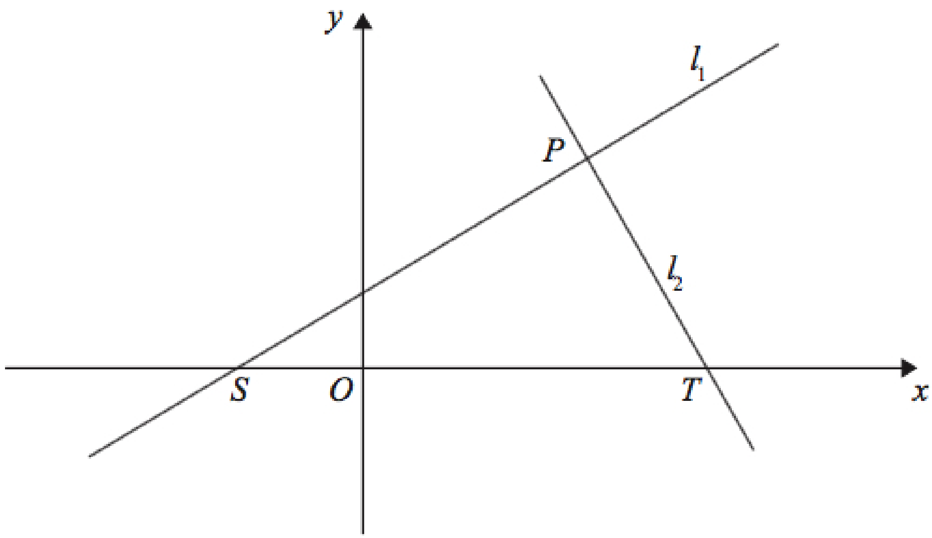

Figure 8: P, S, and T

The point P with x-coordinate 5 lies on l_1. The straight line l_2 is perpendicular to l_1 and passes through P.

(a) Find an equation for l_2, writing your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Solution

$$x = 5 \Rightarrow 5y = 30 \Rightarrow y = 6.$$

Now,

$$5y = 4x + 10 \Rightarrow y = \tfrac{4}{5}x + 2$$

and so

$$\text{gradient of } l_2 = -\tfrac{5}{4}.$$

Finally,

$$\begin{aligned} y - 6 = -\tfrac{5}{4}(x - 5) &\Rightarrow 4(y - 6) = -5(x - 5) \\ &\Rightarrow 4y - 24 = -5x + 25 \\ &\Rightarrow \underline{\underline{5x + 4y - 49 = 0}}. \end{aligned}$$

The lines l_1 and l_2 cut the x-axis at the points S and T respectively, as shown in Figure 8.

(b) Calculate the area of triangle SPT. (4)

Solution

$S(-2\frac{1}{2}, 0)$ and $T(9\frac{4}{5}, 0)$.

$$\begin{aligned} \text{Area} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times (2\tfrac{1}{2} + 9\tfrac{4}{5}) \times 6 \\ &= \tfrac{1}{2} \times \tfrac{123}{10} \times 6 \\ &= \tfrac{738}{20} \\ &= \underline{\underline{36\tfrac{9}{10} \text{ units}^2}}. \end{aligned}$$