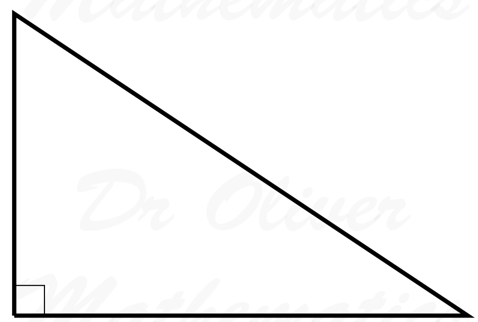
Dr Oliver Mathematics
AQA GCSE Mathematics
2013 November Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

1. This triangle is **drawn accurately**: it a right-angled triangle, 6 cm by 4 cm. (3)



Work out the area of the triangle.

Solution

$$\begin{aligned}\text{Area} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times 6 \times 4 \\ &= 3 \times 4 \\ &= \underline{\underline{12 \text{ cm}^2}}.\end{aligned}$$

2. Theatre tickets cost £T each.
Cinema tickets cost £C each. (2)

Write down an expression for the total cost of 20 theatre tickets and 16 cinema tickets.

Solution

$$\text{Total cost} = \underline{\underline{£(20T + 16C)}}.$$

3. (a) Multiply out (1)

$$3(2c - 1).$$

Solution

$$3(2c-1) = \underline{\underline{6c-3}}.$$

(b) Solve (1)

$$\frac{x}{20} = 10.$$

Solution

$$\frac{x}{20} = 10 \Rightarrow \underline{\underline{x = 200}}.$$

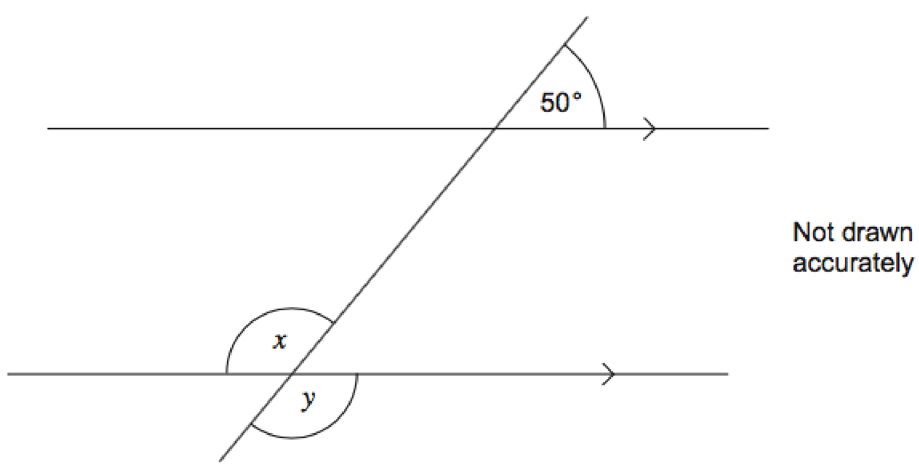
(c) Solve (3)

$$3y + 6 = 30 - 7y.$$

Solution

$$\begin{aligned} 3y + 6 = 30 - 7y &\Rightarrow 10y = 24 \\ &\Rightarrow \underline{\underline{y = 2.4}}. \end{aligned}$$

4. Here is a figure.



(a) Work out the size of angle x. (1)

Solution

$$x = 180 - 50 = \underline{\underline{130^\circ}}.$$

(b) Which one of these describes angles x and y? (1)
Circle your answer.

alternate angles corresponding angles interior angles vertically opposite angles

Solution

Vertically opposite angles.

5.

$$E = mv^2.$$

(a) Work out the value of E when $m = 3$ and $v = 10$. (2)

Solution

$$\begin{aligned} E &= 3 \times 10^2 \\ &= 3 \times 100 \\ &= \underline{\underline{300}}. \end{aligned}$$

Julie and Phil rearrange

$$E = mv^2$$

to make v the subject.
Here are their answers.

Julie	**Phil**
$E = mv^2$	$E = mv^2$
$\frac{E}{m} = v^2$	$\sqrt{E} = mv$
$\sqrt{\frac{E}{m}} = v$	$\frac{\sqrt{E}}{m} = v$
$v = \sqrt{\frac{E}{m}}$	$v = \frac{\sqrt{E}}{m}$

Which student has rearranged the formula correctly?
Tick a box.

☐ Julie ☐ Phil

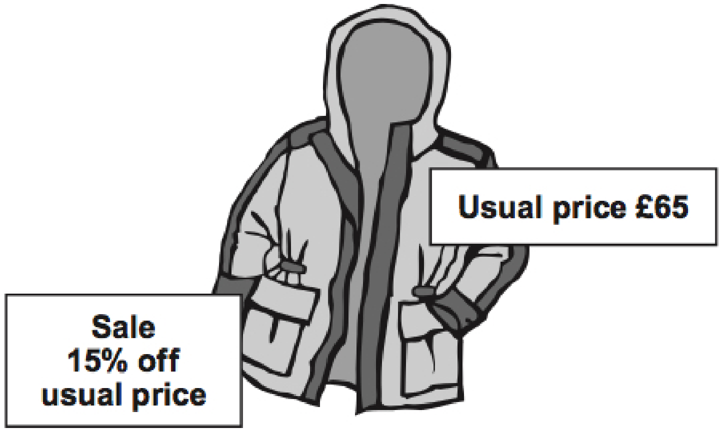
(b) What mistake has the other student made? (2)

Solution

Julie is right. Phil is wrong:

$$\begin{aligned} E = mv^2 &\Rightarrow \sqrt{E} = \sqrt{mv^2} \\ &\Rightarrow \sqrt{E} = \sqrt{m} \times \sqrt{v^2} \\ &\Rightarrow \sqrt{E} = \sqrt{m}\, v. \end{aligned}$$

6. A jacket has been reduced in a sale. (3)



Work out the sale price.

Solution

$$
\begin{aligned}
\text{Sale price} &= \frac{(100 - 15)}{100} \times 65 \\
&= \frac{85}{100} \times 65 \\
&= \frac{17}{20} \times 65 \\
&= 17 \times 3.25
\end{aligned}
$$

×	3	0.2	0.05
10	30	2	0.5
7	21	1.4	0.35

$$= \underline{\underline{£55.25}}.$$

7. There are 24 counters in a bag. (3)
One-third of the counters are blue.
5 red, 5 white, and 5 blue counters are added to the bag.

Tom says, "The probability of taking a blue counter from the bag is still $\frac{1}{3}$."

Is he correct?
Tick a box.

☐ Yes ☐ No ☐ Cannot tell

Give a reason for your answer.

Solution

Initially, there are

$$\tfrac{1}{3} \times 24 = 8$$

blue counters. There are fifteen added and that gives

$$\begin{aligned} \text{P(blue)} &= \frac{8+5}{24+15} \\ &= \tfrac{13}{39} \\ &= \tfrac{1}{3}; \end{aligned}$$

hence, Tom is correct.

8. Which of these fractions is closest to $\frac{3}{4}$? (3)

$$\tfrac{2}{3} \qquad \tfrac{3}{5} \qquad \tfrac{7}{10} \qquad \tfrac{13}{20}$$

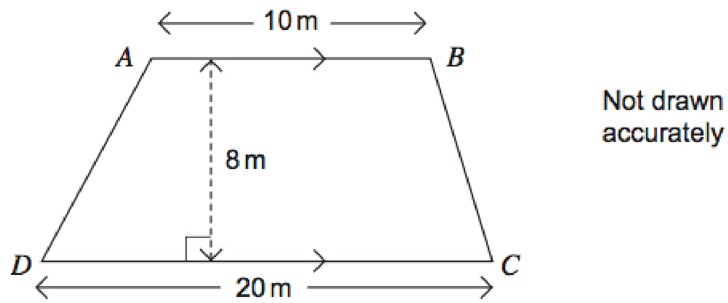
Solution

We convert each fraction in to a percentage:

$$\begin{aligned} \tfrac{3}{4} &\leftrightarrow 75\% \\ \tfrac{2}{3} &\leftrightarrow 66\tfrac{2}{3}\% \\ \tfrac{3}{5} &\leftrightarrow 60\% \\ \tfrac{7}{10} &\leftrightarrow 70\% \\ \tfrac{13}{20} &\leftrightarrow 65\% \end{aligned}$$

Clearly the four fractions are each less than 75% and so the fraction is closest to $\frac{3}{4}$ is $\frac{7}{10}$.

9. (a) $ABCD$ is a trapezium. (3)

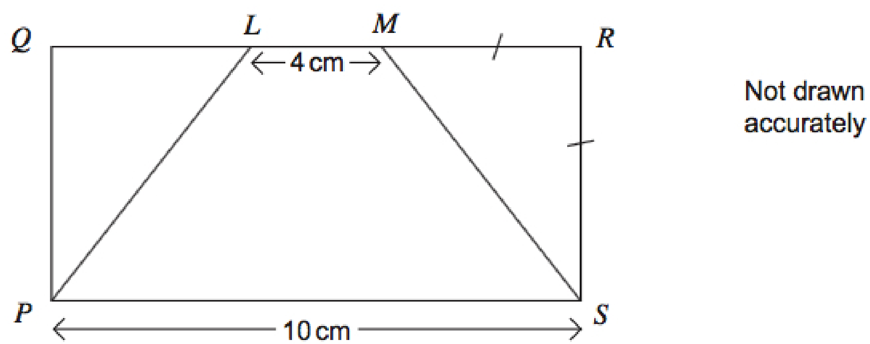


Calculate the area of $ABCD$.
State the units of your answer.

Solution

$$\begin{aligned}\text{Area} &= \tfrac{1}{2}(a+b)h \\ &= \tfrac{1}{2} \times (20+10) \times 8 \\ &= \tfrac{1}{2} \times 30 \times 8 \\ &= 15 \times 8 \\ &= \underline{\underline{120 \text{ m}^2}}.\end{aligned}$$

(b) $PQRS$ is a rectangle. (3)
$LM = 4$ cm.
$PS = 10$ cm.
$MR = RS$.



The area of $PLMS$ is 21 cm^2.
Show that

$$QL = MR.$$

Solution

Let the vertical height be h cm. Then

$$\begin{aligned} A = \tfrac{1}{2}(a+b)h &\Rightarrow 21 = \tfrac{1}{2}(10+4)h \\ &\Rightarrow 21 = \tfrac{1}{2}(14)h \\ &\Rightarrow 21 = 7h \\ &\Rightarrow h = 3. \end{aligned}$$

Now,

$$\begin{aligned} QL + LM + MR = 10 &\Rightarrow QL + 4 + 3 = 10 \\ &\Rightarrow QL + 7 = 10 \\ &\Rightarrow QL = 3. \end{aligned}$$

Hence,

$$\underline{\underline{QL = MR}}.$$

10. A fruit drink is made by mixing juice and lemonade in the ratio

$$\text{juice} : \text{lemonade} = 1 : 4.$$

Juice costs £6.00 per litre.
Lemonade costs 50p per litre.

(a) Show that 1 litre of the fruit drink costs £1.60 to make. (3)

Solution

$$\begin{aligned} \text{1 litre of the fruit drink} &= \text{800 g of juice} + \text{200 g of lemonade} \\ &= (\tfrac{1}{5} \times 1\,000 \text{ g of juice}) + (\tfrac{4}{5} \times 1\,000 \text{ g of lemonade}) \\ &= (\tfrac{1}{5} \times 6) + (\tfrac{4}{5} \times 0.5) \\ &= 1.2 + 0.4 \\ &= \underline{\underline{£1.60}}, \end{aligned}$$

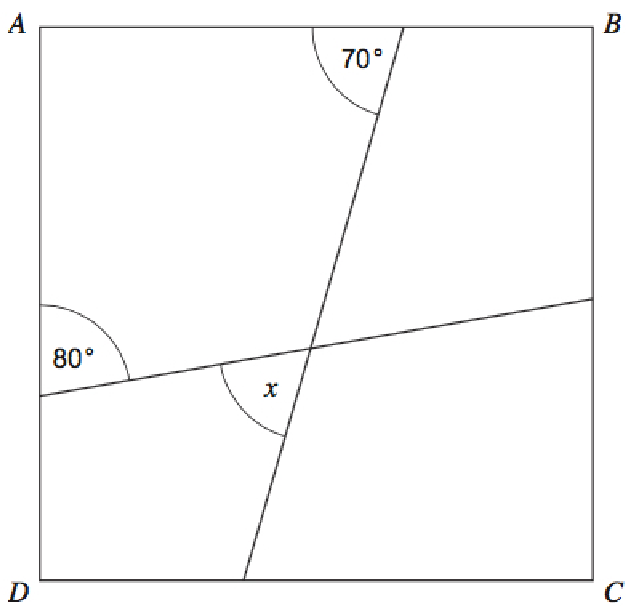
as required.

(b) The fruit drink is sold for £2 a litre. (2)
Work out the percentage profit.

Solution

$$
\begin{aligned}
\text{Percentage profit} &= \left(\frac{2 - 1.6}{1.6}\right) \times 100\% \\
&= \frac{0.4}{1.6} \times 100\% \\
&= \tfrac{1}{4} \times 100\% \\
&= \underline{\underline{25\%}}.
\end{aligned}
$$

11. $ABCD$ is a square. (4)



Not drawn accurately

Work out the size of angle x.
You must show your working, which may be on the diagram.

Solution

Let us make E the point on AD where the 80° emerges, F the point on AB where the 70° emerges, and G where the two diagonals cross. Well, $\angle EAF = 90°$ and

$$\begin{aligned} & \angle EAF + \angle AFG + \angle FGE + \angle GEA = 360 \\ \Rightarrow \quad & 90 + 70 + \angle FGE + 80 = 360 \\ \Rightarrow \quad & \angle FGE + 240 = 360 \\ \Rightarrow \quad & \angle FGE = 120°. \end{aligned}$$

Finally, due to supplementary angles,

$$x + 120 = 180 \Rightarrow \underline{\underline{x = 60°}}.$$

12. Jo teaches the violin. (4)
Half of her students take violins home to practise.
She wants to investigate the following hypothesis.

"Students who take violins home to practise score higher marks in violin exams."

Use the data handling cycle to describe how Jo could carry out this investigation and test her hypothesis.

Solution

E.g., Record all of the examination scores for those who do take the violin home.
Record all of the examination scores for those who do not take the violin home.
Record results in a way that allows a comparison: for example, average score for each group.
Refers to comparing the results for their chosen method

13. Solve the simultaneous equations (4)

$$\begin{aligned} 2x - 3y &= 7 \\ 3x + 4y &= 2. \end{aligned}$$

You **must** show your working.
Do **not** use trial and improvement.

Solution

$$\begin{aligned} 2x - 3y &= 7 \quad (1) \\ 3x + 4y &= 2 \quad (2) \end{aligned}$$

Do, for example, $4 \times (1)$ and $3 \times (2)$:

$$\begin{aligned} 8x - 12y &= 28 \quad (3) \\ 9x + 12y &= 6 \quad (4) \end{aligned}$$

Add $(3) + (4)$:

$$\begin{aligned} 17x = 34 &\Rightarrow \underline{\underline{x = 2}} \\ &\Rightarrow 2(2) - 3y = 7 \\ &\Rightarrow 4 - 3y = 7 \\ &\Rightarrow -3y = 3 \\ &\Rightarrow \underline{\underline{y = -1}}. \end{aligned}$$

14. A holiday park has three different areas to stay in.
Each area has three different types of home.
The table shows the number of families staying in the holiday park during the summer of 2013.

	Forest	Fields	Beach
Economy	55	50	60
Super	35	20	15
Luxury	10	30	25
Total	100	100	100

The manager sends a questionnaire to 60 families to ask them about their holiday.
The sample of size 60 is stratified by **type of home** and **area**.

(a) How many families who stayed in a **Luxury** home in the **Forest** are sent a questionnaire? (2)

Solution

$$\frac{60}{300} = \underline{\underline{\tfrac{1}{5}}}.$$

(b) How many families who stayed in a **Super** home are sent a questionnaire? (2)

Solution

$$\begin{aligned}\frac{35+20+15}{5} &= \frac{70}{5} \\ &= \underline{\underline{14}}.\end{aligned}$$

15. (a) Expand and simplify (2)

$$(2x+1)(3x-4).$$

Solution

$\times$	$2x$	$+1$
$3x$	$6x^2$	$+3x$
-4	$-8x$	-4

$$(2x+1)(3x-4) = \underline{\underline{6x^2 - 5x - 4}}.$$

(b) Factorise (2)

$$6x^2 - 23x - 4.$$

Solution

$$\left.\begin{array}{ll}\text{add to:} & -23 \\ \text{multiply to:} & (+6) \times (-4) = -24\end{array}\right\} -24,\ +1$$

For example,

$$\begin{aligned}6x^2 - 23x - 4 &= 6x^2 - 24x + x - 4 \\ &= 6x(x-4) + 1(x-4) \\ &= \underline{\underline{(6x+1)(x-4)}}.\end{aligned}$$

16. A bag contains triangles and quadrilaterals in the ratio of the number of sides of each shape.

(a) Explain why the least number of shapes that could be in the bag is 7. (1)

Solution

Triangles are 3-sided shapes and quadrilaterals are 4-sided shapes and 3 and 4 do not have any common factors (apart from 1):

$$3 + 4 = \underline{\underline{7}}.$$

(b) A shape is taken at random from the bag and **replaced**. (4)
Another shape is then taken from the bag.

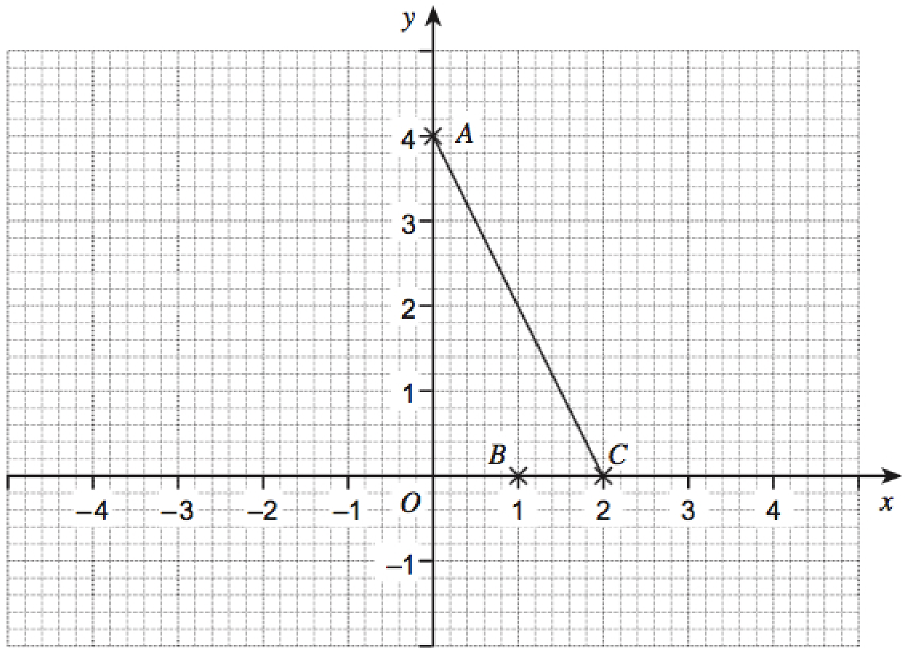
Work out the probability that the two shapes taken from the bag are of the same type.

Solution

$$\begin{aligned} \text{P(same type)} &= \text{P(triangles)} + \text{P(quadrilaterals)} \\ &= \left(\tfrac{3}{7} \times \tfrac{3}{7}\right) + \left(\tfrac{4}{7} \times \tfrac{4}{7}\right) \\ &= \tfrac{9}{49} + \tfrac{16}{49} \\ &= \underline{\underline{\tfrac{25}{49}}}. \end{aligned}$$

17. Show clearly that the equation of the line through B parallel to AC is (3)

$$2x + y = 2.$$



Solution

Well,

$$\begin{aligned}\text{gradient} &= \frac{4-0}{0-2} \\ &= -2\end{aligned}$$

and, using $A(0,4)$, the y-intercept is 4. Hence, the equation of the line which passes through A and C is

$$y = -2x + 4 \Rightarrow 2x + y = 4.$$

Clearly, any line parallel to AC has the equation

$$2x + y = c,$$

for some constant c. Finally, using $B(2,0)$,

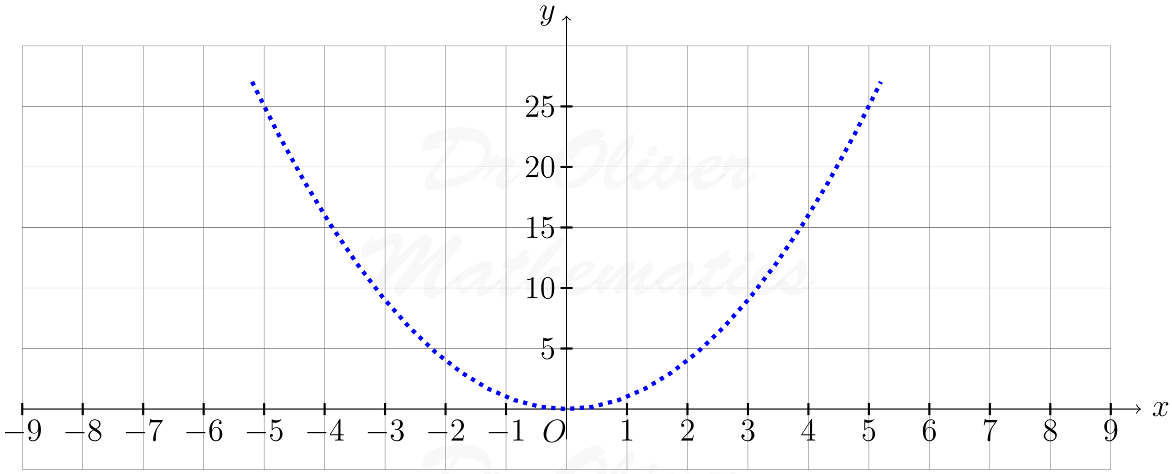
$$2(1) + 0 = 2$$

and the equation of the line through B parallel to AC is

$$\underline{\underline{2x + y = 2}}.$$

18. This graph is a sketch of

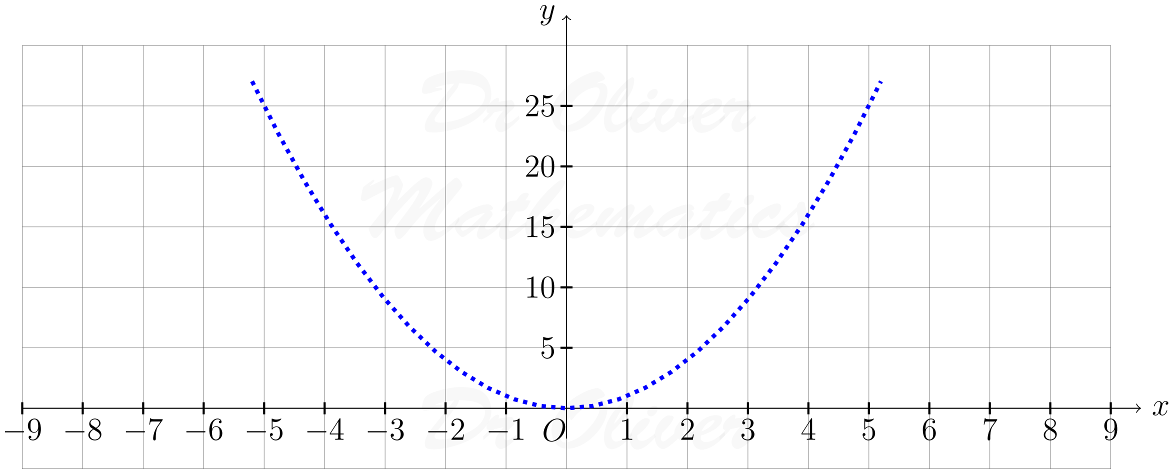
$$y = x^2.$$



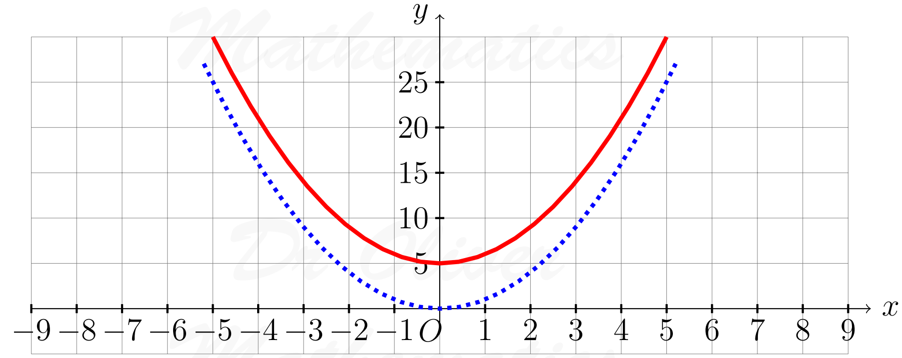
(a) Sketch the graph of (1)

$$y = x^2 + 5$$

on the grid.



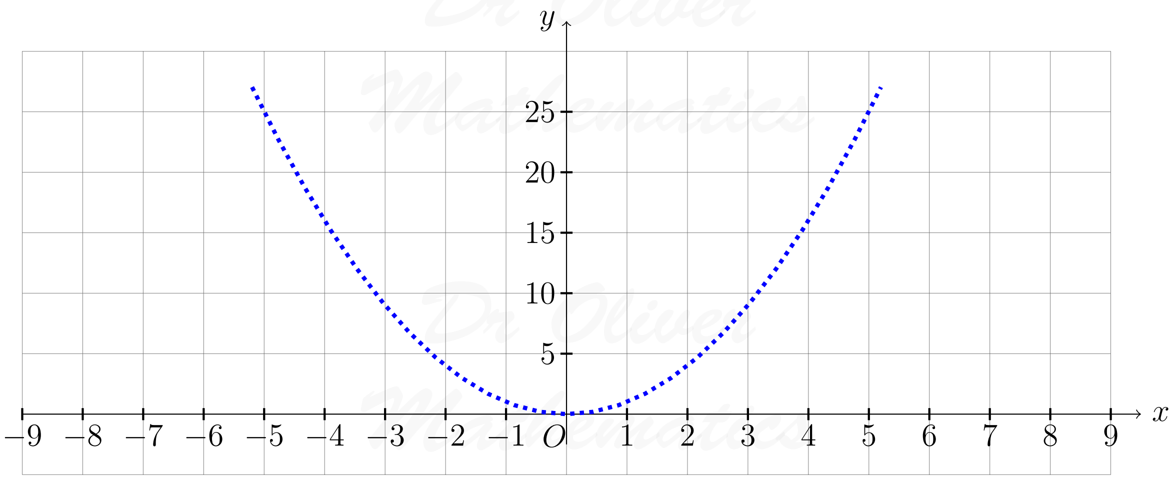
Solution



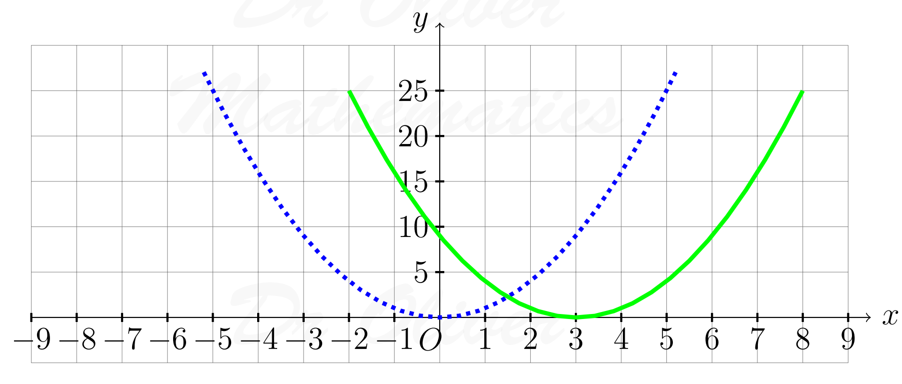
(b) Sketch the graph of (1)

$$y = (x - 3)^2$$

on the grid.



Solution



19. Solve (4)

$$x^2 + 8x + 6 = 0$$

by completing the square. Give your answer in the form $a \pm \sqrt{b}$, where a and b are integers.

Solution

$$\begin{aligned} x^2 + 8x + 6 = 0 &\Rightarrow x^2 + 8x = -6 \\ &\Rightarrow x^2 + 8x + 16 = -6 + 16 \\ &\Rightarrow (x + 4)^2 = 10 \\ &\Rightarrow x + 4 = \pm\sqrt{10} \\ &\Rightarrow \underline{\underline{x = -4 \pm \sqrt{10}}}; \end{aligned}$$

hence, $\underline{\underline{a = -4}}$ and $\underline{\underline{b = 10}}$.