Dr Oliver Mathematics
GCSE Mathematics
2016 Paper 2H: Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Chloe recorded the test marks of 20 students.

$$22 \quad 29 \quad 38 \quad 16 \quad 36 \quad 18 \quad 30 \quad 21 \quad 27 \quad 43$$
$$14 \quad 41 \quad 25 \quad 38 \quad 46 \quad 19 \quad 48 \quad 34 \quad 23 \quad 46$$

(a) Show this information in an ordered stem and leaf diagram. (3)

Solution

First, we will do the 'stem'.

$$\begin{array}{c|cccccc} 1 & 6 & 8 & 4 & 9 & & \\ 2 & 2 & 9 & 1 & 7 & 5 & 3 \\ 3 & 8 & 6 & 0 & 8 & 4 & \\ 4 & 3 & 1 & 6 & 8 & 6 & \end{array}$$

Second, we will do the 'leaf'.

$$\begin{array}{c|cccccc} 1 & 4 & 6 & 8 & 9 & & \\ 2 & 1 & 2 & 3 & 5 & 7 & 9 \\ 3 & 0 & 4 & 6 & 8 & 8 & \\ 4 & 1 & 3 & 6 & 6 & 8 & \end{array}$$

Key : 1|4 means 14 marks

One of these students is going to be chosen at random.

(b) Find the probability that this student has a test mark less than 28. (2)

Solution

$\frac{9}{20}$.

2. (a) Simplify $3a \times 5b \times 2c$. (1)

> **Solution**
> $$3a \times 5b \times 2c = \underline{30abc}.$$

(b) Factorise $3y + 6$. (1)
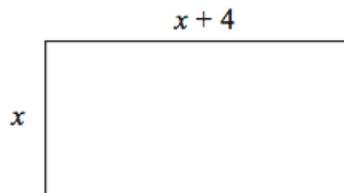
> **Solution**
> $$3y + 6 = \underline{\underline{3(y + 2)}}.$$

(c) Expand $x(x - 3)$. (1)

> **Solution**
> $$x(x - 3) = \underline{\underline{x^2 - 3x}}.$$

3. The diagram shows a rectangle. (3)

All measurements are given in centimetres.
The perimeter of the rectangle is 45 cm.
Work out the value of x.

> **Solution**
> $$x + (x + 4) + x + (x + 4) = 45 \Rightarrow 4x + 8 = 45$$
> $$\Rightarrow 4x = 37$$
> $$\Rightarrow \underline{\underline{x = 9\tfrac{1}{4} \text{ cm}}}.$$

4. A shop sells bags of crisps in different size packs. (4)
   There are

(a) 18 bags of crisps in a small pack,

(b) 20 bags of crisps in a medium pack, and

(c) 26 bags of crisps in a large pack.

Crisps 18 bags	Crisps 20 bags	Crisps 26 bags
£4 each pack	£4.99 each pack	£6.00 each pack

Which size pack is the best value for money?
You must show all your working.

Solution

18 bags:
$$4 \div 18 = 0.222\ldots$$

20 bags:
$$4.99 \div 20 = 0.2495$$

26 bags:
$$6 \div 26 = 0.230\ldots$$

So, the one with <u>18 bags</u> is the best buy.

5. There are only blue counters, green counters, red counters and yellow counters in a bag. (3)
Olga is going to take at random a counter from the bag.
The table shows the probability that Olga will take a blue counter and the probability that she will take a yellow counter.

Colour	Blue	Green	Red	Yellow
Probability	0.4			0.15

The number of red counters in the bag is 4 times the number of green counters in the bag.
Complete the table.

Solution

$$P(\text{green or red}) = 1 - 0.4 - 0.15 = 0.45.$$

So,

$$P(\text{red}) = 4\,P(\text{green}) \Rightarrow 5\,P(\text{green}) = 0.45$$
$$\Rightarrow P(\text{green}) = 0.09$$
$$\Rightarrow P(\text{red}) = 0.36.$$

Hence, the final table looks like this.

Colour	Blue	Green	Red	Yellow
Probability	0.4	0.09	0.36	0.15

6. The body mass index, B, for a person of mass m kg and height h metres is given by the formula

$$B = \frac{m}{h^2}.$$

Usman has a mass of 50 kg.
He has a height of 1.57 m.

(a) Work out Usman's body mass index. (2)
Give your answer correct to one decimal place.

Solution

$$B = \frac{50}{1.57^2}$$
$$= 20.284\,798\,57 \ (\text{FCD})$$
$$= 20.3 \ (1 \text{ dp}).$$

Tom's height is 1.80 m.
He wants his body mass index to be 21.

(b) Work out the mass that will give Tom a body mass index of 21. (2)

Solution

$$21 = \frac{m}{1.80^2} \Rightarrow m = 21 \times 1.80^2$$
$$\Rightarrow m = 68.04.$$

4

Tom is a ski jumper.
The maximum length of skis he can use is 145% of his height. Tom?s height is 1.80 m.

(c) Work out the maximum length of skis Tom can use. (3)

Solution
$$\text{Length} = 1.80 \times 1.45 = \underline{\underline{2.61}}.$$

7. The equation (4)
$$x^3 - 5x = 34$$

has a solution between 3 and 4.
Use a trial and improvement method to find this solution.
Give your answer correct to 1 decimal place.
You must show all your working.

Solution

x	f(x)	Comment
3.75	33.984	too low
3.80	35.872	too high

Clearly,
$$3.75 < x < 3.8$$

and the answer is
$$\underline{\underline{x = 3.8 \text{ (1 dp)}}}.$$

8. Emma has a digital photo.

540 pixels

720 pixels

The photo has a width of 720 pixels.
The photo has a height of 540 pixels.

(a) Write down the ratio of the width of the photo to the height of the photo. (2)
Give your ratio in its simplest form.

Solution
$$720 : 540 = 72 : 54 = \underline{4 : 3}.$$

Emma wants the ratio of the width of the photo to the height of the photo to be $3 : 2$.
She reduces the number of pixels in the height of the photo.
The width of the photo is still 720 pixels.
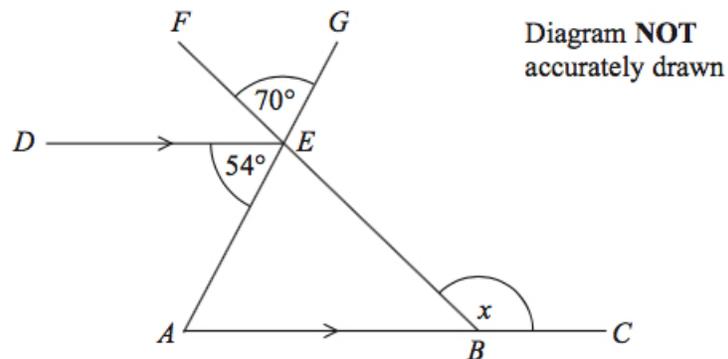The ratio of the width of the photo to the new height of the photo is $3 : 2$.

(b) Work out the new height of the photo. (2)

Solution
Let x pixels be the new height of the photo. Then
$$720 : x = 3 : 2 \Rightarrow \frac{720}{x} = \frac{3}{2}$$
$$\Rightarrow x = \frac{720 \times 2}{3}$$
$$\Rightarrow \underline{\underline{x = 480}}.$$

9. ABC and DE are parallel lines. (4)

Diagram **NOT** accurately drawn

AEG and BEF are straight lines.
Angle $AED = 54°$.
Angle $FEG = 70°$.
Work out the size of the angle marked x.
Give a reason for each stage of your working.

6

10. The table gives information about the heights of 50 trees. (4)

Height (h metres)	Frequency
$0 < h \leqslant 4$	8
$4 < h \leqslant 8$	21
$8 < h \leqslant 12$	12
$12 < h \leqslant 16$	7
$16 < h \leqslant 20$	2

Work out an estimate for the mean height of the trees.

Solution

$$\begin{aligned}
\text{Mean height} &\approx \frac{(2 \times 8) + (6 \times 21) + (10 \times 12) + (14 \times 7) + (18 \times 2)}{50} \\
&= \frac{16 + 126 + 120 + 98 + 36}{50} \\
&= \frac{396}{50} \\
&= \underline{\underline{7.92}}.
\end{aligned}$$

11. Colin works on 5 days each week. (5)
Each day he drives from his home to work and from work to his home.
Colin pays £3.50 each day to use the car park at work.
The distance from Colin's home to work is 18 miles.
Colin's car uses one gallon of petrol every 45.2 miles.
1 litre of petrol costs 136.9p.
1 gallon = 4.546 litres.
Work out the total cost for Colin to use his car for work each week.
You must show all your working.

12. The diagram shows a regular pentagon. (4)
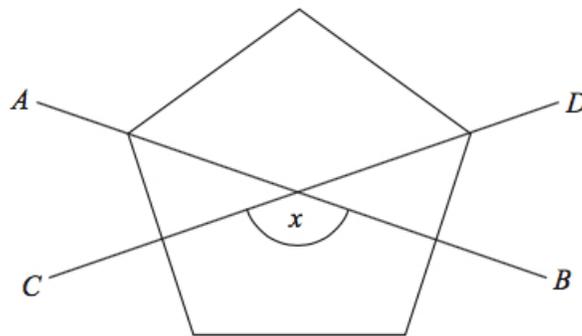
   AB and CD are two of the lines of symmetry of the pentagon.
   Work out the size of the angle marked x.
   You must show all your working.

   Solution

   The internal angles are all $108°$ which means half that is $54°$.
   Now, the diagonal makes an angle of $90°$ and that means

   $$54 + 90 = \underline{\underline{144°}} \text{ (co-interior angles add up to 180).}$$

13. (a) Complete the table of values for $y = x^3 - 3x + 1$. (2)

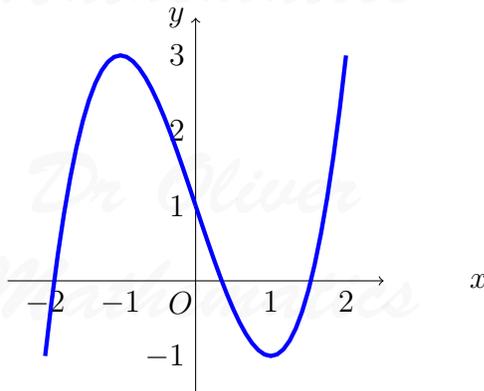 | x | -2 | -1 | 0 | 1 | 2 |
   |-----|------|------|-----|-----|-----|
   | y | | 3 | | | 3 |

Solution

x	-2	-1	0	1	2
y	$\underline{-1}$	3	$\underline{1}$	$\underline{-1}$	3

(b) Draw the graph of $y = x^3 - 3x + 1$ for values of x from -2 to 2. (2)
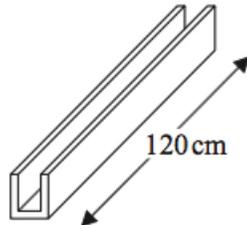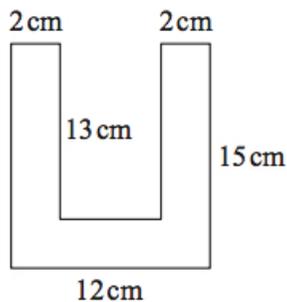
Solution

14. The diagram shows a metal bar in the shape of a prism. (5)

120 cm

The length of the metal bar is 120 cm.
The cross section of the metal bar is shown below.

2 cm   2 cm

13 cm

15 cm

12 cm

9

All corners are right angles.

The metal bar is made from steel with density 8 g/cm³. Sean has a trolley.

The trolley can carry a maximum mass of 250 kg.

How many metal bars can the trolley carry at the same time?

You must show your working.

Solution

$$\text{Area of the cross-section} = (12 \times 15) - (13 \times 8) = 76$$

and the

$$\text{volume} = 76 \times 120 = 9\,120 \text{ cm}^3.$$

Now,

$$\text{mass} = 9\,120 \times 8 = 72\,960 \text{ g} = 72.96 \text{ kg}.$$

Finally,

$$\text{metal bars} = \frac{250}{72.96} = 3\tfrac{389}{912}$$

and so the answer is $\underline{\underline{3}}$.

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15. This notice was in a car magazine: "Most new cars lose more than half of their value in the first three years." (4)

Paul bought a new car.

The value of the car was £15 000.

In the first year, the value of the car depreciated by 23%.

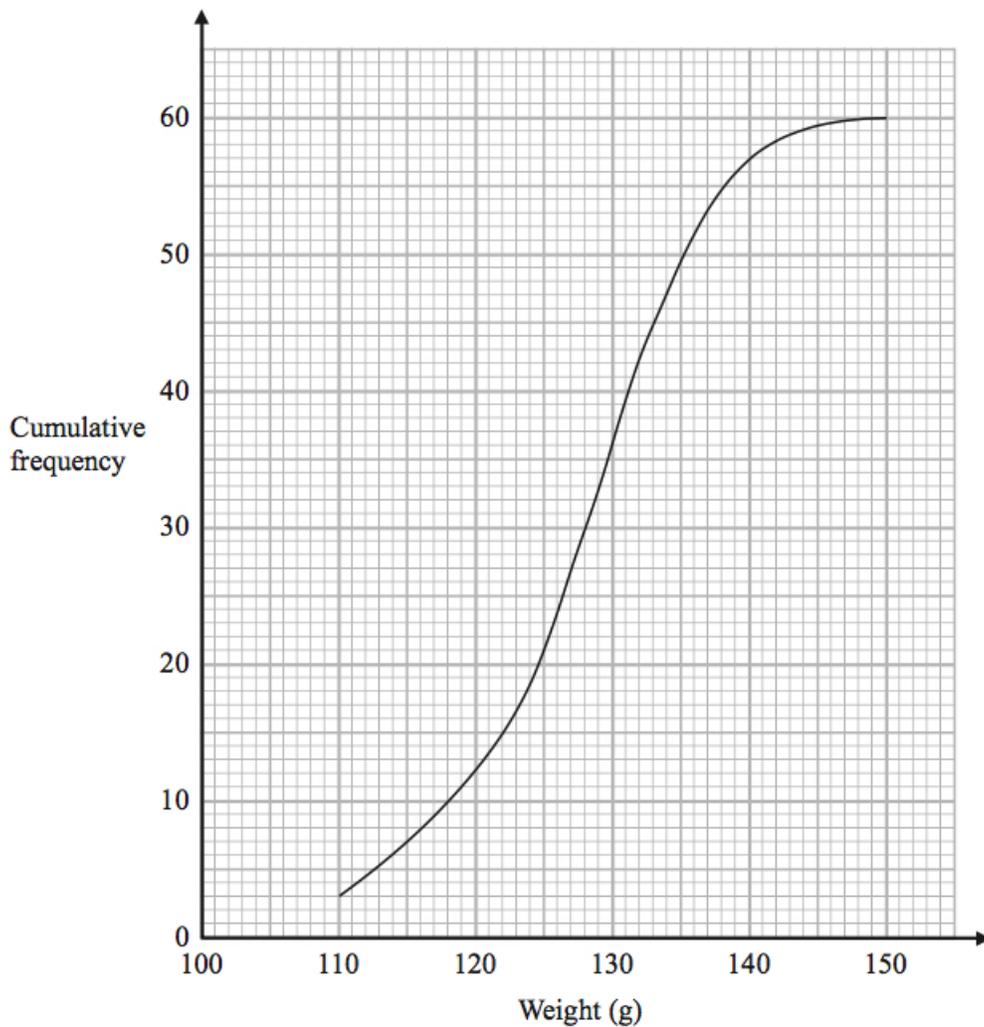After the first year, the value of the car depreciated by 18% each year.

Work out if Paul's car lost more than half of its value by the end of three years.

Solution

$$\text{Car is worth} = 15\,000 \times 0.77 \times 0.82^2 = £7\,766.22;$$

so, $\underline{\underline{\text{no}}}$, the car is worth more.

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16. The cumulative frequency graph shows information about the weights of 60 apples.

Cumulative frequency

Weight (g)

(a) Use the graph to find an estimate for the median weight. (1)

Solution

$$30\text{th piece of information} = \underline{\underline{128}} \text{ g}.$$

(b) Use the graph to find an estimate for the interquartile range of the weights. (2)

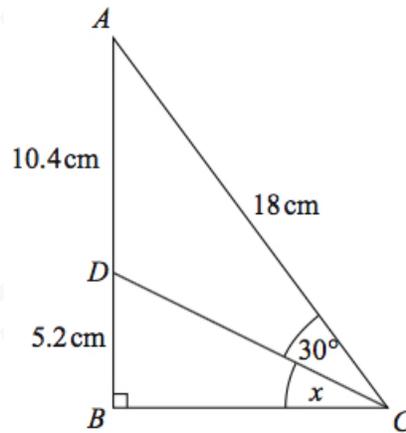**Solution**

$$LQ = 15\text{th piece of information} = 122 \text{ g},$$
$$UQ = 45\text{th piece of information} = 133 \text{ g},$$

and

$$IQR = 133 - 122 = \underline{\underline{11}} \text{ g}.$$

17. ABC is a right-angled triangle.   (4)

D is a point on AB.
Angle $ACD = 30°$.
$AD = 10.4$ cm.
$DB = 5.2$ cm.
$AC = 18$ cm.
Work out the size of the angle marked x.
Give your answer correct to 1 decimal place.

Solution

$$\sin(x + 30) = \frac{10.4 + 5.2}{18} \Rightarrow x + 30 = 60.073\,565\,13 \text{ (FCD)}$$
$$\Rightarrow x = 30.073\,565\,13 \text{ (FCD)}$$
$$\Rightarrow x = 30.1 \text{ (1 dp)}.$$

18. (a) Simplify $2a^3b \times 5a^2b^3$.   (2)

Solution
$$2a^3b \times 5a^2b^3 = 10a^5b^4.$$

(b) Make y the subject of the formula   (3)

$$p = \sqrt{\frac{x + y}{5}}.$$

Solution

$$p = \sqrt{\frac{x+y}{5}} \Rightarrow p^2 = \frac{x+y}{5}$$
$$\Rightarrow 5p^2 = x+y$$
$$\Rightarrow \underline{\underline{y = 5p^2 - x}}.$$

19. The table gives information about 234 students in a school. (2)

Year	Female	Male
12	77	51
13	53	31
14	13	9

Sadia is doing a survey of these students.
She is using a sample of 50 students stratified by year group and by gender.
Work out the number of Year 12 male students in the sample.

Solution

$$\frac{51}{234} \times 50 = 10\frac{35}{39}$$

so she wants <u>11 Year 12 male students</u>.

20. Solve (3)
$$3x^2 + 6x - 2 = 0.$$

Give your solutions correct to 2 decimal places.

Solution

$$x = \frac{-6 \pm \sqrt{(-6)^2 - 4 \times 3 \times (-2)}}{6}$$
$$= \frac{-6 \pm \sqrt{60}}{6}$$
$$= -2.290\,994\,449 \text{ or } 0.290\,994\,448\,7 \text{ (FCD)}$$
$$= -2.29 \text{ or } 0.29 \text{ (2 dp).}$$

21. (3)
$$I = 5(v - u).$$

$v = 14$ correct to 2 significant figures.
$u = 8.7$ correct to 2 significant figures.
Work out the upper bound for the value of I.
You must show your working.

Solution

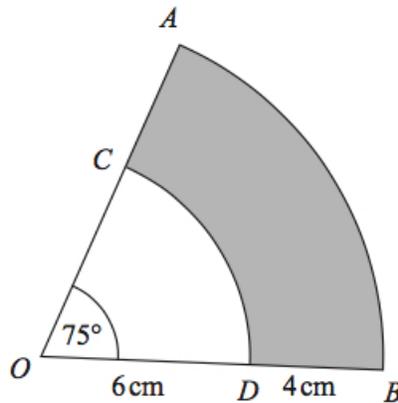$$13.5 \leqslant v < 14.5 \text{ and } 8.65 \leqslant u < 8.75.$$

So
$$I = 5(14.5 - 8.65) = \underline{\underline{29.25}}.$$

22. OAB is a sector of a circle, centre O. (3)
OCD is a sector of a circle, centre O.
OCA and ODB are straight lines.

Angle $AOB = 75°$.

$OD = 6$ cm.

$DB = 4$ cm.

Calculate the perimeter of the shaded region.

Give your answer correct to 3 significant figures.

Solution

$$\text{Perimeter} = \left(\tfrac{75}{360} \times 2\pi \times 10\right) + \left(\tfrac{75}{360} \times 2\pi \times 6\right) + 4 + 4$$
$$= 28.943\,951\,02 \text{ (FCD)}$$
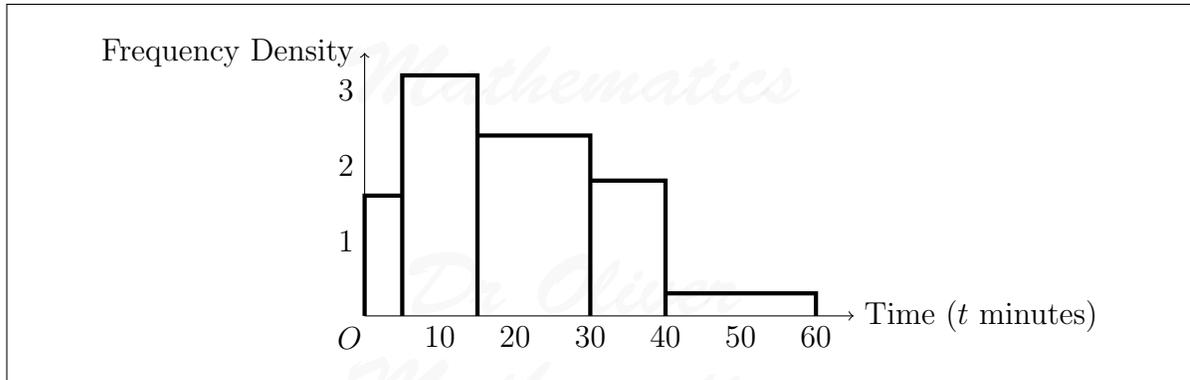$$= \underline{\underline{28.9 \text{ cm (3 sf)}}}.$$

23. The table gives information about the lengths of time some people were in a supermarket. (3)

Time (t minutes)	Frequency
$0 < t \leqslant 5$	8
$5 < t \leqslant 15$	32
$15 < t \leqslant 30$	36
$30 < t \leqslant 40$	18
$40 < t \leqslant 60$	6

Draw a histogram for the information in the table.

Solution

Time (t minutes)	Frequency	Width	Frequency Density
$0 < t \leqslant 5$	8	5	1.6
$5 < t \leqslant 15$	32	10	3.2
$15 < t \leqslant 30$	36	15	2.4
$30 < t \leqslant 40$	18	10	1.8
$40 < t \leqslant 60$	6	20	0.3

Frequency Density

Time (t minutes)

24. (a) Simplify fully (2)

$$\frac{3 - x}{3x^2 - 5x - 12}.$$

Solution

$$\frac{3 - x}{3x^2 - 5x - 12} = \frac{3 - x}{(3x + 4)(x - 3)}$$
$$= -\frac{1}{3x + 4}.$$

(b) Write (3)

$$\frac{x}{x - 1} - \frac{x}{x + 1}$$

as a single fraction in its simplest form.

Solution

$$\frac{x}{x - 1} - \frac{x}{x + 1} = \frac{x(x + 1) - x(x - 1)}{(x - 1)(x + 1)}$$
$$= \frac{(x^2 x + x) - (x^2 x - x)}{(x - 1)(x + 1)}$$
$$= \frac{2x}{(x - 1)(x + 1)}.$$

25. ABC is an acute-angled triangle. (6)
$BA = 7$ cm.
$BC = 8$ cm.

16
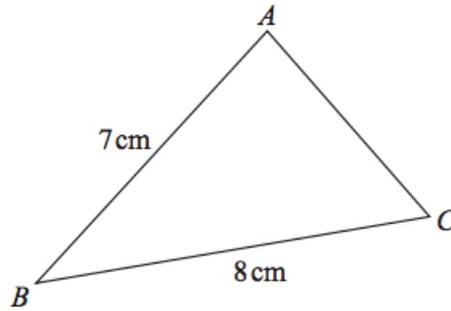
The area of triangle ABC is 18 cm^2.

Work out the size of angle BAC.

Give your answer correct to 3 significant figures.

You must show all your working.

Solution

Area of a triangle:

$$18 = \tfrac{1}{2} \times 7 \times 8 \sin ABC \Rightarrow \sin ABC = \tfrac{9}{14}$$

$$\Rightarrow \cos ABC = \tfrac{\sqrt{115}}{14}.$$

Cosine rule:

$$b^2 = 7^2 + 8^2 - 2 \times 7 \times 8 \times \cos ABC \Rightarrow b^2 = 27.209\,557\,64 \text{ (FCD)}$$

$$\Rightarrow b = 5.216\,278\,141 \text{ (FCD)}.$$

Sine rule:

$$\frac{\sin BAC}{8} = \frac{\sin ABC}{5.216\ldots} \Rightarrow \sin BAC = \frac{8 \times \tfrac{9}{14}}{5.216\ldots}$$

$$\Rightarrow BAC = 80.375\,501\,14 \text{ (FCD)}$$

$$\Rightarrow BAC = 80.4° \text{ (3 sf)}.$$