

Dr Oliver Mathematics

Worked Examples

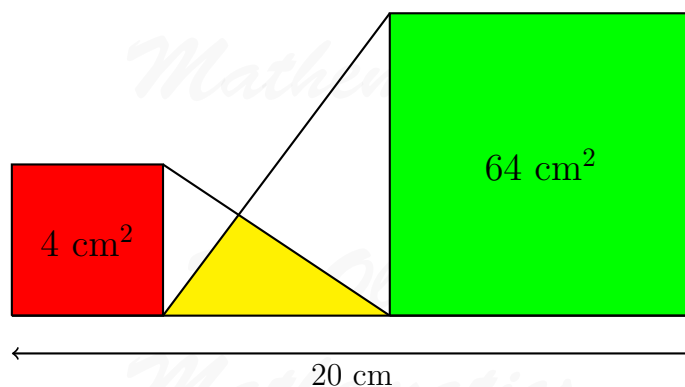
Area 8

From: PreMath, 14 January 2024

1. We have the following:

- a red square, with area 4 cm^2 ,
- a yellow triangle,
- a green square, with area 64 cm^2 , and
- the total length of these three shapes is 20 cm .

The diagram is **NOT**
drawn accurately



Find the area of the yellow triangle.

Solution

Red square: 4 cm^2 is $2 \text{ cm} \times 2 \text{ cm}$.

Green square: 64 cm^2 is $8 \text{ cm} \times 8 \text{ cm}$.

Now, the length of the whole things is 20 cm :

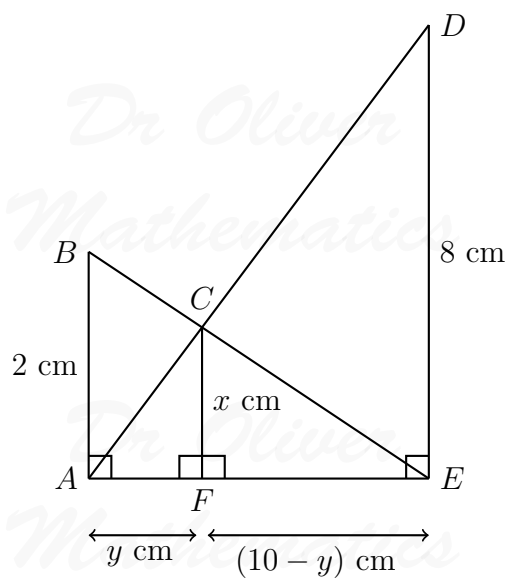
$$\begin{aligned} & \text{red square} + \text{yellow triangle} + \text{green square} = 20 \\ \Rightarrow & 2 + \text{yellow triangle} + 8 = 20 \\ \Rightarrow & \text{yellow triangle} = 10, \end{aligned}$$

and so the base of the yellow triangle is 10 cm.

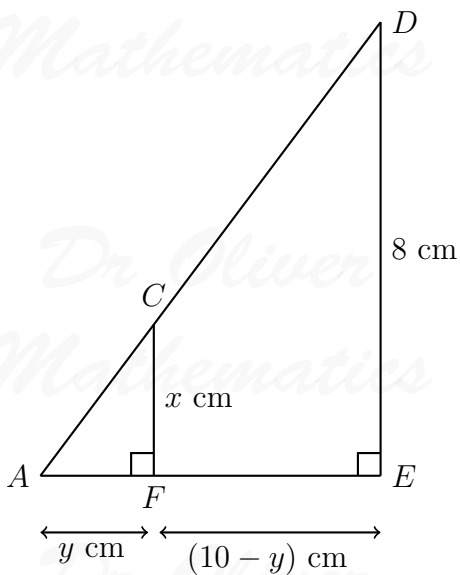
Now, we re-draw the picture, sticking labels on points.

- Let $CF = x$ cm be the height of the triangle.
- Let $AF = y$ cm such that $\angle AFC = 90^\circ$.

Then



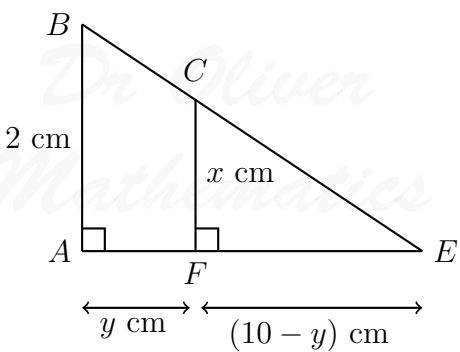
Triangles ACF and ADE are similar:



So

$$\frac{y}{10} = \frac{x}{8} \Rightarrow y = \frac{5}{4}x \quad (1).$$

Triangles CEF and ABC are similar:



So

$$\begin{aligned} \frac{10 - y}{10} &= \frac{x}{2} \Rightarrow 10 - y = 5x \\ &\Rightarrow y = 10 - 5x \quad (2). \end{aligned}$$

Do (1) = (2):

$$\begin{aligned} \frac{5}{4}x &= 10 - 5x \Rightarrow \frac{25}{4}x = 10 \\ &\Rightarrow x = \frac{8}{5}; \end{aligned}$$

so, the height of the green triangle is $\frac{8}{5}$ cm.

Finally,

$$\begin{aligned}\text{area of the yellow triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 10 \times \frac{8}{5} \\ &= \underline{\underline{8 \text{ cm}^2}}.\end{aligned}$$