Dr Oliver Mathematics
Mathematics: Higher
2013 Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

Section A

1. The functions f and g are defined by (2)

$$\mathrm{f}(x) = x^2 + 1 \text{ and } \mathrm{g}(x) = 3x - 4,$$

on the set of real numbers.

Find $\mathrm{g}(\mathrm{f}(x))$.

A. $3x^2 - 1$

B. $9x^2 - 15$

C. $9x^2 + 17$

D. $3x^3 - 4x^2 + 3x - 4$

2. The point $P(5, 12)$ lies on the curve with equation (2)

$$y = x^2 - 4x + 7.$$

What is the gradient of the tangent to this curve at P?

A. 2

B. 6

C. 12

D. 13

3. Calculate the discriminant of the quadratic equation (2)

$$2x^2 + 4x + 5 = 0.$$

A. -32

B. -24

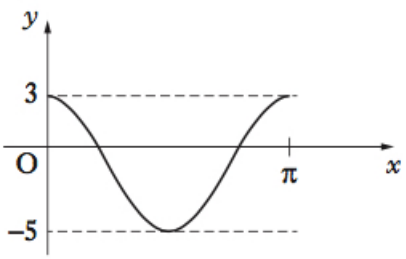
C. 48

D. 56

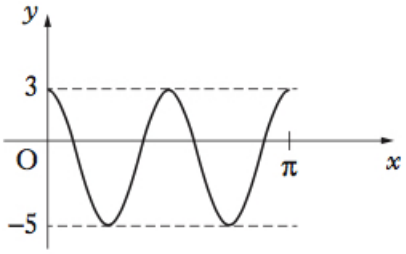
4. Which of the following shows the graph of (2)

$$y = 4\cos 2x - 1, \text{ for } 0 \leqslant x \leqslant \pi?$$

A



B



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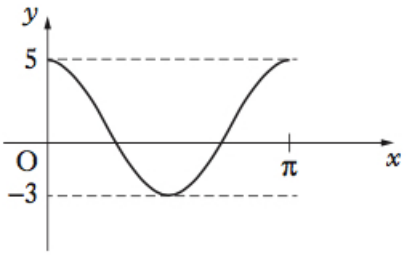

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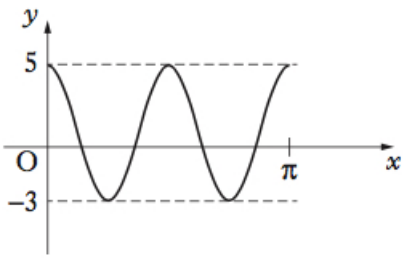


5. The line L passes through the point $(-2, -1)$ and is parallel to the line with equation (2)

$$5x + 3y - 6 = 0.$$

What is the equation of L?

A. $3x + 5y - 11 = 0$

B. $3x + 5y + 11 = 0$

C. $5x + 3y - 13 = 0$

D. $5x + 3y + 13 = 0$

6. What is the remainder when (2)

$$x^3 + 3x^2 - 5x - 6$$

is divided by $(x - 2)$?

A. 0

B. 3

C. 4

D. 8

7. Find (2)

$$\int x(3x + 2)\,\mathrm{d}x.$$

A. $x^3 + c$

B. $x^3 + x^2 + c$

C. $\frac{1}{2}x^2(\frac{3}{2}x^2 + 2x) + c$

D. $3x^2 + 2x + c$

8. A sequence is defined by the recurrence relation (2)

$$u_{n+1} = 0.1u_n + 8, \text{ with } u_1 = 11.$$

Here are two statements about this sequence:

(1) $u_0 = 9.1$;
(2) The sequence has a limit as $u \to \infty$.

Which of the following is true?

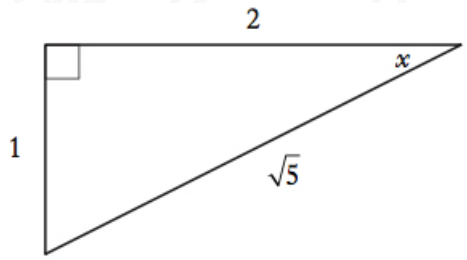
A. Neither statement is correct.

B. Only statement (1) is correct.

C. Only statement (2) is correct.

D. Both statements are correct.

9. The diagram shows a right-angled triangle with sides and angles as marked. (2)



Find the value of $\sin 2x$.

A. $\frac{4}{5}$

B. $\frac{2}{5}$

C. $\frac{2}{\sqrt{5}}$

D. $\frac{1}{\sqrt{5}}$

10. If $0 < a < 90$, which of the following is equivalent to $\cos(270 - a)^\circ$? (2)

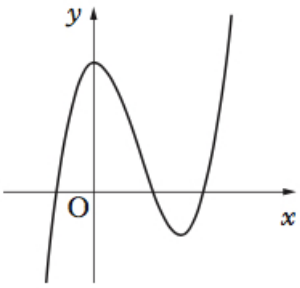
A. $\cos a^\circ$

B. $\sin a^\circ$

C. $-\cos a^\circ$

D. $-\sin a^\circ$

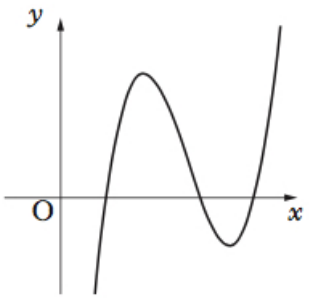
11. The diagram shows a cubic curve with equation $y = \mathrm{f}(x)$. (2)



Which of the following diagrams could show the curve with equation

$$y = -\mathrm{f}(x - k),\ k > 0?$$

A



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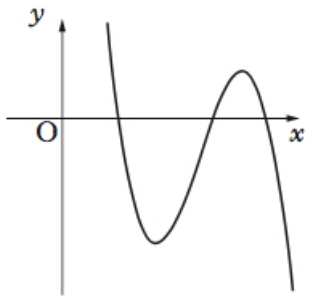

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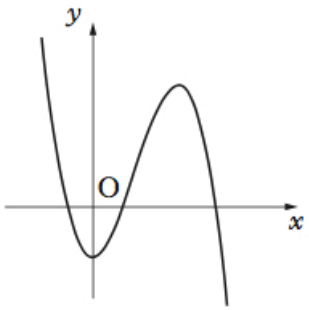

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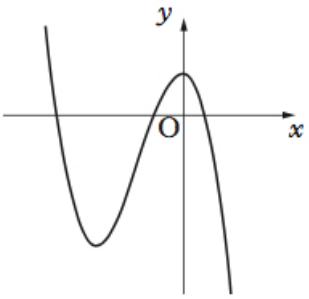


12. If (2)

$$\mathbf{f} = 3\mathbf{i} + 2\mathbf{k} \text{ and } \mathbf{g} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k},$$

find

$$|\mathbf{f} + \mathbf{g}|.$$

A. $\sqrt{14}$ units

B. $\sqrt{42}$ units

C. $\sqrt{66}$ units

D. $\sqrt{70}$ units

13. A function f is defined on a suitable domain by (2)

$$\mathrm{f}(x) = \frac{x+2}{x^2 - 7x + 12}.$$

What value(s) of x cannot be in this domain?

A. 3 and 4

B. -3 and -4

C. -2

D. 0

14. Given that (2)

$$|\mathbf{a}| = 3,\ |\mathbf{b}| = 2,\ \text{and}\ \mathbf{a}.\mathbf{b} = 5,$$

what is the value of

$$\mathbf{a}.(\mathbf{a} + \mathbf{b})?$$

A. 11

B. 14

C. 15

D. 21

15. Solve (2)

$$\tan\left(\frac{x}{2}\right) = -1$$

for $0 \leqslant x < 2\pi$.

A. $\frac{1}{2}\pi$

B. $\frac{7}{8}\pi$

C. $\frac{3}{2}\pi$

D. $\frac{15}{8}\pi$

16. Find (2)

$$\int (1 - 6x)^{-\frac{1}{2}}\, \mathrm{d}x,$$

where $x < \frac{1}{6}$.

A. $\frac{1}{9}(1-6x)^{-\frac{3}{2}}+c$

B. $3(1-6x)^{-\frac{3}{2}}+c$

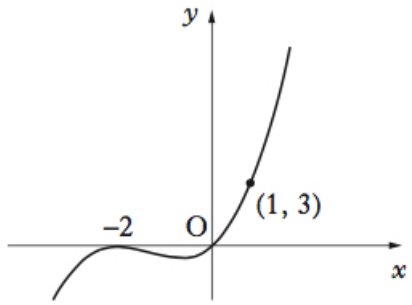
C. $-\frac{1}{3}(1-6x)^{\frac{1}{2}}+c$

D. $-3(1-6x)^{\frac{1}{2}}+c$

17. The diagram shows a curve with equation of the form (2)

$$y = kx(x+a)^2,$$

which passes through the points $(-2, 0)$, $(0, 0)$, and $(1, 3)$.



What are the values of a and k?

A. $a = -2$ and $k = \frac{1}{3}$

B. $a = -2$ and $k = 3$

C. $a = 2$ and $k = \frac{1}{3}$

D. $a = 2$ and $k = 3$

18. Given that (2)

$$y = \sin(x^2 - 3),$$

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

A. $\sin 2x$

B. $\cos 2x$

C. $2x\sin(x^2-3)$

D. $2x\cos(x^2-3)$

19. Solve (2)

$$1 - 2x - 3x^2 > 0,$$

where x is a real number.

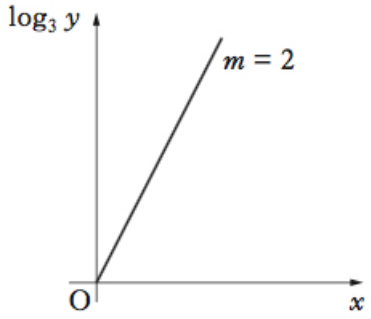
A. $x < -1$ or $x > \frac{1}{3}$

B. $-1 < x < \frac{1}{3}$

C. $x < -\frac{1}{3}$ or $x > 1$

D. $-\frac{1}{3} < x < 1$

20. The graph of $\log_3 y$ plotted against x is a line through the origin with gradient 2, as shown. (2)



Express y in terms of x.

A. $y = 2x$

B. $y = 9x$

C. $y = 6^x$

D. $y = 9^x$

Section B

21. Express (3)

$$2x^2 + 12x + 1$$

in the form

$$a(x + b)^2 + c.$$

22. A circle C_1 has equation

$$x^2 + y^2 + 2x + 4y - 27 = 0.$$

(a) Write down the centre and calculate the radius of C_1. (2)

The point $P(3, 2)$ lies on the circle C_1.

(b) Find the equation of the tangent at P. (3)

A second circle C_2 has centre $(10, -1)$. The radius of C_2 is half of the radius of C_1.

(c) Show that the equation of C_2 is (3)

$$x^2 + y^2 - 20x + 2y + 93 = 0.$$

(d) Show that the tangent found in part (b) is also a tangent to circle C_2. (4)

23. (a) The expression (4)

$$\sqrt{3}\sin x^\circ - \cos x^\circ$$

can be written

$$k\sin(x - a)^\circ,$$

where $k > 0$ and $0 \leqslant a < 360$.

Calculate the values of k and a.

(b) Determine the maximum value of (2)

$$4 + 5\cos x^\circ - 5\sqrt{3}\sin x^\circ,$$

where $0 \leqslant x < 360$.

24. (a) (i) Show that the points $A(-7, -8, 1)$, $T(3, 2, 5)$, and $B(18, 17, 11)$ are collinear. (4)
(ii) Find the ratio in which T divides AB.

The point C lies on the x-axis.

(b) If TB and TC are perpendicular, find the coordinates of C. (5)