Dr Oliver Mathematics
Advanced Subsidiary Paper 1: Pure Mathematics
June 2022: Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.
Inexact answers should be given to three significant figures unless otherwise stated.

1. Find **(4)**

$$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right) \mathrm{d}x,$$

giving your answer in simplest form.

2.

$$\mathrm{f}(x) = 2x^3 + 5x^2 + 2x + 15.$$

(a) Use the factor theorem to show that $(x+3)$ is a factor of $\mathrm{f}(x)$. **(2)**

(b) Find the constants a, b, and c such that **(2)**

$$\mathrm{f}(x) = (x+3)(ax^2 + bx + c).$$

(c) Hence show that $\mathrm{f}(x) = 0$ has only one real root. **(2)**

(d) Write down the real root of the equation **(1)**

$$\mathrm{f}(x-5) = 0.$$

3. The triangle PQR is such that $\overrightarrow{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 13\mathbf{i} - 15\mathbf{j}$.

(a) Find $\overrightarrow{QR}$. **(2)**

(b) Hence find $|\overrightarrow{QR}|$ giving your answer as a simplified surd. **(2)**

The point S lies on the line segment QR so that

$$QS : SR = 3 : 2.$$

(c) Find $\overrightarrow{PS}$. **(2)**

4. Figure 1 shows a sketch of triangle ABC with

- $AB = (x+2)$ cm,
- $BC = (3x+10)$ cm,

- $AC = 7x$ cm,
- angle $BAC = 60°$, and
- angle $ACB = \theta°$.

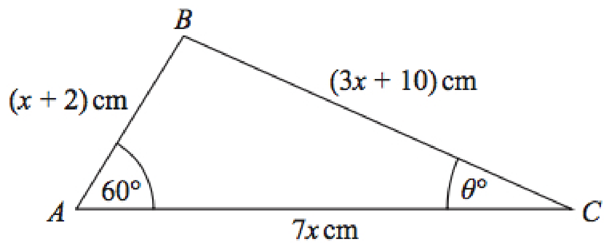

Figure 1: a sketch of triangle ABC

(a) (i) Show that (3)

$$17x^2 - 35x - 48 = 0.$$

(ii) Hence find the value of x. (1)

(b) Hence find the value of θ giving your answer to one decimal place. (2)

5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t,$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5.$$

(a) Use this relationship to find a complete equation for the model in the form (4)

$$A = pq^t,$$

giving the value of p and the value of q each to 4 significant figures.

(b) With reference to the model, interpret (2)

(i) the value of the constant p,

(ii) the value of the constant q.

(c) Find, according to the model, (3)

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(d) State one reason why this may not be a realistic model in the long term. (1)

6. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of (4)

$$\left(3 - \frac{2x}{9}\right)^8,$$

giving each term in simplest form.

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8.$$

(b) Find the coefficient of x^2 in the series expansion of $f(x)$, giving your answer as a simplified fraction. (2)

7. (a) Factorise completely (2)

$$9x - x^3.$$

The curve C has equation

$$y = 9x - x^3.$$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis. (2)

The line l has equation $y = k$, where k is a constant.

Given that C and l intersect at 3 distinct points,

(c) find the range of values for k, writing your answer in set notation. (3)
Solutions relying on calculator technology are not acceptable.

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t},\ t \in \mathbb{R},\ t \geqslant 0,$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm^2,

(a) state the value of k. (1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm^2. (3)
Give your answer in minutes to one decimal place.

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture. (2)
Give your answer in kg/cm^2 per minute to 3 significant figures.

9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p, (2)

(i) $\log_3 \left(\dfrac{x}{9}\right)$

(ii) $\log_3 (\sqrt{x})$.

(b) Hence, or otherwise, solve (4)

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11,$$

giving your answer as a simplified fraction.
Solutions relying on calculator technology are not acceptable.

10. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \tfrac{1}{3}x^2 - 2\sqrt{x} + 3,\ x \geqslant 0.$$

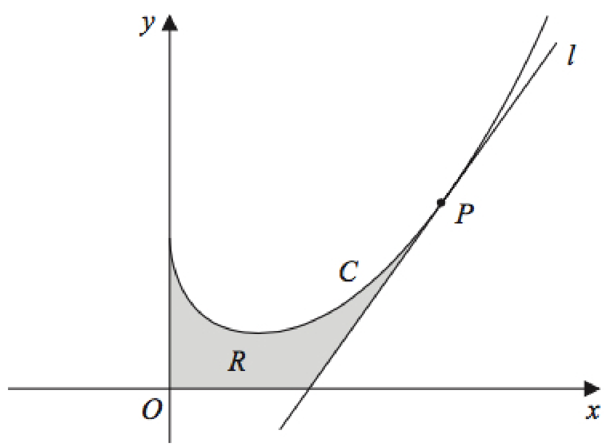

Figure 2: a sketch of part of the curve C

The point P lies on C and has x-coordinate 4.

The line l is the tangent to C at P.

(a) Show that l has equation (5)

$$13x - 6y - 26 = 0.$$

The region R, shown shaded in Figure 2, is bounded by the y-axis, the curve C, the line l, and the x-axis.

(b) Find the exact area of R. (5)

11. Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0.$$

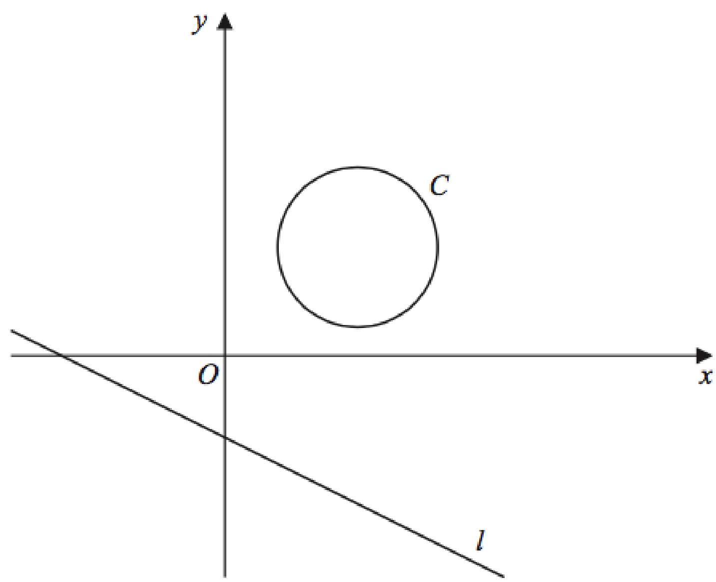

Figure 3: $x^2 + y^2 - 10x - 8y + 32 = 0$

(a) Find (3)

(i) the coordinates of the centre of C,

(ii) the radius of C.

(b) Find the shortest distance between C and l. (5)

12. A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is 355 cm^3.

The metal used

- for the circular base and the curved side costs 0.04 pence/cm^2 and
- for the circular top costs 0.09 pence/cm^2

Both metals used are of negligible thickness.

(a) Show that the total cost, C pence, of the metal for one container is given by (4)

$$C = 0.13\pi r^2 + \frac{28.4}{r}.$$

(b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures. (4)

(c) Using $\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}$ prove that the cost is minimised for the value of r found in part (b). (2)

(d) Hence find the minimum value of C, giving your answer to the nearest integer. (2)

13. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that (3)

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1-\sin\theta},\ \theta \neq (2n+1)90^\circ,\ n \in \mathbb{Z}.$$

Given that $\cos 2x \neq 0$,

(b) solve for $0^\circ < x < 90^\circ$, (5)

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x,$$

giving your answers to one decimal place.

14. (a) A student states: (1)

"if x^2 is greater than 9, then x must be greater than 3."

Determine whether or not this statement is true, giving a reason for your answer.

(b) Prove that for all positive integers n, (3)

$$n^3 + 3n^2 + 2n$$

is divisible by 6.