

Dr Oliver Mathematics
Mathematics
Linear Equations, Quadratic Equations, and Quadratic
Inequalities
Past Examination Questions

This booklet consists of 33 questions across a variety of examination topics.
The total number of marks available is 230.

1. Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

- (a) express $f(x)$ in the form $(x - a)^2 + b$, where a and b are integers. (3)

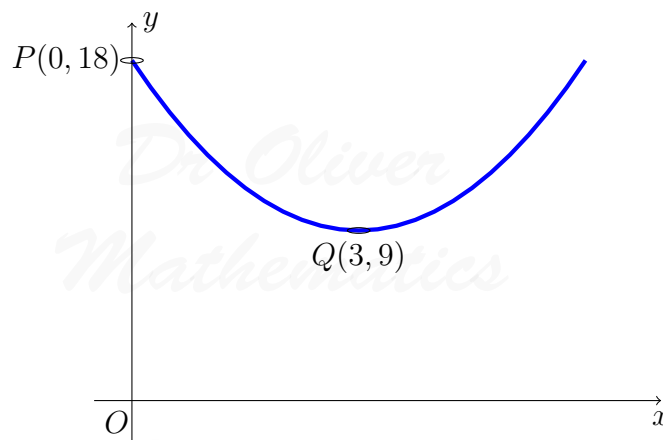
Solution

$$\begin{aligned} f(x) &= (x^2 - 6x + 9) + 9 \\ &= \underline{\underline{(x - 3)^2 + 9}}. \end{aligned}$$

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y -axis at P and has a minimum point at Q .

- (b) Sketch the graph of C , showing the coordinates of P and Q . (4)

Solution



The line $y = 41$ meets C at the point R .

- (c) Find the x -coordinate of R , giving your answer in the form $p + q\sqrt{2}$, where p and q are integers. (5)

Solution

$$\begin{aligned}x^2 - 6x + 18 = 41 &\Rightarrow (x - 3)^2 + 9 = 41 \\&\Rightarrow (x - 3)^2 = 32 \\&\Rightarrow (x - 3)^2 = \sqrt{32} \text{ (as } x \text{ is non-negative)} \\&\Rightarrow x - 3 = 4\sqrt{2} \\&\Rightarrow \underline{\underline{x = 3 + 4\sqrt{2}}}.\end{aligned}$$

2. Given that the equation (4)

$$kx^2 + 12x + k = 0,$$

where k is a positive constant, has equal roots, find the value of k .

Solution

$a = k$, $b = 12$, and $c = k$:

$$\begin{aligned}b^2 - 4ac = 0 &\Rightarrow 12^2 - 4k^2 = 0 \\&\Rightarrow 4k^2 = 144 \\&\Rightarrow k^2 = 36 \\&\Rightarrow \underline{\underline{k = 6}}.\end{aligned}$$

- 3.

$$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$

where a and b are constants.

- (a) Find the value of a and the value of b . (3)

Solution

$$\begin{aligned}x^2 - 8x - 29 &\equiv (x^2 - 8x + 16) - 45 \\&\equiv \underline{\underline{(x - 4)^2 - 45}}.\end{aligned}$$

- (b) Hence, or otherwise, show that the roots of (3)

$$x^2 - 8x - 29 = 0$$

are $c \pm d\sqrt{5}$, where c and d are integers to be found.

Solution

$$\begin{aligned}x^2 - 8x - 29 = 0 &\Rightarrow (x - 4)^2 - 45 = 0 \\&\Rightarrow (x - 4)^2 = 45 \\&\Rightarrow x - 4 = \pm 3\sqrt{5} \\&\Rightarrow \underline{x = 4 \pm 3\sqrt{5}};\end{aligned}$$

hence, $c = 4$ and $d = 3$.

4. Find the set of values of x for which

- (a) $3(2x + 1) > 5 - 2x$, (2)

Solution

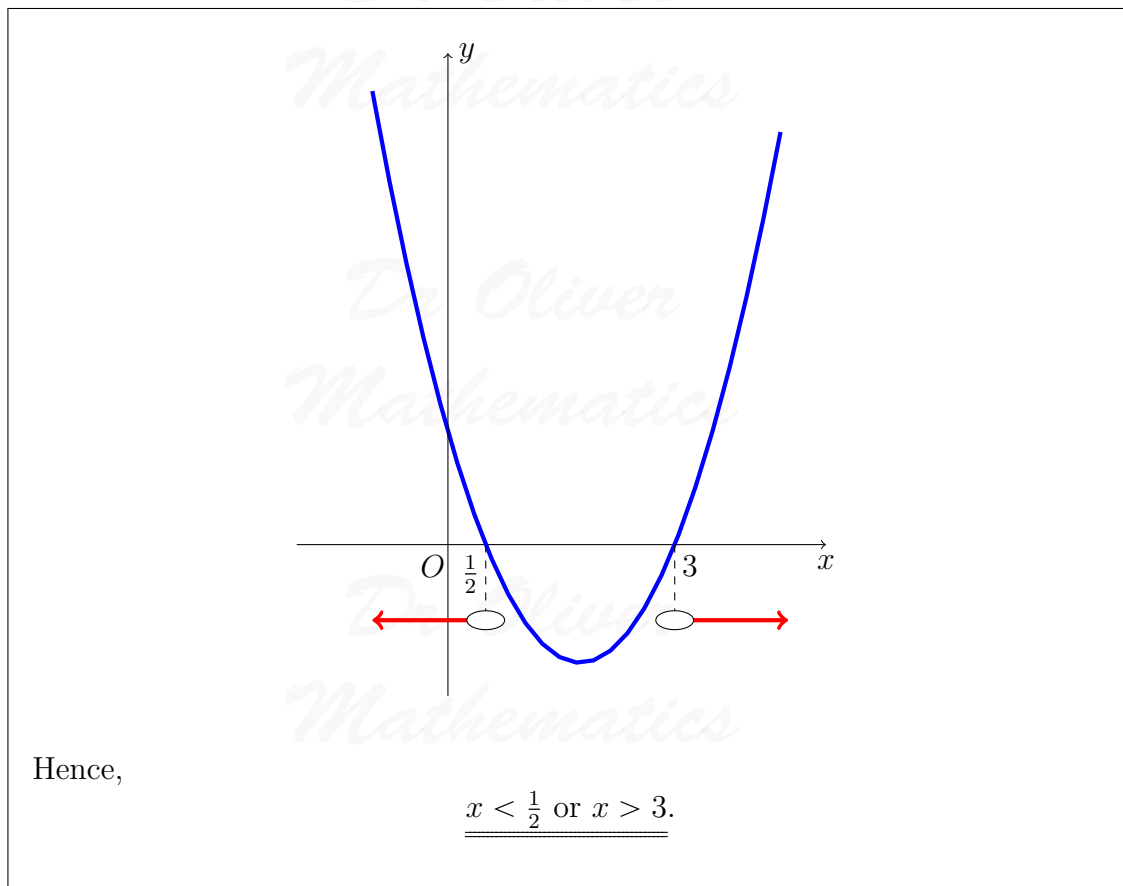
$$\begin{aligned}3(2x + 1) > 5 - 2x &\Rightarrow 6x + 3 > 5 - 2x \\&\Rightarrow 8x > 2 \\&\Rightarrow \underline{x > \frac{1}{4}}.\end{aligned}$$

- (b) $2x^2 - 7x + 3 > 0$, (4)

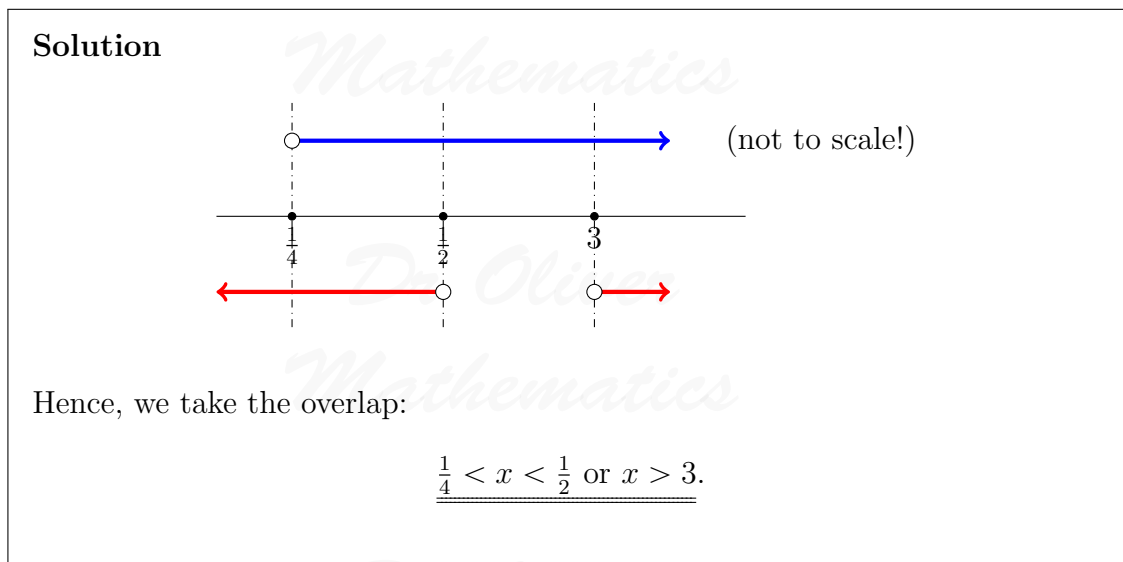
Solution

$$\begin{aligned}2x^2 - 7x + 3 > 0 &\Rightarrow 2x^2 - 6x - x + 3 > 0 \\&\Rightarrow 2x(x - 3) - (x - 3) > 0 \\&\Rightarrow (2x - 1)(x - 3) > 0.\end{aligned}$$

The critical values are $x = \frac{1}{2}$ and $x = 3$.



(c) **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0$. (2)



5.

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

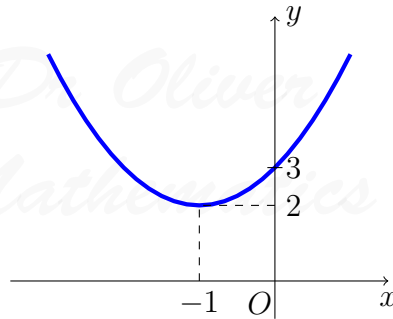
- (a) Find the values of the constants a and b . (2)

Solution

$$\begin{aligned}x^2 + 2x + 3 &\equiv (x^2 + 2x + 1) + 2 \\ &\equiv \underline{\underline{(x + 1)^2 + 2}}.\end{aligned}$$

- (b) In the space provided below, sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes. (3)

Solution



- (c) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (b). (2)

Solution

The discriminant is

$$b^2 - 4ac = 2^2 - 4 \times 1 \times 3 = 4 - 12 = \underline{\underline{-8}}$$

and the quadratic has no real roots.

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

- (d) Find the set of possible values of k , giving your answer in surd form. (4)

Solution

$a = 1$, $b = k$, and $c = 3$:

$$b^2 - 4ac < 0 \Rightarrow k^2 - 4 \times 1 \times 3 < 0$$

$$\Rightarrow k^2 < 12$$

$$\Rightarrow \underline{\underline{-2\sqrt{3} < k < 2\sqrt{3}}}.$$

6. Find the set of values of x for which

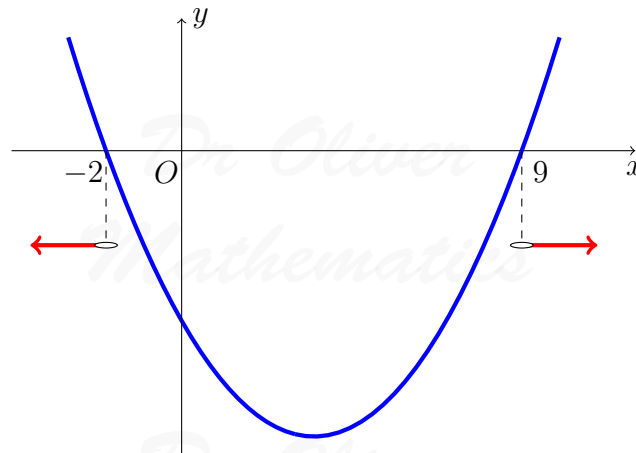
$$x^2 - 7x - 18 > 0.$$

(4)

Solution

$$x^2 - 7x - 18 > 0 \Rightarrow (x - 9)(x + 2) > 0.$$

We will now draw the graph.



Hence,

$$\underline{\underline{x < -2 \text{ or } x > 9.}}$$

7. The equation

$$x^2 + 2px + (3p + 4) = 0,$$

where p is a positive constant, has equal roots.

(a) Find the value of p .

(4)

Solution

$a = 1$, $b = 2p$, and $c = 3p + 4$:

$$\begin{aligned}b^2 - 4ac = 0 &\Rightarrow (2p)^2 - 4 \times 1 \times (3p + 4) = 0 \\&\Rightarrow 4p^2 - 4(3p + 4) = 0 \\&\Rightarrow 4p^2 - 12p - 16 = 0 \\&\Rightarrow 4(p^2 - 3p - 4) = 0 \\&\Rightarrow 4(p - 4)(p + 1) = 0 \\&\Rightarrow \underline{\underline{p = 4}},\end{aligned}$$

as $p \neq -1$.

(b) For this value of p , solve the equation

(2)

$$x^2 + 2px + (3p + 4) = 0.$$

Solution

$$\begin{aligned}x^2 + 8x + 16 = 0 &\Rightarrow (x + 4)^2 = 0 \\&\Rightarrow \underline{\underline{x = -4}} \text{ (with multiplicity of two).}\end{aligned}$$

8. The equation

(4)

$$2x^2 - 3x - (k + 1) = 0,$$

where k is a constant, has no real roots. Find the set of possible values of k .

Solution

$a = 2$, $b = -3$, and $c = -(k + 1)$:

$$\begin{aligned}b^2 - 4ac < 0 &\Rightarrow (-3)^2 - 4 \times 2 \times [-(k + 1)] < 0 \\&\Rightarrow 9 + 8(k + 1) < 0 \\&\Rightarrow 8(k + 1) < -9 \\&\Rightarrow k + 1 < -1\frac{1}{8} \\&\Rightarrow \underline{\underline{k < -2\frac{1}{8}}}.\end{aligned}$$

9. The equation

$$x^2 + kx + (k + 3) = 0,$$

where k is a constant, has different real roots.

(a) Show that $k^2 - 4k - 12 > 0$.

(2)

Solution

$a = 1$, $b = k$, and $c = k + 3$:

$$\begin{aligned} b^2 - 4ac > 0 &\Rightarrow k^2 - 4 \times 1 \times (k + 3) > 0 \\ &\Rightarrow \underline{\underline{k^2 - 4k - 12 > 0}}. \end{aligned}$$

(b) Find the set of possible values of k .

(4)

Solution

$$\begin{aligned} k^2 - 4k - 12 > 0 &\Rightarrow (k - 6)(k + 2) > 0 \\ &\Rightarrow \underline{\underline{k < -2}} \text{ or } \underline{\underline{k > 6}}. \end{aligned}$$

10. The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x .

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

Solution

$a = 1$, $b = k$, and $c = 8 - k$:

$$\begin{aligned} b^2 - 4ac < 0 &\Rightarrow k^2 - 4 \times 1 \times (8 - k) < 0 \\ &\Rightarrow \underline{\underline{k^2 + 4k - 32 < 0}}. \end{aligned}$$

(b) Hence find the set of possible values of k .

(4)

Solution

$$\begin{aligned} k^2 + 4k - 32 < 0 &\Rightarrow (k + 8)(k - 4) < 0 \\ &\Rightarrow \underline{\underline{-8 < k < 4}}. \end{aligned}$$

11. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,
(a) show that $q^2 + 8q < 0$. (2)

Solution

$a = 2q$, $b = q$, and $c = -1$:

$$\begin{aligned} b^2 - 4ac < 0 &\Rightarrow q^2 - 4 \times (2q) \times (-1) < 0 \\ &\Rightarrow \underline{q^2 + 8q < 0}. \end{aligned}$$

- (b) Hence find the set of possible values of q . (3)

Solution

$$\begin{aligned} q^2 + 8q < 0 &\Rightarrow q(q + 8) < 0 \\ &\Rightarrow \underline{-8 < q < 0}. \end{aligned}$$

12. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x .

- (a) Show that k satisfies $k^2 - 5k + 4 > 0$. (3)

Solution

$a = k$, $b = 4$, and $c = 5 - k$:

$$\begin{aligned} b^2 - 4ac > 0 &\Rightarrow 4^2 - 4 \times k \times (5 - k) > 0 \\ &\Rightarrow 16 - 4k(5 - k) > 0 \\ &\Rightarrow 16 - 20k + 5k^2 > 0 \\ &\Rightarrow 5(k^2 - 5k + 4) > 0 \\ &\Rightarrow \underline{k^2 - 5k + 4 > 0}. \end{aligned}$$

- (b) Hence find the set of possible values of k . (4)

Solution

$$\begin{aligned} k^2 - 5k + 4 < 0 &\Rightarrow (k - 1)(k - 4) < 0 \\ &\Rightarrow \underline{k < 1} \text{ or } \underline{k > 4}. \end{aligned}$$

13. Find the set of values of x for which

(a) $4x - 3 > 7 - x$,

(2)

Solution

$$\begin{aligned}4x - 3 > 7 - x &\Rightarrow 5x > 10 \\ &\Rightarrow \underline{\underline{x > 2.}}\end{aligned}$$

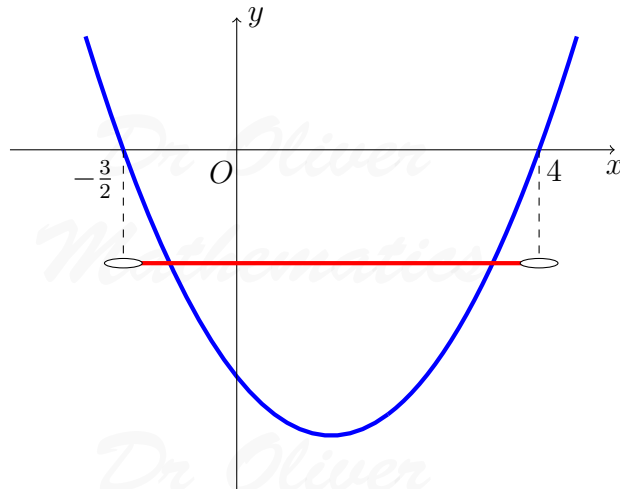
(b) $2x^2 - 5x - 12 < 0$,

(4)

Solution

$$\begin{aligned}2x^2 - 5x - 12 < 0 &\Rightarrow 2x^2 - 8x + 3x - 12 < 0 \\ &\Rightarrow 2x(x - 4) + 3(x - 4) < 0 \\ &\Rightarrow (2x + 3)(x - 4) < 0.\end{aligned}$$

The critical values are $x = -\frac{3}{2}$ and $x = 4$.



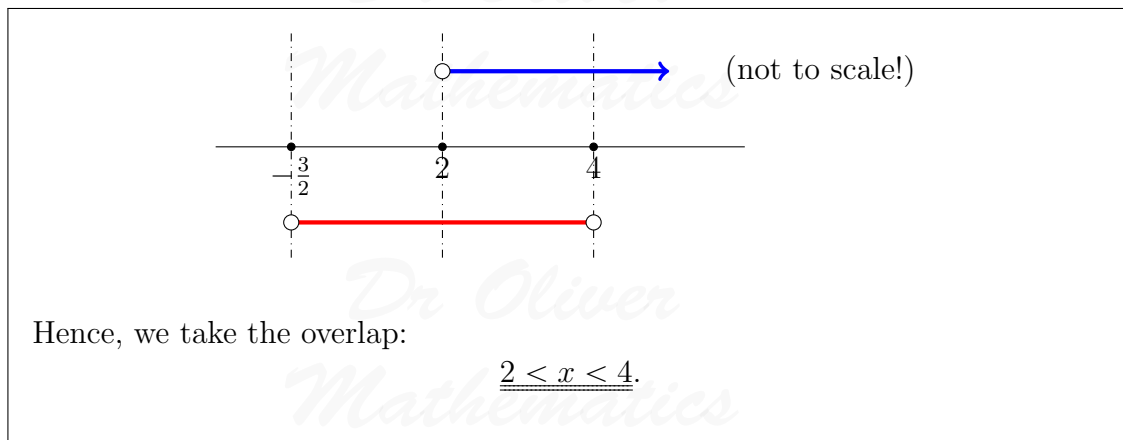
Hence,

$$\underline{\underline{-\frac{3}{2} < x < 4.}}$$

(c) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$.

(1)

Solution



14. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots. Find the value of p . (4)

Solution

$a = 1$, $b = 3p$, and $c = p$:

$$\begin{aligned} b^2 - 4ac = 0 &\Rightarrow (3p)^2 - 4 \times 1 \times p = 0 \\ &\Rightarrow 9p^2 - 4p = 0 \\ &\Rightarrow p(9p - 4) = 0 \\ &\Rightarrow \underline{\underline{p = \frac{4}{9}}}, \end{aligned}$$

as $p \neq 0$.

- 15.

$$f(x) = x^2 + 4kx + (3 + 11k),$$

where k is a constant.

- (a) Express $f(x)$ in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k . (3)

Solution

$$\begin{aligned} x^2 + 4kx + (3 + 11k) &= (x^2 + 4kx + 4k^2) + (3 + 11k - 4k^2) \\ &= \underline{\underline{(x + 2k)^2 + (3 + 11k - 4k^2)}}; \end{aligned}$$

hence,

$$\underline{\underline{p = 2k}}$$

and

$$\underline{\underline{q = 3 + 11k - 4k^2.}}$$

Given that the equation $f(x) = 0$ has no real roots,

(b) find the set of possible values of k .

(4)

Solution

$a = 1$, $b = 4k$, and $c = 3 + 11k$:

$$\begin{aligned} b^2 - 4ac < 0 &\Rightarrow (4k)^2 - 4 \times 1 \times (3 + 11k) < 0 \\ &\Rightarrow 16k^2 - 44k - 12 < 0 \\ &\Rightarrow 4(4k^2 - 11k - 3) < 0 \\ &\Rightarrow 4k^2 - 11k - 3 < 0 \\ &\Rightarrow (4k + 1)(k - 3) < 0 \\ &\Rightarrow \underline{\underline{-\frac{1}{4} < k < 3.}} \end{aligned}$$

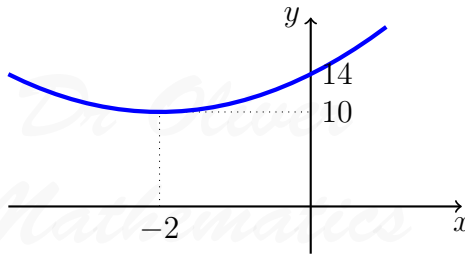
Given that $k = 1$,

(c) sketch the graph of $y = f(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)

Solution

The graph is $y = x^2 + 4x + 14 = (x + 2)^2 + 10$.



16. Find the set of values of x for which

(a) $3(x - 2) < 8 - 2x$,

(2)

Solution

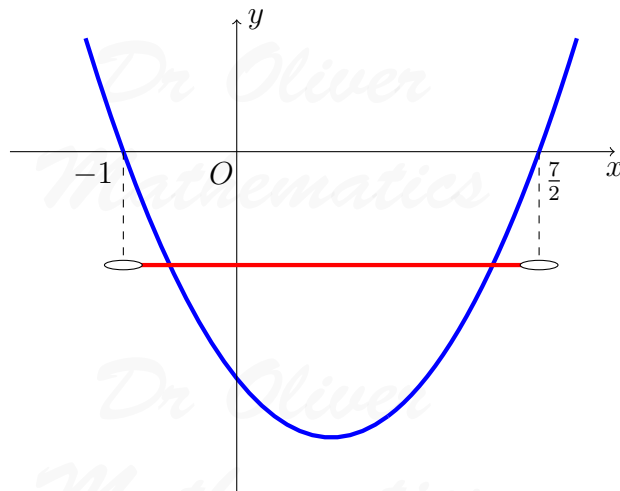
$$\begin{aligned}3(x - 2) < 8 - 2x &\Rightarrow 3x - 6 < 8 - 2x \\ &\Rightarrow 5x < 14 \\ &\Rightarrow \underline{\underline{x < \frac{14}{5}}}.\end{aligned}$$

(b) $(2x - 7)(1 + x) < 0,$

(3)

Solution

The critical values are $x = -1$ and $x = \frac{7}{2}$.



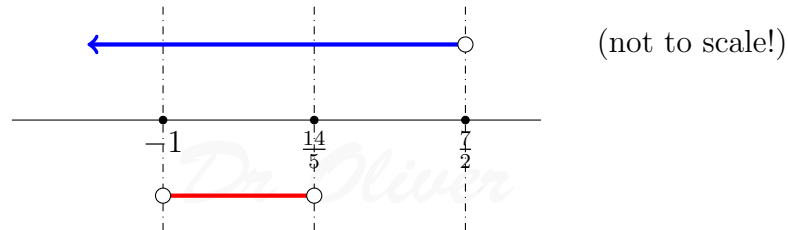
Hence,

$$\underline{\underline{-1 < x < \frac{7}{2}}}.$$

(c) **both** $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0.$

(1)

Solution



Hence, we take the overlap:

$$\underline{\underline{-1 < x < \frac{14}{5}}}$$

17. (a) Show that $x^2 + 6x + 11$ can be written as (2)

$$(x + p)^2 + q,$$

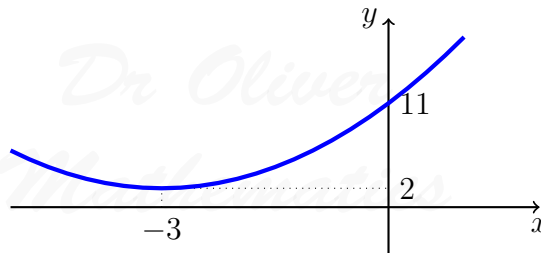
where p and q are constants.

Solution

$$\begin{aligned}x^2 + 6x + 11 &= (x^2 + 6x + 9) + 2 \\ &= \underline{\underline{(x + 3)^2 + 2}}.\end{aligned}$$

- (b) Sketch the graph of $y = x^2 + 6x + 11$, indicating clearly the coordinates of any intersections with the coordinate axes. (2)

Solution



- (c) Find the value of the discriminant of $x^2 + 6x + 11$. (2)

Solution

$a = 1$, $b = 6$, and $c = 11$:

$$\begin{aligned}b^2 - 4ac &= 6^2 - 4 \times 1 \times (-11) \\ &= 36 - 44 \\ &= \underline{\underline{-8}}.\end{aligned}$$

18. The equation

$$x^2 + (k - 3)x + (3 - 2k) = 0,$$

where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0.$$

(3)

Solution

$a = 1$, $b = k - 3$, and $c = 3 - 2k$:

$$\begin{aligned} b^2 - 4ac > 0 &\Rightarrow (k - 3)^2 - 4 \times 1 \times (3 - 2k) > 0 \\ &\Rightarrow (k^2 - 6k + 9) - (12 - 8k) > 0 \\ &\Rightarrow \underline{\underline{k^2 + 2k - 3 > 0}}. \end{aligned}$$

(b) Hence find the set of possible values of k .

(4)

Solution

$$\begin{aligned} k^2 + 2k - 3 < 0 &\Rightarrow (k + 3)(k - 1) < 0 \\ &\Rightarrow \underline{\underline{k < -3}} \text{ or } \underline{\underline{k > 1}}. \end{aligned}$$

19.

$$f(x) = x^2 + (k + 3)x + k,$$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k .

(2)

Solution

$a = 1$, $b = k + 3$, and $c = k$:

$$\begin{aligned} b^2 - 4ac &= (k + 3)^2 - 4 \times 1 \times k \\ &= (k^2 + 6k + 9) - 4k \\ &= \underline{\underline{k^2 + 2k + 9}}. \end{aligned}$$

(b) Show that the discriminant of $f(x)$ can be expressed in the form

(2)

$$(k + a)^2 + b,$$

where a and b are integers to be found.

Solution

$$\begin{aligned}k^2 + 2k + 9 &= (k^2 + 2k + 1) + 8 \\ &= \underline{\underline{(k + 1)^2 + 8}}.\end{aligned}$$

(c) Show that, for all values of k , the equation $f(x) = 0$ has real roots. (2)

Solution

For all k , $(k + 1)^2 + 8 > 0$; this is what we need to establish real roots.

20. Find the set of values of x for which

(a) $4x - 5 > 15 - x$, (2)

Solution

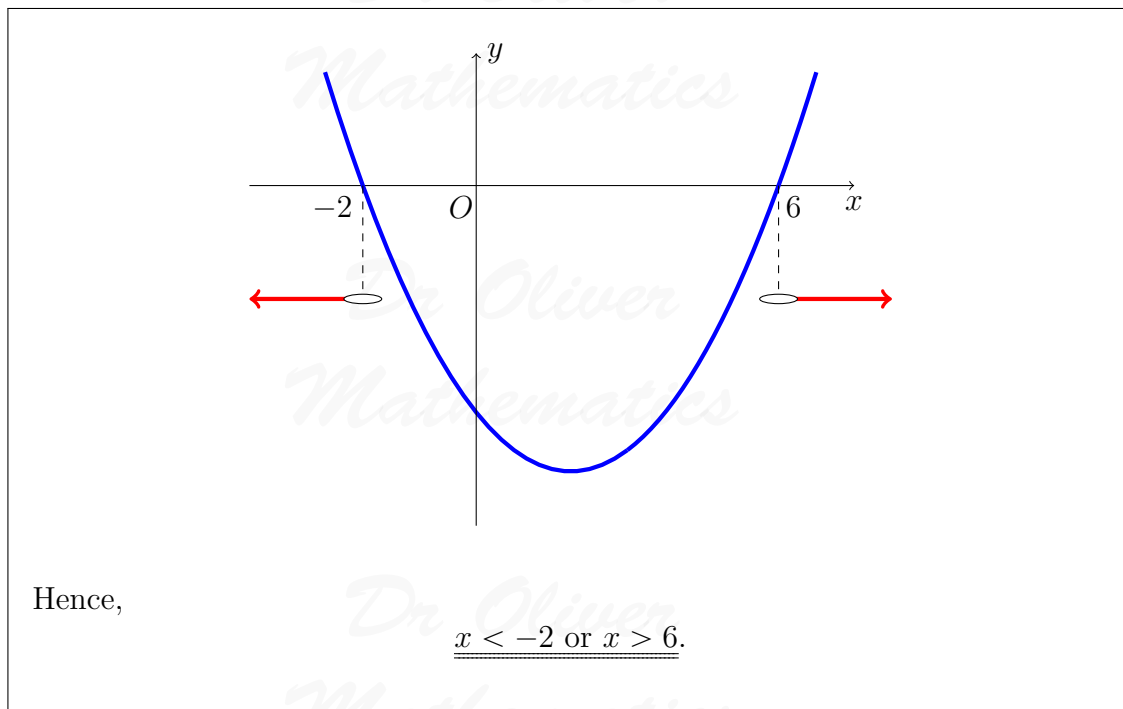
$$\begin{aligned}4x - 5 > 15 - x &\Rightarrow 5x > 20 \\ &\Rightarrow \underline{\underline{x > 4}}.\end{aligned}$$

(b) $x(x - 4) > 12$. (4)

Solution

$$\begin{aligned}x(x - 4) > 12 &\Rightarrow x^2 - 4x - 12 > 0 \\ &\Rightarrow (x - 6)(x + 2) > 0.\end{aligned}$$

The critical values are $x = -2$ and $x = 6$.



21.

$$4x - 5 - x^2 = q - (x + p)^2,$$

where p and q are integers.

(a) Find the value of p and the value of q .

(3)

Solution

$$\begin{aligned} 4x - 5 - x^2 &= -1 - (x^2 - 4x + 4) \\ &= \underline{\underline{-1 - (x - 2)^2}}; \end{aligned}$$

hence, $p = -2$ and $q = -1$.

(b) Calculate the discriminant of $4x - 5 - x^2$.

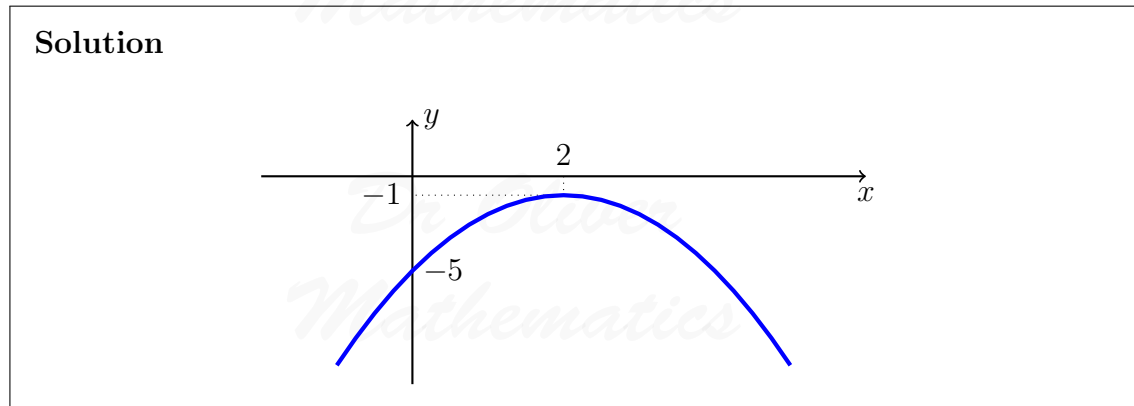
(2)

Solution

$a = -1$, $b = 4$, and $c = -5$:

$$\begin{aligned} b^2 - 4ac &= 4^2 - 4 \times (-1) \times (-5) \\ &= 16 - 20 \\ &= \underline{\underline{-4}}. \end{aligned}$$

- (c) Sketch the graph of $y = 4x - 5 - x^2$, showing the coordinates of any point at which the graph crosses the coordinate axes. (3)

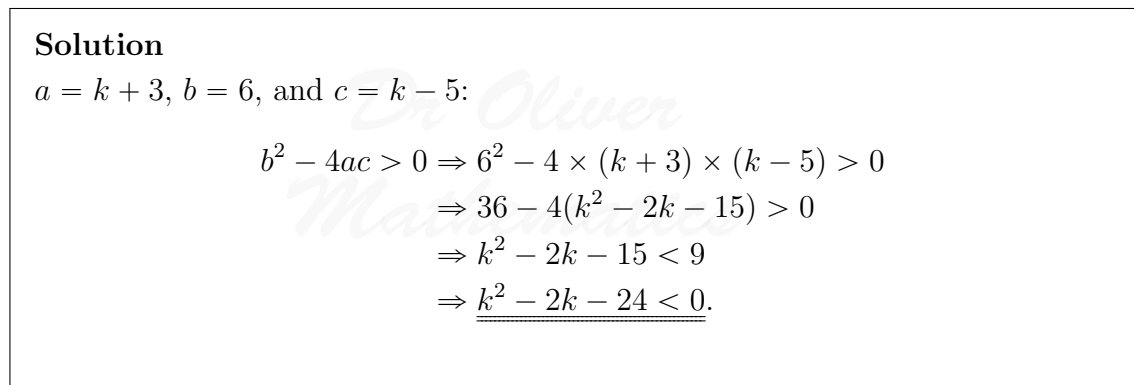


22. The equation

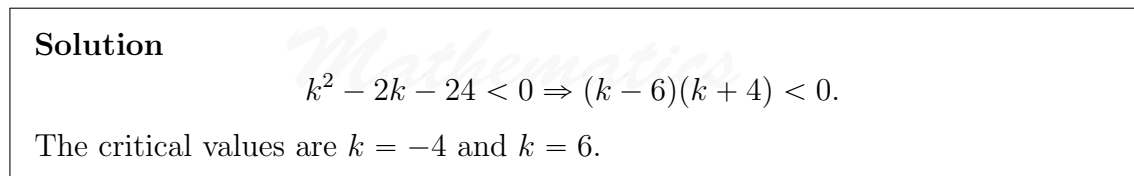
$$(k + 3)x^2 + 6x + k = 6,$$

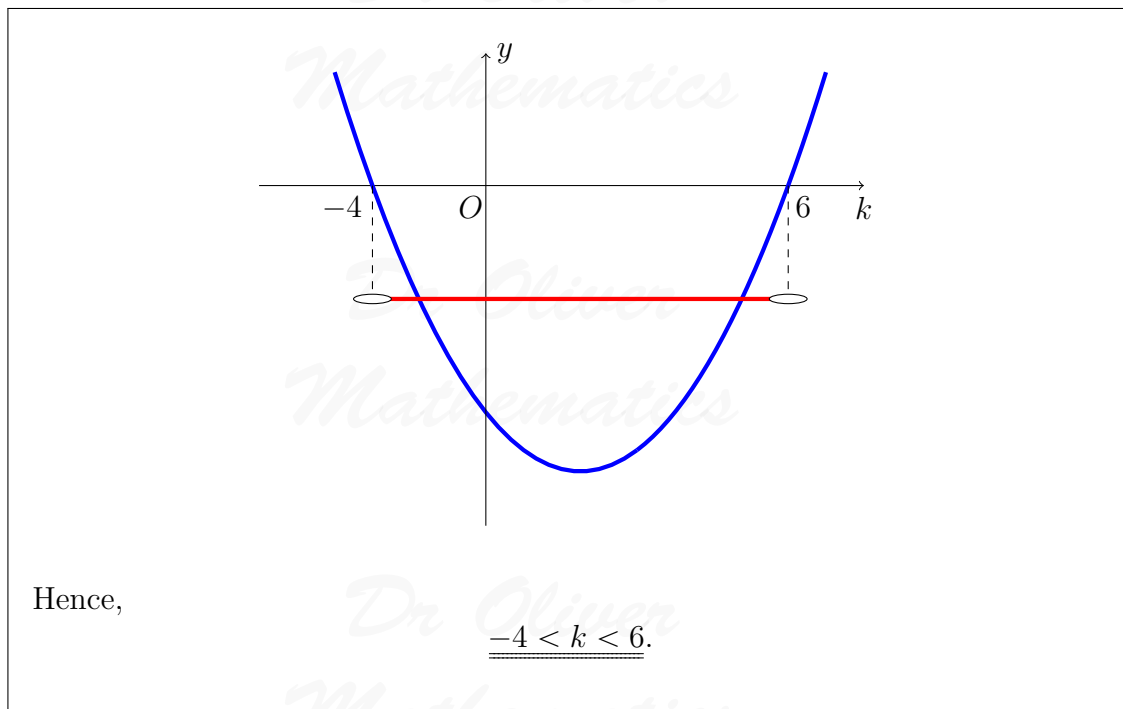
where k is a constant, has two distinct real solutions for x .

- (a) Show that k satisfies $k^2 - 2k - 24 < 0$. (4)



- (b) Hence find the set of possible values of k . (3)





23.

$$4x^2 + 8x + 3 \equiv a(x + b)^2 + c.$$

(a) Find the values of the constants a , b , and c .

(3)

Solution

$$\begin{aligned} 4x^2 + 8x + 3 &\equiv 4[x^2 + 2x] + 3 \\ &\equiv 4[(x^2 + 2x + 1) - 1] + 3 \\ &\equiv \underline{\underline{4(x + 1)^2 - 1;}} \end{aligned}$$

hence,

$$\underline{\underline{a = 4, b = 1, \text{ and } c = -1.}}$$

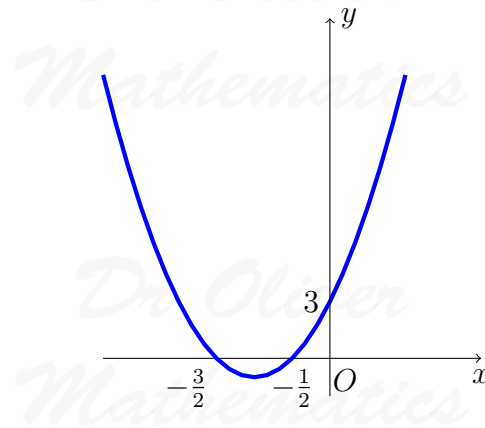
(b) Sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)

Solution

$$\begin{aligned}4(x+1)^2 - 1 = 0 &\Rightarrow 4(x+1)^2 = 1 \\&\Rightarrow (x+1)^2 = \frac{1}{4} \\&\Rightarrow x+1 = \pm\frac{1}{2} \\&\Rightarrow x = -\frac{3}{2} \text{ or } x = -\frac{1}{2}.\end{aligned}$$

Here is the graph.



24. Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$,

(2)

Solution

$$\begin{aligned}2(3x + 4) > 1 - x &\Rightarrow 6x + 8 > 1 - x \\&\Rightarrow 7x > -7 \\&\Rightarrow \underline{\underline{x > -1}}.\end{aligned}$$

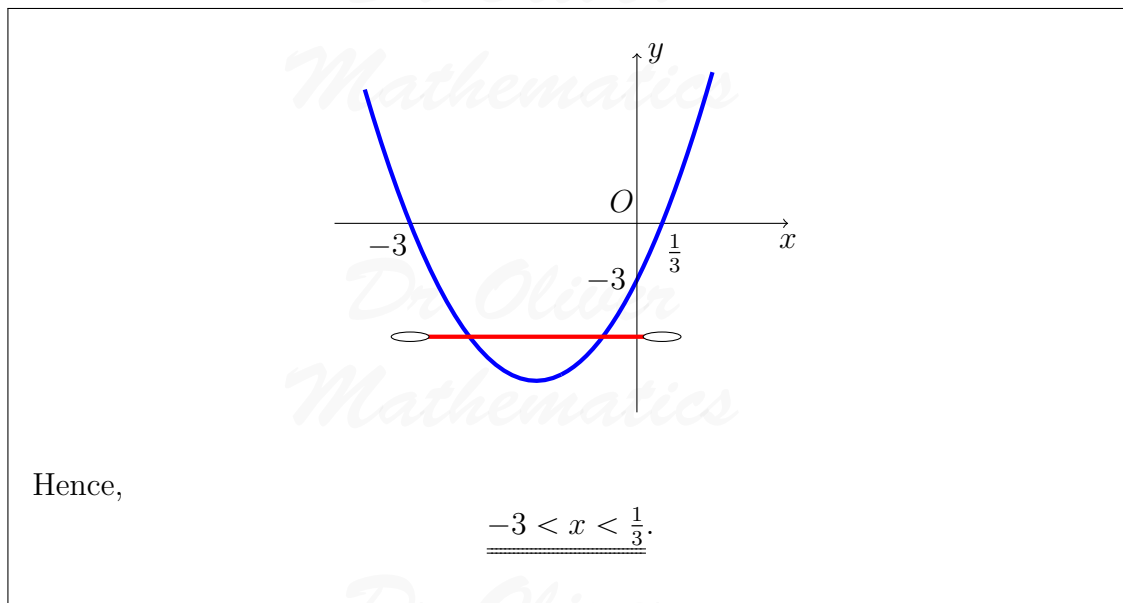
(b) $3x^2 + 8x - 3 < 0$.

(4)

Solution

$$\begin{aligned}3x^2 + 8x - 3 < 0 &\Rightarrow 3x^2 + 9x - x - 3 < 0 \\&\Rightarrow 3x(x + 3) - (x + 3) < 0 \\&\Rightarrow (3x - 1)(x + 3) < 0.\end{aligned}$$

The critical values are $x = -3$ and $x = \frac{1}{3}$.



25. Given the simultaneous equations

$$\begin{aligned} 2x + y &= 1 \\ x^2 - 4ky + 5k &= 0, \end{aligned}$$

where k is a non-zero constant,

(a) show that

$$x^2 + 8kx + k = 0. \tag{2}$$

Solution

$$\begin{aligned} y = 1 - 2x &\Rightarrow x^2 - 4k(1 - 2x) + 5k = 0 \\ &\Rightarrow \underline{\underline{x^2 + 8kx + k = 0}}. \end{aligned}$$

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k . (3)

Solution

$a = 1$, $b = 8k$, and $c = k$:

$$\begin{aligned}b^2 - 4ac = 0 &\Rightarrow (8k)^2 - 4 \times 1 \times k = 0 \\&\Rightarrow 64k^2 - 4k = 0 \\&\Rightarrow 4k(16k - 1) = 0.\end{aligned}$$

Since $k \neq 0$, $k = \frac{1}{16}$.

(c) For this value of k , find the solution of the simultaneous equations. (3)

Solution

$$k = \frac{1}{16}:$$

$$\begin{aligned}x^2 + \frac{1}{2}x + \frac{1}{16} = 0 &\Rightarrow \left(x + \frac{1}{4}\right)^2 = 0 \\&\Rightarrow \underline{\underline{x = -\frac{1}{4}}}\end{aligned}$$

and

$$y = 1 - 2 \times \left(-\frac{1}{4}\right) = \underline{\underline{1\frac{1}{2}}}.$$

26. A rectangular room has a width of x m. The length of the room is 4 m longer than its width. Given that the perimeter of the room is greater than 19.2 m,

(a) show that $x > 2.8$. (3)

Solution

$$\begin{aligned}x + (x + 4) + x + (x + 4) > 19.2 &\Rightarrow 4x + 8 > 19.2 \\&\Rightarrow 4x > 11.2 \\&\Rightarrow \underline{\underline{x > 2.8}}.\end{aligned}$$

Given also that the area of the room is less than 21 m^2 ,

(b) (i) write down an inequality, in terms of x , for the area of the room. (4)

Solution

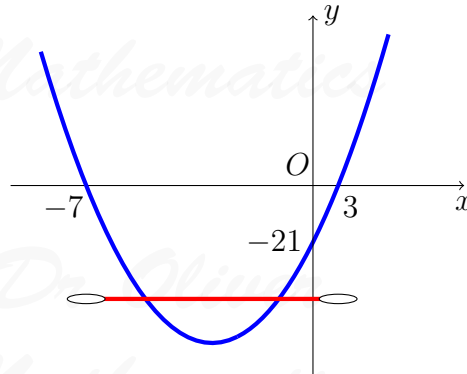
$$\underline{\underline{x(x + 4) < 21}}.$$

(ii) Solve this inequality.

Solution

$$\begin{aligned}x(x + 4) < 21 &\Rightarrow x^2 + 4x - 21 < 0 \\ &\Rightarrow (x + 7)(x - 3) < 0.\end{aligned}$$

The critical values are $x = -7$ and $x = 3$.



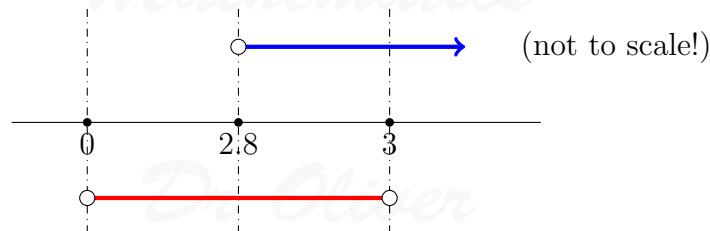
But $x > 0$ (why?) and

$$\underline{\underline{0 < x < 3.}}$$

(c) Hence find the range of possible value for x .

(1)

Solution



Hence, we take the overlap:

$$\underline{\underline{2.8 < x < 3.}}$$

27. Find the set of values of x for which

(a) $3x - 7 > 3 - x$,

(2)

Solution

$$\begin{aligned}3x - 7 &> 3 - x \Rightarrow 4x > 10 \\ &\Rightarrow \underline{\underline{x > 2\frac{1}{2}}}.\end{aligned}$$

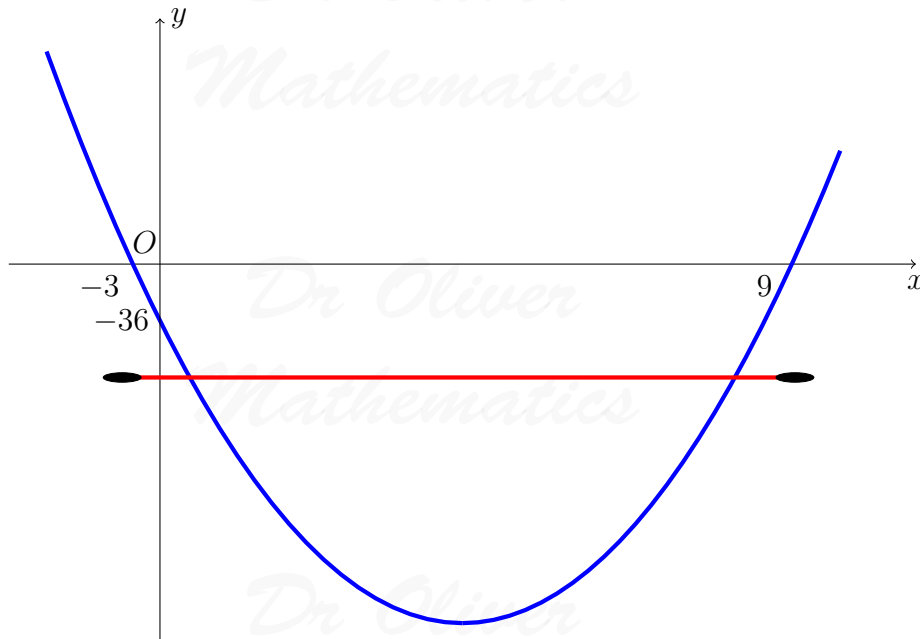
(b) $x^2 - 9x \leq 36$,

(4)

Solution

$$\begin{aligned}x^2 - 9x \leq 36 &\Rightarrow x^2 - 9x - 36 \leq 0 \\ &\Rightarrow (x - 12)(x + 3) \leq 0.\end{aligned}$$

The critical values are $x = -3$ and $x = 12$.



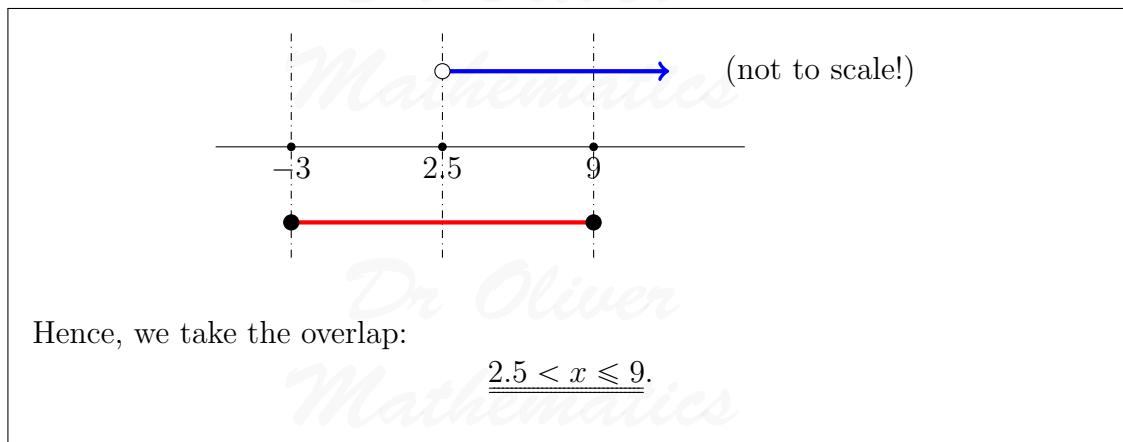
Hence,

$$\underline{\underline{-3 \leq x \leq 9.}}$$

(c) **both** $3x - 7 > 3 - x$ and $x^2 - 9x \leq 36$.

(1)

Solution



28. Given that

$$f(x) = 2x^2 + 8x + 3,$$

- (a) find the value of the discriminant of $f(x)$. (2)

Solution

$a = 2$, $b = 8$, and $c = 3$:

$$\begin{aligned} b^2 - 4ac &= 8^2 - 4 \times 2 \times 3 \\ &= 64 - 24 \\ &= \underline{\underline{40}}. \end{aligned}$$

- (b) Express $f(x)$ in the form $p(x + q)^2 + r$, where p , q , and r are integers to be found. (3)

Solution

$$2x^2 + 8x + 3 = 2(x^2 + 4x + 4) - 5 = \underline{\underline{2(x + 2)^2 - 5}}.$$

The line $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = f(x)$.

- (c) Calculate the value of c . (5)

Solution

$$2x^2 + 8x + 3 = 4x + c \Rightarrow 2x^2 + 4x + (3 - c) = 0.$$

$a = 2$, $b = 4$, and $d = 3 - c$:

$$\begin{aligned}b^2 - 4ad &= 0 \Rightarrow 4^2 - 4 \times 2 \times (3 - c) = 0 \\&\Rightarrow 16 - 8(3 - c) = 0 \\&\Rightarrow 8c = 8 \\&\Rightarrow \underline{c = 1}.\end{aligned}$$

29. Figure 1 shows the plan of a garden.

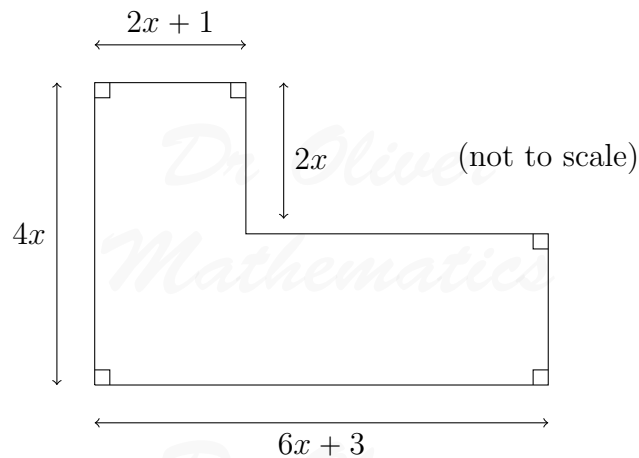


Figure 1: the plan of a garden

The marked angles are right angles. The six edges are straight lines. The lengths shown in the diagram are given in metres. Given that the perimeter of the garden is greater than 40 m,

(a) show that $x > 1.7$.

(3)

Solution

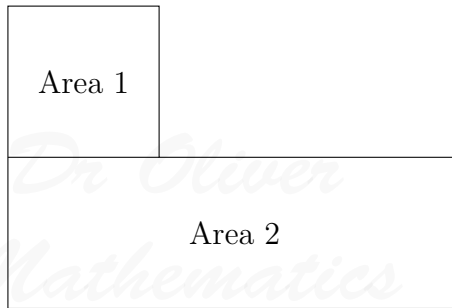
$$\begin{aligned}4x + (2x + 1) + 2x + (4x + 2) + 2x + (6x + 3) &> 40 \\ \Rightarrow 20x + 6 &> 40 \\ \Rightarrow 20x &> 34 \\ \Rightarrow \underline{x > 1.7}.\end{aligned}$$

Given that the area of the garden is less than 120 m^2 ,

(b) form and solve a quadratic in equality in x .

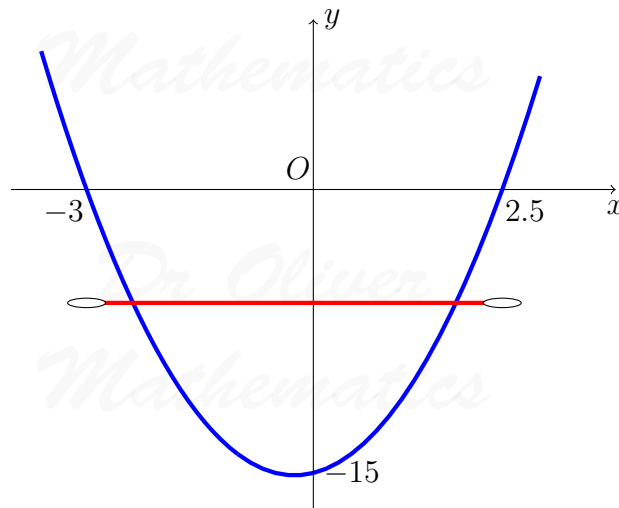
(5)

Solution



$$\begin{aligned} \text{Area 1} + \text{Area 2} &< 120 \\ \Rightarrow 2x(2x + 1) + 2x(6x + 3) &< 120 \\ \Rightarrow 4x^2 + 2x + 12x^2 + 6x - 120 &< 0 \\ \Rightarrow 16x^2 + 8x - 120 &< 0 \\ \Rightarrow 2x^2 + x - 15 &< 0 \\ \Rightarrow 2x^2 + 6x - 5x - 15 &< 0 \\ \Rightarrow 2x(x + 3) - 5(x + 3) &< 0 \\ \Rightarrow (2x - 5)(x + 3) &< 0. \end{aligned}$$

The critical values are $x = -3$ and $x = 2.5$.

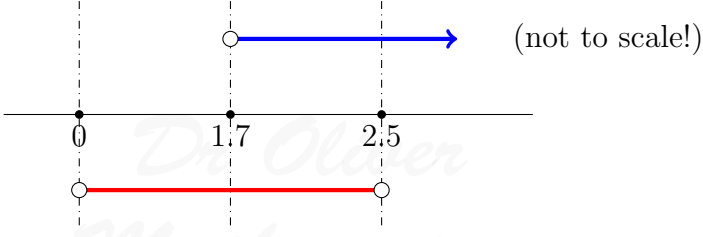


But $x > 0$ (why?) and

$$\underline{\underline{0 < x < 2.5.}}$$

- (c) Hence state the range of the possible values of x . (1)

Solution



Hence, we take the overlap:

$$\underline{1.7 < x < 2.5.}$$

30. The equation

$$(p - 1)x^2 + 4x + (p - 5) = 0,$$

where p is a constant, has no real roots.

- (a) Show that p satisfies $p^2 - 6p + 1 > 0$. (3)

Solution

$a = p - 1$, $b = 4$, and $c = p - 5$:

$$\begin{aligned} b^2 - 4ac < 0 &\Rightarrow 4^2 - 4 \times (p - 1) \times (p - 5) < 0 \\ &\Rightarrow 0 < 4(p - 1)(p - 5) - 16 \\ &\Rightarrow 0 < (p - 1)(p - 5) - 4 \\ &\Rightarrow 0 < (p^2 - 6p + 5) - 4 \\ &\Rightarrow \underline{p^2 - 6p + 1 > 0.} \end{aligned}$$

- (b) Hence find the set of possible values of p . (4)

Solution

$$\begin{aligned} p^2 - 6p + 1 > 0 &\Rightarrow p^2 - 6p + 9 > 8 \\ &\Rightarrow (p - 3)^2 > 8 \\ &\Rightarrow p - 3 < -2\sqrt{2} \text{ or } p - 3 > 2\sqrt{2} \\ &\Rightarrow \underline{p < 3 - 2\sqrt{2} \text{ or } p > 3 + 2\sqrt{2}.} \end{aligned}$$

31. The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation

$$y = 2px^2 - 6px + 4p,$$

where p is a constant.

- (a) Show that $4p^2 - 20p + 9 < 0$.

(4)

Solution

$$2px^2 - 6px + 4p = 3x - 7 \Rightarrow 2px^2 - (6p + 3)x + (4p + 7) = 0.$$

Now, $a = 2p$, $b = -(6p + 3)$, and $c = 4p + 7$:

$$\begin{aligned} b^2 - 4ac < 0 &\Rightarrow (6p + 3)^2 - 4 \times 2p \times (4p + 7) < 0 \\ &\Rightarrow (36p^2 + 36p + 9) - 8p(4p + 7) < 0 \\ &\Rightarrow 36p^2 + 36p + 9 - 32p^2 - 56p < 0 \\ &\Rightarrow \underline{\underline{4p^2 - 20p + 9 < 0.}} \end{aligned}$$

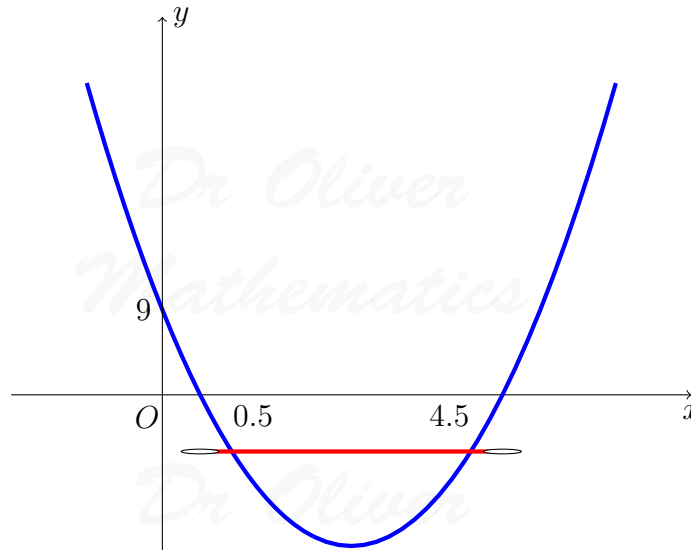
- (b) Hence find the set of possible values of p .

(4)

Solution

$$\begin{aligned} 4p^2 - 20p + 9 < 0 &\Rightarrow 4p^2 - 18p - 2p + 9 < 0 \\ &\Rightarrow 2p(2p - 9) - 1(2p - 9) < 0 \\ &\Rightarrow (2p - 9)(2p - 1) < 0. \end{aligned}$$

The critical values are $x = 0.5$ and $x = 4.5$.



Hence,

$$\underline{\underline{0.5 < p < 4.5.}}$$

32.

$$f(x) = x^2 - 8x + 13.$$

(2)

Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants.

Solution

$$\begin{aligned}x^2 - 8x + 13 &= (x^2 - 8x + 16) + 3 \\ &= \underline{\underline{(x - 4)^2 + 3;}}\end{aligned}$$

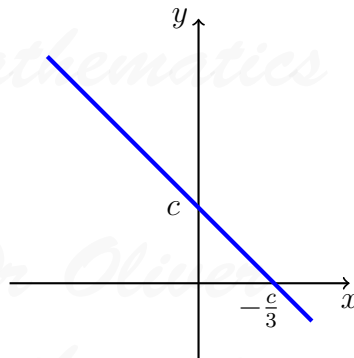
hence, $a = -4$ and $b = 3$.

33. (a) On each sketch, show the coordinates of any point at which the graph crosses the y -axis and the equation of any horizontal asymptote.

(4)

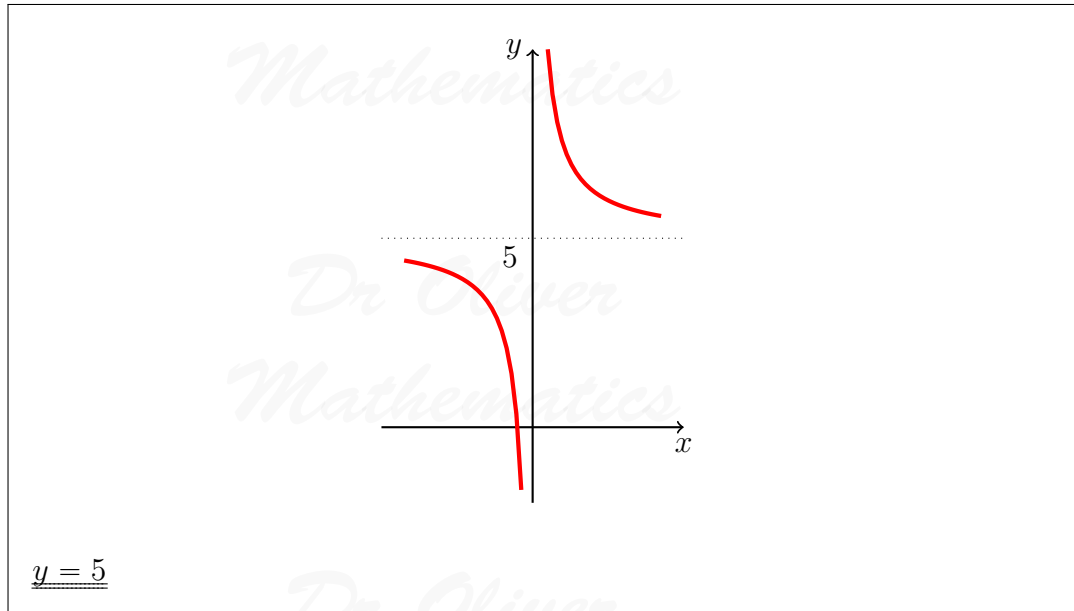
(i) $y = -3x + c$, where c is a positive constant,

Solution



(ii) $y = \frac{1}{x} + 5$.

Solution



Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \frac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 12$. (3)

Solution

$$\begin{aligned} -3x + c &= \frac{1}{x} + 5 \Rightarrow -3x^2 + cx = 1 + 5x \\ &\Rightarrow 3x^2 + (5 - c)x + 1 = 0. \end{aligned}$$

$a = 3$, $b = 5 - c$, and $c = 1$:

$$\begin{aligned} b^2 - 4ac > 0 &\Rightarrow (5 - c)^2 - 4 \times 3 \times 1 > 0 \\ &\Rightarrow \underline{\underline{(5 - c)^2 > 12}}. \end{aligned}$$

(c) Hence find the range of possible values for c . (4)

Solution

$$\begin{aligned} (5 - c)^2 > 12 &\Rightarrow 5 - c < -2\sqrt{3} \text{ or } 5 - c > 2\sqrt{3} \\ &\Rightarrow \underline{\underline{5 + 2\sqrt{3} < c}} \text{ or } \underline{\underline{5 - 2\sqrt{3} > c > 0}} \end{aligned}$$

as c is a positive constant.