

Dr Oliver Mathematics

Base n

In this note, we will investigate base n .

1 Why do we need to know this?

Why do we need to? Well, is it not always base 10, is it?

- (a) feet and inches: e.g., 4 feet 2 inches equals 50 inches,
- (b) time (minutes and seconds): e.g., 2 minutes 5 seconds equals 125 seconds,
- (c) time (days and hours): e.g., 5 days 11 hours equals 131 hours,
- (d) time (weeks and days): e.g., 3 weeks 4 days equals 25 days,
- (e) pounds and ounces: e.g., 6 pounds 10 ounces equals 106 ounces,
- (f) stones and pounds: e.g., 11 stones 2 pounds equals 156 pounds,
- (g) furlongs: 8 furlongs equals 1 mile,

and so on. It turns out that we need to do it constantly, even though it is (for most people) automatic.

2 Bases

The *base* is a non-negative number that is greater than or equal to 1. Bases, from 2 to 16, are recorded below.

Base	Measure
2	Binary
3	Ternary
4	Quaternary
5	Quinary
6	Senary
7	Septenary
8	Octal
9	Nonary
10	Decimal
11	Undecimal
12	Duodecimal
13	Tridecimal
14	Tetradecimal
15	Pentadecimal
16	Hexadecimal

- (a) 0, 1, 2, ... 9 (as normal),
 (b) "A", "B", "C", ..., "Z" as 10, 11, 12, ..., 35,
 (c) "a", "b", "c", ..., "z" as 36, 37, 38, ..., 61,
 and you can go on (and on and on ...).

In base n , we count

- (a) 0, 1, 2, ..., $(n - 1)$,
 (b) 10, 11, 12, ..., $1(n - 1)$,
 (c) 20, 21, 22, ..., $2(n - 1)$,
 (d) 30, 31, 32, ..., $3(n - 1)$,

and so on.

For example,

Base	Counting
2	0, 1, 10, 11, 100, 101, 110, 111, ...
3	0, 1, 2, 10, 11, 12, 20, 21, 22, 100, ...
4	0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, ...

3 Changing from base n to base 10

Example 1

Express 10110_2 in base 10.

Solution

$$\begin{aligned}10110_2 &= [(1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)]_{10} \\ &= [16 + 0 + 4 + 1 + 0]_{10} \\ &= \underline{\underline{21}}_{10}.\end{aligned}$$

Example 2

Express 2012_3 in base 10.

Solution

$$\begin{aligned}2012_3 &= [(2 \times 3^3) + (0 \times 3^2) + (1 \times 3^1) + (2 \times 3^0)]_{10} \\ &= [54 + 0 + 3 + 2]_{10} \\ &= \underline{\underline{59}}_{10}.\end{aligned}$$

Example 3

Express $2C5A_{14}$ in base 10.

Solution

$$\begin{aligned}2C5A_{14} &= [(2 \times 14^3) + (12 \times 14^2) + (5 \times 14^1) + (10 \times 14^0)]_{10} \\ &= [5488 + 2352 + 70 + 10]_{10} \\ &= \underline{\underline{7920}}_{10}.\end{aligned}$$

4 Changing from base 10 to base n

Example 4

Express 2873_{10} in base 5.

Solution Well, it is quite hard to see what you need to do to. So, we take a logarithm to base 5:

$$\log_5 2873 = 4.947\dots$$

and so you can be sure that 5^4 is less than 2873. Now, we need to know what the quotient and remainder is on dividing 2873 by 5^4 :

$$2873 = 4 \times 5^4 + 373.$$

Next, we need to know what the quotient and remainder is on dividing 2873 by 5^4 :

$$373 = 2 \times 5^3 + 123,$$

and we keep going, from 5^2 , 5^1 , and 5^0 :

$$2873 = 4 \times 5^4 + 373$$

$$373 = 2 \times 5^3 + 123$$

$$123 = 4 \times 5^2 + 23$$

$$23 = 4 \times 5^1 + 3$$

$$3 = 3 \times 5^0 + 0$$

and hence

$$2873_{10} = \underline{\underline{42443}}_5.$$

What?! Explain, please!

Certainly!

$$\begin{aligned} 2873 &= (4 \times 5^4) + 373 \\ &= (4 \times 5^4) + (2 \times 5^3) + 123 \\ &= (4 \times 5^4) + (2 \times 5^3) + (4 \times 5^2) + 23 \\ &= (4 \times 5^4) + (2 \times 5^3) + (4 \times 5^2) + (4 \times 5^1) + 3 \\ &= (4 \times 5^4) + (2 \times 5^3) + (4 \times 5^2) + (4 \times 5^1) + (3 \times 5^0) \\ &= \underline{\underline{42443}}_5. \end{aligned}$$

Example 5

Express 250_{10} in base 9.

Solution

$$\log_9 250 = 2.512 \dots$$

and so we need 9^2 , 9^1 , and 9^0 .

$$250 = 3 \times 9^2 + 7$$

$$7 = 0 \times 9^1 + 7$$

$$7 = 7 \times 9^0 + 0$$

and

$$250_{10} = \underline{307}_9.$$

Is there another way? Yes, there is.

Example 6

Express 134_{10} in base 5.

Solution

$$134 = 26 \times 5 + 4$$

$$26 = 5 \times 5 + 1$$

$$5 = 1 \times 5 + 0$$

$$1 = 0 \times 5 + 1$$

and we read upwards: hence,

$$134_{10} = \underline{1014}_5.$$

Explain, please!

$$134 = 26 \times 5 + 4$$

$$= 5(5 \times 5 + 1) + 4$$

$$= (5^2 \times 5) + (5 \times 1) + 4$$

Well, there is a problem:

$$5^2 \times 5 \text{ is the same as } 5^3 \times 1$$

and so we convert the former to the latter:

$$= (5^3 \times 1) + (5^2 \times 0) + (5 \times 1) + 4$$

$$= \underline{1014}_5.$$

Example 7

Express 247_{10} in base 5.

Solution

$$247 = 49 \times 5 + 2$$

$$49 = 9 \times 5 + 4$$

$$9 = 1 \times 5 + 4$$

$$1 = 0 \times 5 + 1$$

and we read upwards: hence,

$$247_{10} = \underline{144}_5.$$

Example 8

Express 498_{10} in hexadecimal notation.

Solution

$$498 = 31 \times 16 + 2$$

$$31 = 1 \times 16 + 15$$

$$1 = 0 \times 16 + 1$$

and we read upwards: hence,

$$498_{10} = \underline{1F2}_{16},$$

On balance, I prefer the second approach (but it is up to you).

5 Changing from base n to base m

Example 9

Express $81A4_{15}$ in base 9.

Solution

First, change $81A4_{15}$ into base 10 and then convert this number into base 9.

$$\begin{aligned} 81A4_{15} &= [(8 \times 15^3) + (1 \times 15^2) + (10 \times 15^1) + (4 \times 15^0)]_{10} \\ &= [27000 + 225 + 15 + 4]_{10} \\ &= 27244_{10} \end{aligned}$$

and

$$27244 = 3027 \times 9 + 1$$

$$3027 = 336 \times 9 + 3$$

$$336 = 37 \times 9 + 3$$

$$37 = 4 \times 9 + 1$$

$$4 = 0 \times 9 + 4$$

and we read upwards: hence,

$$81A4_{15} = \underline{41331}_9.$$

6 Problems

Here are some problems for you to try.

1. Express 2713_8 in base 10.

Solution

$$\begin{aligned}2713_8 &= [(2 \times 8^3) + (7 \times 8^2) + (1 \times 8^1) + (3 \times 8^0)]_{10} \\ &= [1024 + 448 + 8 + 3]_{10} \\ &= \underline{\underline{1483}}_{10}.\end{aligned}$$

2. Express 7895_{11} in base 9.

Solution

$$\begin{aligned}7895_{11} &= [(7 \times 11^3) + (8 \times 11^2) + (9 \times 11^1) + (5 \times 11^0)]_{10} \\ &= [9317 + 968 + 99 + 5]_{10} \\ &= 10389_{10}.\end{aligned}$$

Now,

$$10389 = 1154 \times 9 + 3$$

$$1154 = 128 \times 9 + 2$$

$$128 = 14 \times 9 + 2$$

$$14 = 1 \times 9 + 5$$

$$1 = 0 \times 9 + 1$$

and we read upwards: hence,

$$7895_{11} = \underline{\underline{15223}}_9$$

3. Express 517618_{10} in base 7.

Solution

Now,

$$517618 = 73945 \times 7 + 3$$

$$73945 = 10563 \times 7 + 4$$

$$10563 = 1509 \times 7 + 0$$

$$1509 = 215 \times 7 + 4$$

$$215 = 30 \times 7 + 5$$

$$30 = 4 \times 7 + 2$$

$$4 = 0 \times 7 + 4$$

and we read upwards: hence,

$$517618_{10} = \underline{\underline{4254043_9}}.$$

4. Express 1211_3 in base 2.

Solution

$$\begin{aligned} 1211_3 &= [(1 \times 3^3) + (2 \times 3^2) + (1 \times 3^1) + (1 \times 3^0)]_{10} \\ &= [27 + 18 + 3 + 1]_{10} \\ &= 49_{10}. \end{aligned}$$

Now,

$$49 = 24 \times 2 + 1$$

$$24 = 12 \times 2 + 0$$

$$12 = 6 \times 2 + 0$$

$$6 = 3 \times 2 + 0$$

$$3 = 1 \times 2 + 1$$

$$1 = 0 \times 2 + 1$$

and we read upwards: hence,

$$1211_3 = \underline{\underline{110001_2}}.$$