Dr Oliver Mathematics
GCSE Mathematics
2017 Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. The table shows the probabilities that a biased dice will land on 2, on 3, on 4, on 5, and on 6. (3)

Number of dice	1	2	3	4	5	6
Probability		0.17	0.18	0.09	0.15	0.1

Neymar rolls the biased dice 200 times.
Work out an estimate for the total number of times the dice will land on 1 or on 3.

Solution

$$\mathrm{P}(1) = 1 - (0.17 + 0.18 + 0.09 + 0.15 + 0.1) = 0.31$$

and

$$\mathrm{P}(1 \text{ or } 3) = 0.31 + 0.18 = 0.49.$$

Finally,

$$\text{number of times} = 200 \times 0.49 = \underline{\underline{98}}.$$

2. On Saturday, some adults and some children were in a theatre. (5)
The ratio of the number of adults to the number of children was 5 : 2.

Each person had a seat in the Circle or had a seat in the Stalls.
$\frac{3}{4}$ of the children had seats in the Stalls.
117 children had seats in the Circle.
There are exactly 2600 seats in the theatre.

On this Saturday, were there people on more than 60% of the seats?
You must show how you get your answer.

Solution

$$3 \times 117 = 351$$

children have seats in the Stalls.

$$351 + 117 = 468$$

children have tickets.

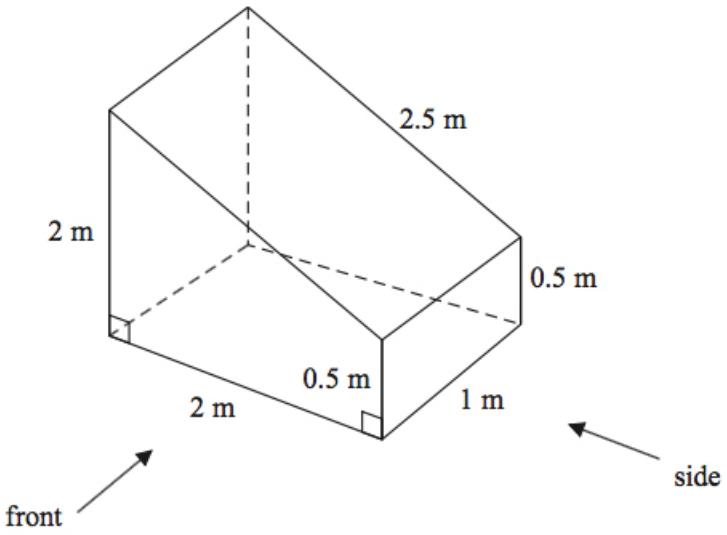
$$\tfrac{5}{2} \times 468 = 1170$$

adults have tickets.

$$\left(\frac{1170 + 468}{2600}\right) \times 100\% = 63\%.$$

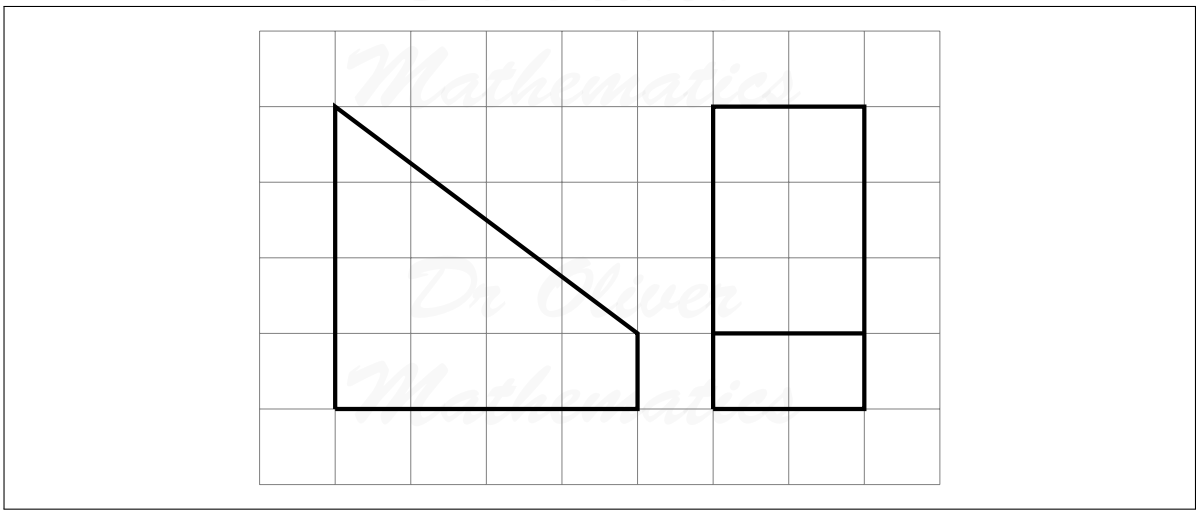
Yes, there were.

3. The diagram shows a prism with a cross section in the shape of a trapezium. (4)



On the centimetre grid below, draw the front elevation and the side elevation of the prism.
Use a scale of 2 cm to 1 m.

Solution



4. Olly drove 56 km from Liverpool to Manchester.
He then drove 61 km from Manchester to Sheffield.
Olly's average speed from Liverpool to Manchester was 70 km/h.
Olly took 75 minutes to drive from Manchester to Sheffield.

(a) Work out Olly's average speed for his total drive from Liverpool to Sheffield. (4)

Solution

$$\text{Time (Liverpool to Manchester)} = \tfrac{56}{70} = 0.8 \text{ hours.}$$

Finally,

$$\text{average speed} = \frac{56+61}{0.8+1.25} = \underline{\underline{57\tfrac{3}{41} \text{ km/h.}}}$$

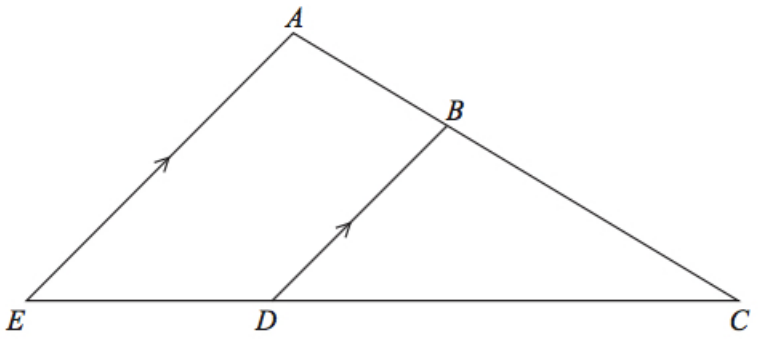
Janie drove from Barnsley to York.
Janie's average speed from Barnsley to Leeds was 80 km/h.
Her average speed from Leeds to York was 60 km/h.
Janie says that the average speed from Barnsley to York can be found by working out the mean of 80 km/h and 60 km/h.

(b) If Janie is correct, what does this tell you about the two parts of Janie's journey? (1)

Solution

<u>They are the same length</u>.

5. ABC and EDC are straight lines.
EA is parallel to DB.



$EC = 8.1$ cm.
$DC = 5.4$ cm.
$DB = 2.6$ cm.

(a) Work out the length of AE. (2)

Solution

$$AE = \frac{8.1 \times 2.6}{5.4} = \underline{\underline{3.9 \text{ cm}}}.$$

$AC = 6.15$ cm.

(b) Work out the length of AB. (2)

Solution

$$ECD = 8.1 - 5.4 = 2.7 \text{ cm}$$

and

$$AB = \frac{6.15 \times 2.7}{8.1} = \underline{\underline{2.05 \text{ cm}}}.$$

6. Anil wants to invest £25 000 for 3 years in a bank. (3)

Personal Bank: Compound Interest, 2% for each year.
Secure Bank: Compound Interest, 4.3% for the first year, 0.9% for each extra year.

Which bank will give Anil the most interest at the end of 3 years?
You must show all your working.

Solution

Personal Bank:

$$25\,000 \times (1.02)^3 = 26\,530.20.$$

Secure Bank:

$$25\,000 \times 1.043 = 26\,075$$
$$26\,075 \times (1.009)^2 = 26\,546.46\ldots$$

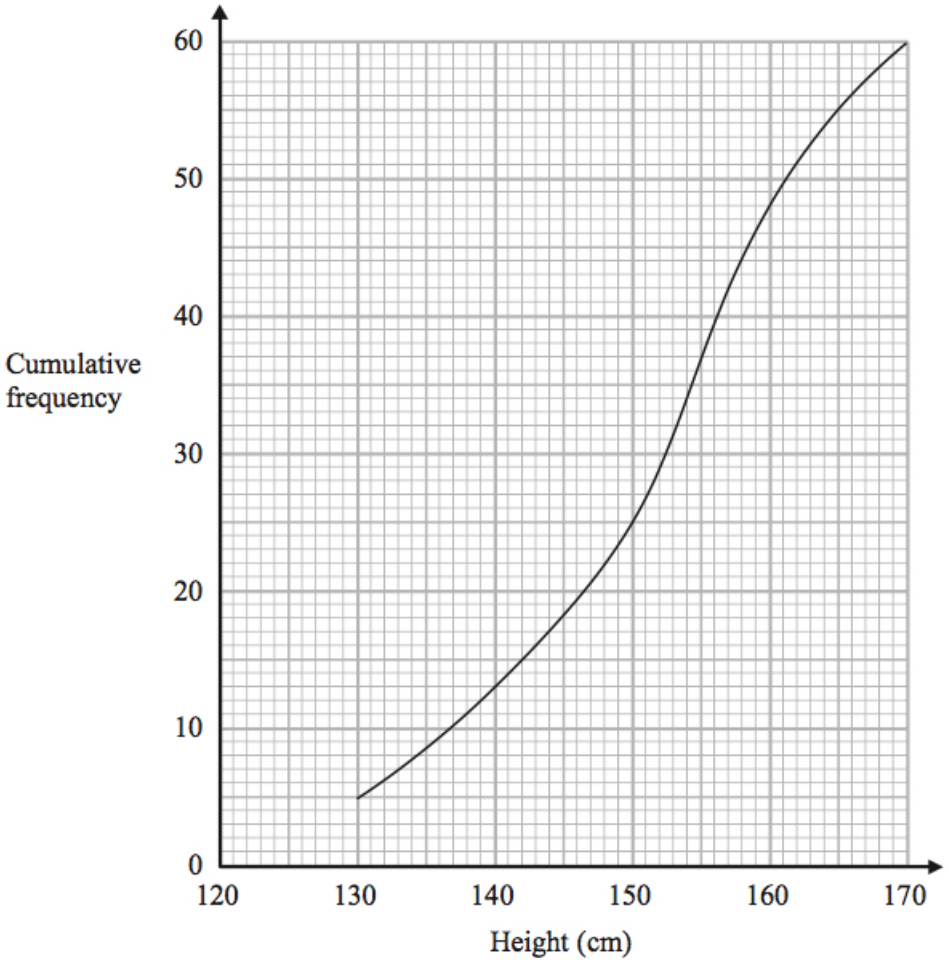
He should bank with the Secure Bank and he will make £16.26... more.

7. A number, n, is rounded to 2 decimal places. (2)
The result is 4.76.
Using inequalities, write down the error interval for n.

Solution

$$4.755 \leqslant n < 4.765.$$

8. The cumulative frequency graph shows some information about the heights, in cm, of 60 students. (2)



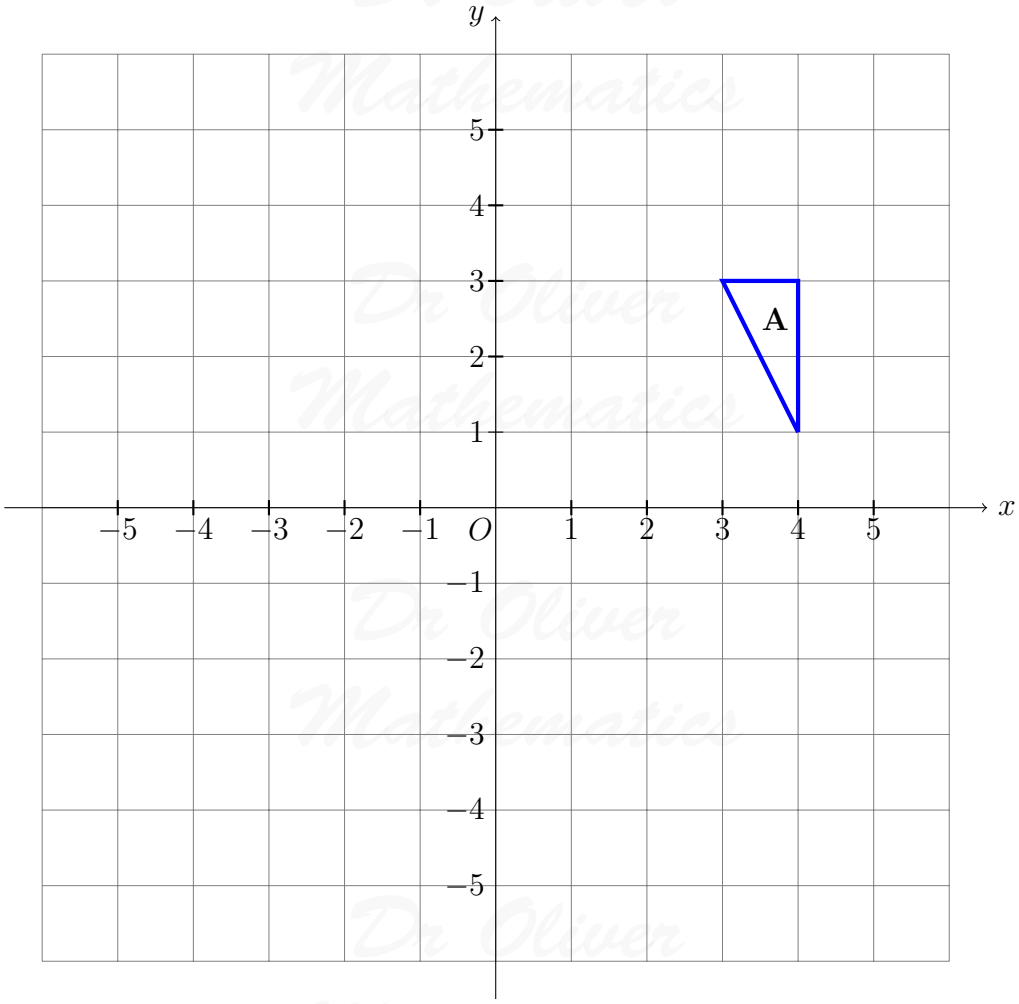
Work out an estimate for the number of these students with a height greater than 160 cm.

Solution

Correct read-off at 160: it is 48.
Then

$$60 - 48 = \underline{\underline{12 \text{ students}}}.$$

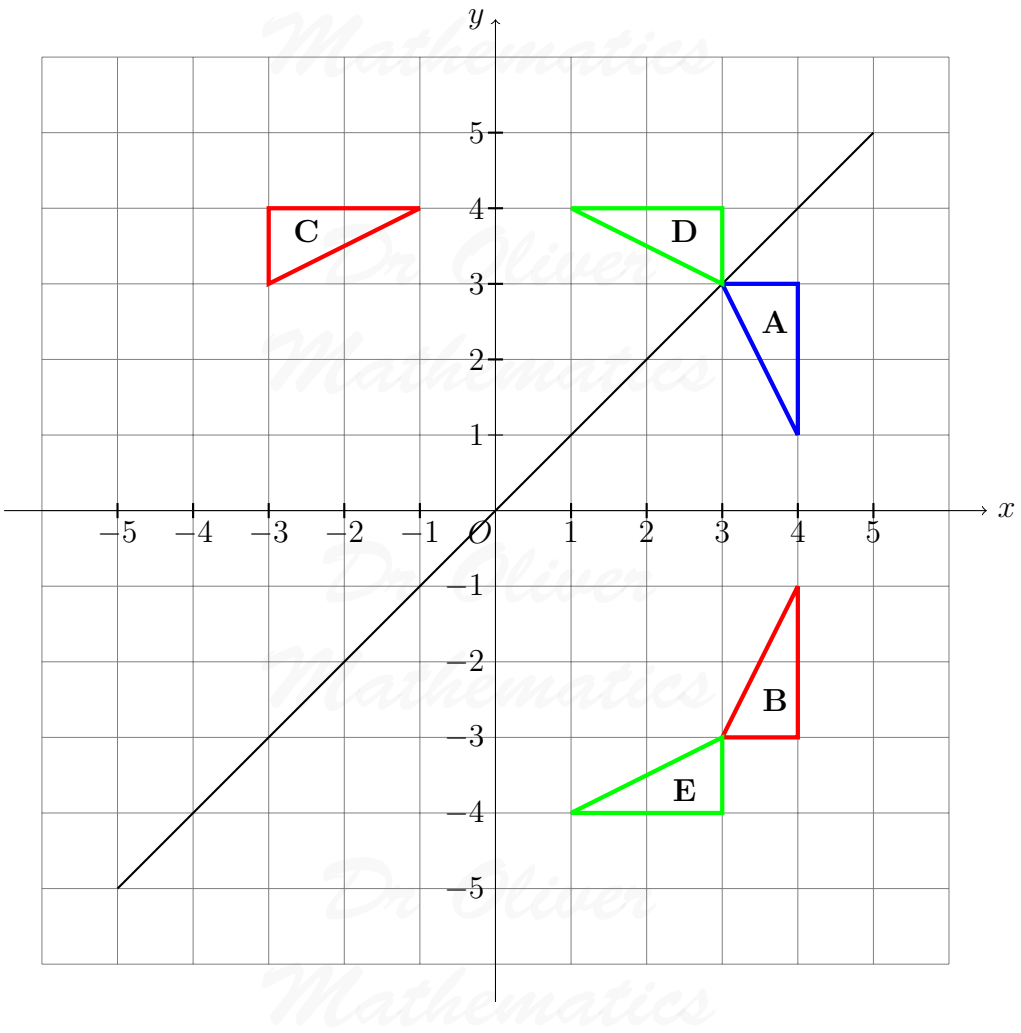
9. The diagram shows triangle **A** drawn on a grid. (3)



Kyle reflects triangle **A** in the x-axis to get triangle **B**.
He then reflects triangle **B** in the line $y = x$ to get triangle **C**.
Amy reflects triangle **A** in the line $y = x$ to get triangle **D**.
She is then going to reflect triangle **D** in the x-axis to get triangle **E**.
Amy says that triangle **E** should be in the same position as triangle **C**.
Is Amy correct?
You must show how you get your answer.

Solution

Let the red shapes denote Kyle and let the green shapes denote Amy.



No, she is not correct.

10. The table shows some information about eight planets.

Planet	Distance from Earth (km)	Mass (kg)
Earth	0	5.97×10^{24}
Jupiter	6.29×10^{8}	1.898×10^{27}
Mars	7.83×10^{7}	6.42×10^{23}
Mercury	9.17×10^{7}	3.302×10^{23}
Neptune	4.35×10^{9}	1.024×10^{26}
Saturn	1.28×10^{9}	5.68×10^{26}
Uranus	2.72×10^{9}	8.683×10^{25}
Venus	4.14×10^{7}	4.869×10^{24}

(a) Write down the name of the planet with the greatest mass. (1)

Solution

Jupiter.

(b) Find the difference between the mass of Venus and the mass of Mercury. (1)

Solution

$$4.869 \times 10^{24} - 3.302 \times 10^{23} = 4.538\,8 \times 10^{24} \text{ kg}.$$

Nishat says that Neptune is over a hundred times further away from Earth than Venus is.

(c) Is Nishat right? (2)
You must show how you get your answer.

Solution

$$\frac{4.35 \times 10^9}{4.14 \times 10^7} = 105\tfrac{5}{69};$$

so Nishat is correct.

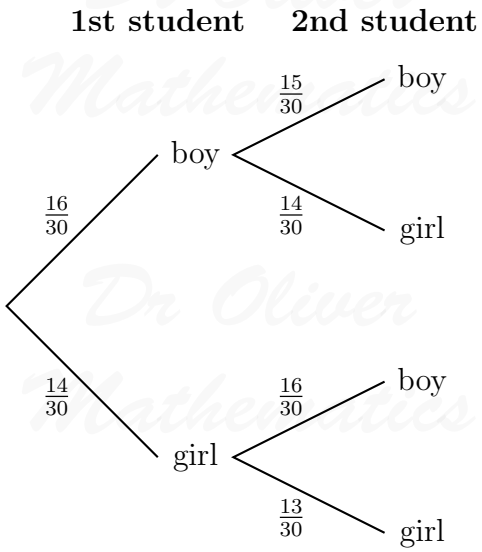
11. Solve (4)

$$\frac{3x-2}{4} - \frac{2x+5}{3} = \frac{1-x}{6}.$$

Solution

$$\begin{aligned}
\frac{3x-2}{4} - \frac{2x+5}{3} = \frac{1-x}{6} &\Rightarrow 3(3x-2) - 4(2x+5) = 2(1-x) \\
&\Rightarrow 9x - 6 - 8x - 20 = 2 - 2x \\
&\Rightarrow 3x = 28 \\
&\Rightarrow x = 9\tfrac{1}{3}.
\end{aligned}$$

12. There are 30 students in Mr Lear's class.
16 of the students are boys.
Two students from the class are chosen at random.
Mr Lear draws this probability tree diagram for this information.



(a) Write down **one** thing that is wrong with the probabilities in the probability tree diagram. (1)

Solution

E.g., in each of the two '2nd student', they only add up to $\frac{29}{30}$.

Owen and Wasim play for the school football team.
The probability that Owen will score a goal in the next match is 0.4.
The probability that Wasim will score a goal in the next match is 0.25.
Mr Slater says, "The probability that both boys will score a goal in the next match is 0.4 + 0.25."

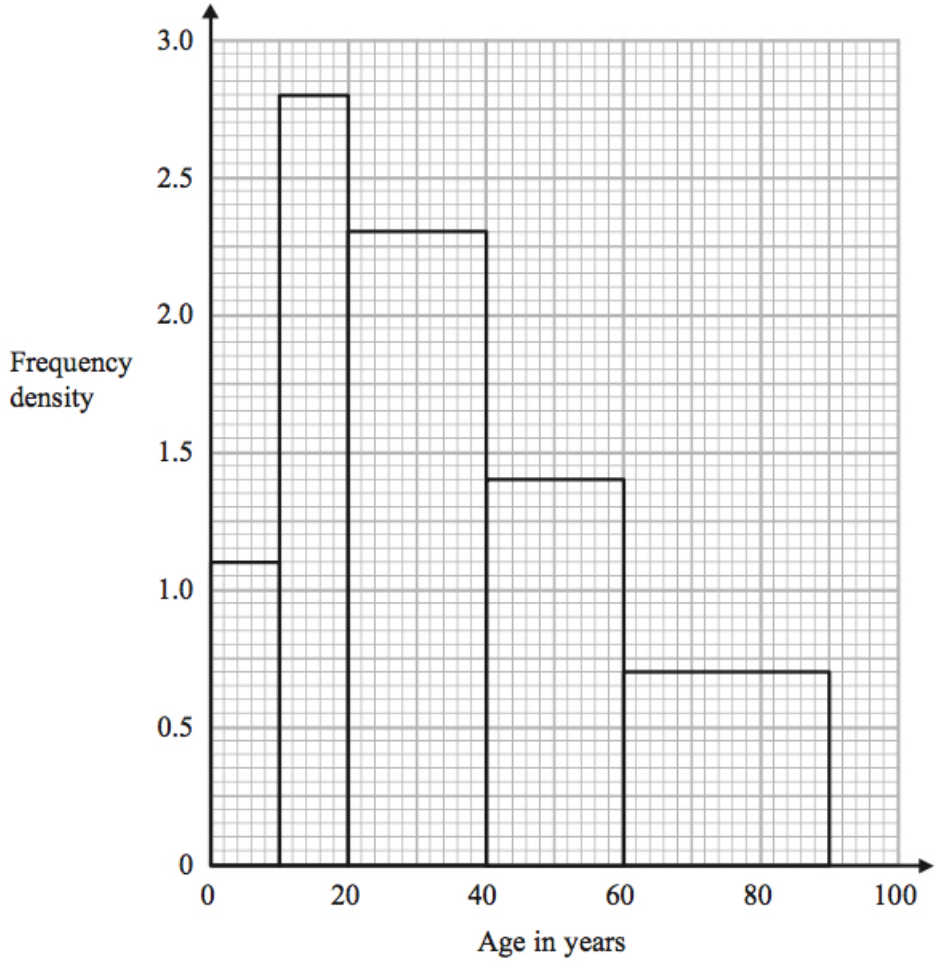
(b) Is Mr Slater right? (1)
Give a reason for your answer.

Solution

No; we should multiply the probabilities:

$$0.4 \times 0.25 = 0.1.$$

13. The histogram shows some information about the ages of the 134 members of a sports club. (3)



20% of the members of the sports club who are over 50 years of age are female.
Work out an estimate for the number of female members who are over 50 years of age.

Solution

Age	Frequency Density	Width	Frequency
$0-10$	1.1	10	$1.1 \times 10 = 11$
$10-20$	2.8	10	$2.8 \times 10 = 26$
$20-40$	2.3	20	$2.3 \times 20 = 46$
$40-60$	1.4	20	$1.4 \times 20 = 28$
$60-90$	0.7	30	$0.7 \times 30 = 21$
Add			134

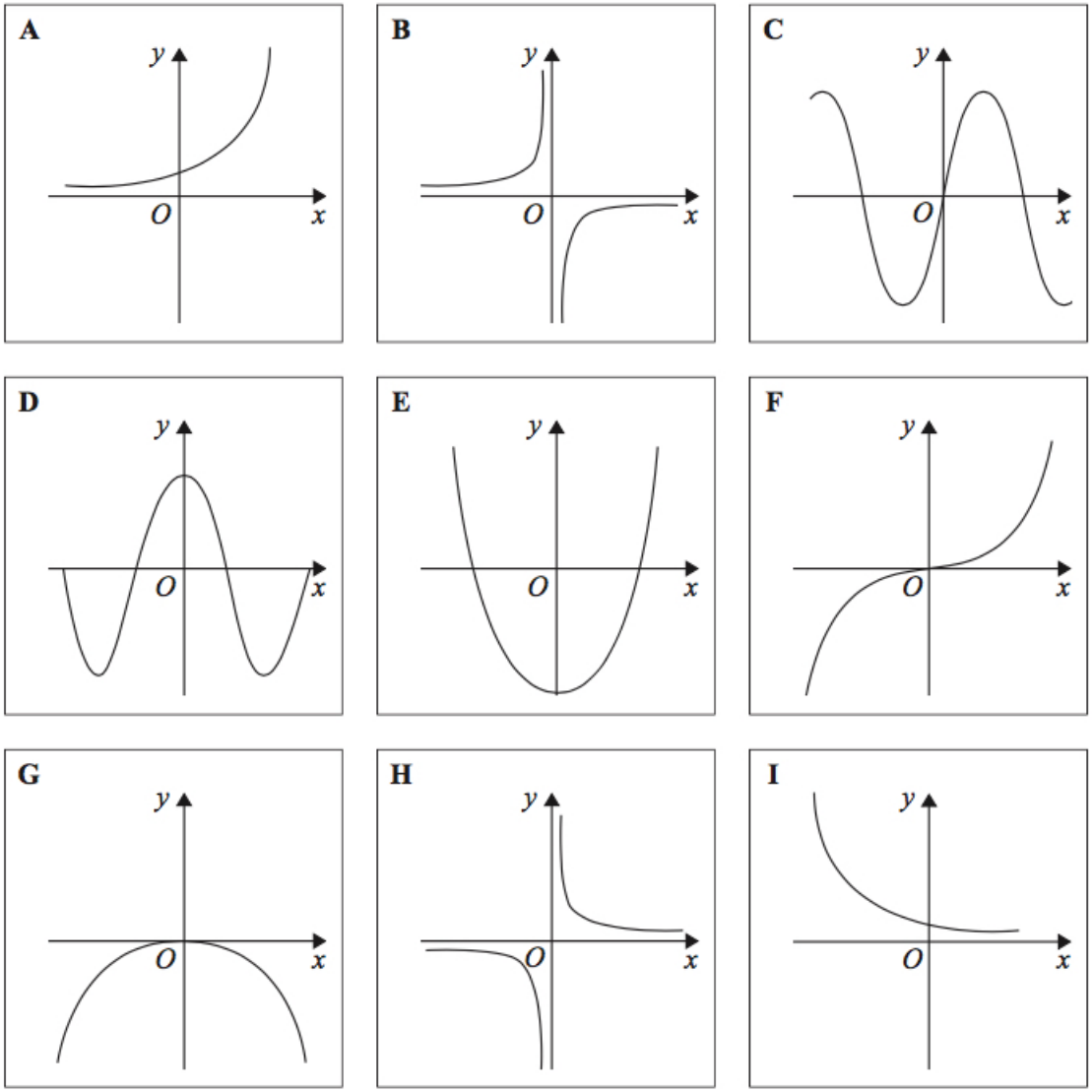
Estimate for $50 - 90$ members:

$$14 + 21 = 35 \text{ members.}$$

Finally,

$$\frac{35}{5} = \underline{\text{7 members.}}.$$

14. Here are some graphs. (3)



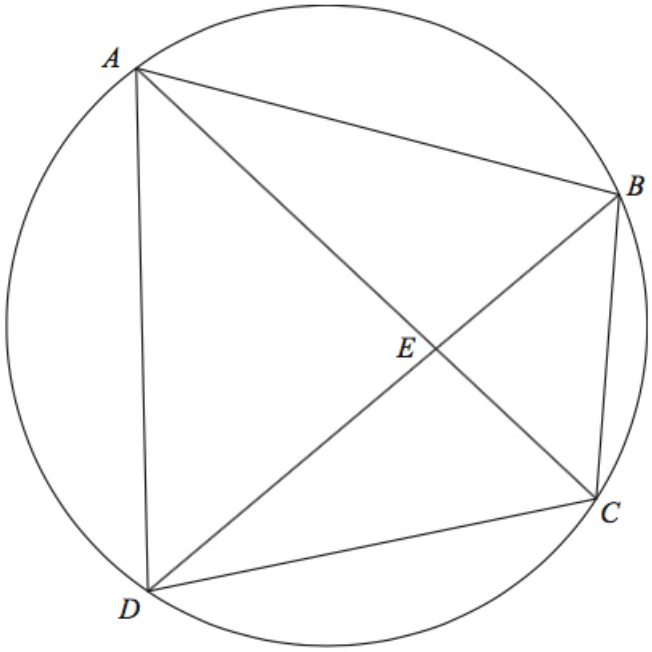
In the table below, match each equation with the letter of its graph.

Equation	Graph
$y = \sin x^\circ$	
$y = x^3 + 4x$	
$y = 2^x$	
$y = \frac{4}{x}$	

Solution

Equation	Graph
$y = \sin x^\circ$	C
$y = x^3 + 4x$	F
$y = 2^x$	A
$y = \frac{4}{x}$	H

15. A, B, C, and D are four points on the circumference of a circle. (3)



AEC and BED are straight lines.
Prove that triangle ABE and triangle DCE are similar.
You must give reasons for each stage of your working.

Solution

$\angle ABE = \angle DCE$ (angles in the same segment)
$\angle AEB = \angle CED$ (opposite angles)
$\angle BAE = \angle CDE$ (third angle is the same)
and so the triangles $\triangle ABE$ and $\triangle CDE$ are <u>similar</u>.

16. Using algebra, prove that $0.1\dot{3}\dot{6} \times 0.\dot{2}$ is equal in value to $\frac{1}{33}$. (3)

Solution

$$\begin{aligned} x = 0.1\dot{3}\dot{6} &\Rightarrow 10x = 1.\dot{3}\dot{6} \\ &\Rightarrow 10x = 1\tfrac{36}{99} \\ &\Rightarrow 10x = 1\tfrac{4}{11} \\ &\Rightarrow 10x = \tfrac{15}{11} \\ &\Rightarrow x = \tfrac{15}{110} \\ &\Rightarrow x = \tfrac{3}{22} \end{aligned}$$

and

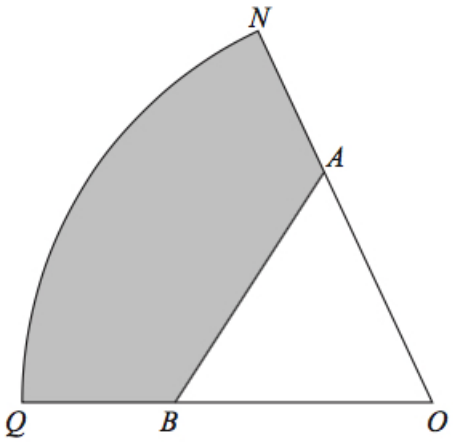
$$0.\dot{2} = \tfrac{2}{9}.$$

Finally,

$$\begin{aligned} 0.1\dot{3}\dot{6} \times 0.\dot{2} &= \tfrac{3}{22} \times \tfrac{2}{9} \\ &= \underline{\underline{\tfrac{1}{33}}}, \end{aligned}$$

as required.

17. ONQ is a sector of a circle with centre O and radius 11 cm. (5)



A is the point on ON and B is the point on OQ such that AOB is an equilateral triangle of side 7 cm.
Calculate the area of the shaded region as a percentage of the area of the sector ONQ.
Give your answer correct to 1 decimal place.

Solution

$$\begin{aligned}\text{Sector} &= \tfrac{1}{6} \times \pi \times 11^2 - \tfrac{1}{2} \times 7^2 \times \sin 60^\circ \\ &= \tfrac{121}{6}\pi - \tfrac{49}{4}\sqrt{3}\end{aligned}$$

and the percentage of the area is

$$\begin{aligned}\frac{\frac{121}{6}\pi - \frac{49}{4}\sqrt{3}}{\frac{121}{6}\pi} \times 100\% &= 66.510\,186\,93 \text{ (FCD)} \\ &= \underline{\underline{66.5\% \text{ (1 dp)}}}.\end{aligned}$$

18. $16^{\frac{1}{5}} \times 2^x = 8^{\frac{3}{4}}$. (3)
Work out the exact value of x.

Solution

$$16^{\frac{1}{5}} = (2^4)^{\frac{1}{5}} = 2^{\frac{4}{5}}$$

and

$$8^{\frac{3}{4}} = (2^3)^{\frac{3}{4}} = 2^{\frac{9}{4}}.$$

Now,

$$2^x = \frac{8^{\frac{3}{4}}}{16^{\frac{1}{5}}} \Rightarrow 2^x = \frac{2^{\frac{9}{4}}}{2^{\frac{4}{5}}}$$
$$\Rightarrow 2^x = 2^{\frac{29}{20}}$$
$$\Rightarrow \underline{\underline{x = 1\tfrac{9}{20}}}.$$

19. (4)

$$2 - \frac{x+2}{x-3} - \frac{x-6}{x+3}$$

can be written as a single fraction in the form

$$\frac{ax+b}{x^2-9}$$

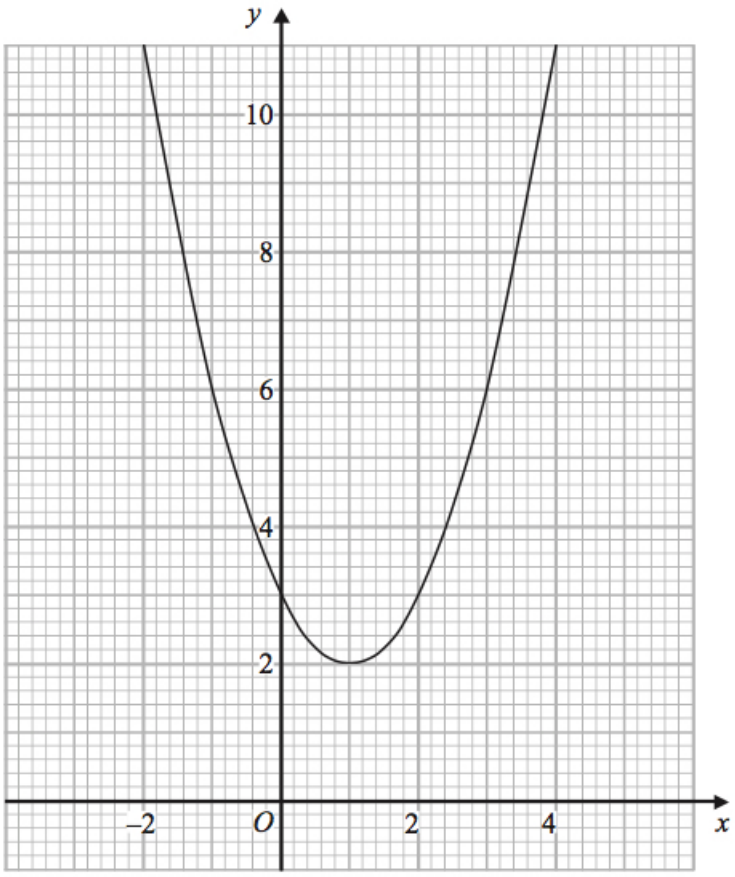
where a and b are integers.
Work out the value of a and the value of b.

Solution

$$2 - \frac{x+2}{x-3} - \frac{x-6}{x+3} \equiv \frac{2(x-3)(x+3) - (x+2)(x+3) - (x-6)(x-3)}{(x-3)(x+3)}$$
$$\equiv \frac{(2x^2-18) - (x^2+5x+6) - (x^2-9x+18)}{(x-3)(x+3)}$$
$$\equiv \frac{4x-42}{(x-3)(x+3)};$$

hence, $\underline{\underline{a = 4}}$ and $\underline{\underline{b = -42}}$.

20. The diagram shows part of the graph of $y = x^2 - 2x + 3$.



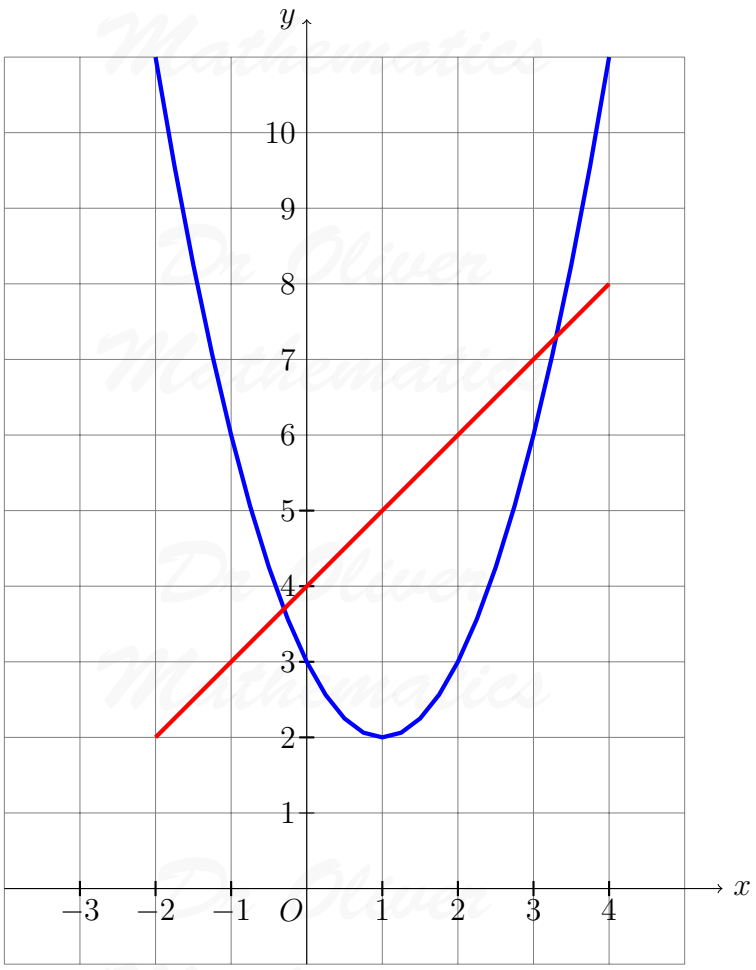
(a) By drawing a suitable straight line, use your graph to find estimates for the solutions of (2)

$$x^2 - 3x - 1 = 0.$$

Solution

$$x^2 - 3x - 1 = 0 \Rightarrow x^2 - 2x + 3 = x + 4$$

and so draw the $y = x + 4$ on to the graph.

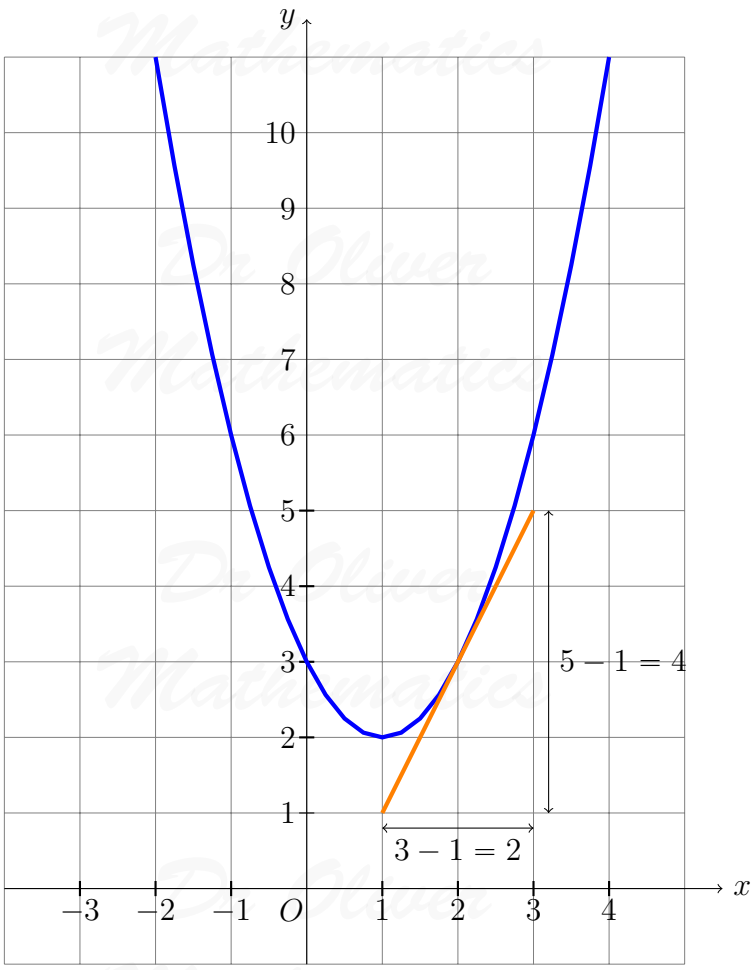


Read off: approximately $\underline{\underline{x = -0.3}}$ and $\underline{\underline{x = 3.3}}$.

P is the point on the graph of $y = x^2 - 2x + 3$ where $x = 2$.

(b) Calculate an estimate for the gradient of the graph at the point P. (3)

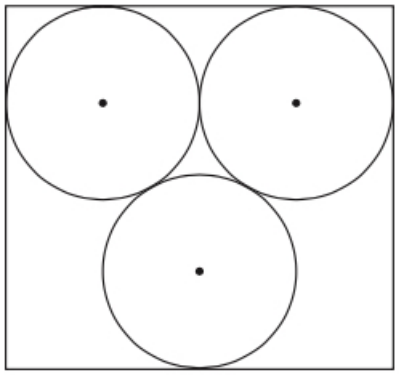
Solution



The gradient is

$$\frac{4}{2} = \underline{\underline{2}}.$$

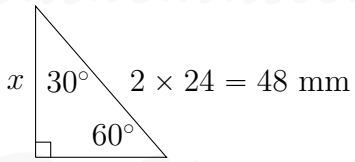
21. The diagram shows 3 identical circles inside a rectangle. (4)
Each circle touches the other two circles and the sides of the rectangle, as shown in the diagram.



The radius of each circle is 24 mm.
Work out the area of the rectangle.
Give your answer correct to 3 significant figures.

Solution

The length is $4 \times 24 = 96$ mm.



Now,

$$x = 48\cos 30^\circ$$

and so height is

$$24 + 48\cos 30^\circ + 24 = 48(1 + \cos 30^\circ).$$

Finally,

$$\begin{aligned} \text{area} &= 96 \times 48(1 + \cos 30^\circ) \\ &= 8\,598.645\,061 \text{ (FCD)} \\ &= \underline{\underline{8\,600 \text{ mm}^2 \text{ (3 sf)}}}. \end{aligned}$$

22. Here are the first five terms of a sequence. (3)

4 11 22 37 56

Find an expression, in terms of n, for the nth term of this sequence.

Solution

Is it a linear sequence? No. Why? The differences are getting bigger.

Is it a quadratic sequence? Yes. Let the

$$n\text{th term} = an^2 + bn + c.$$

Write down the sequence:	4	11	22	37	56
First line of differences:	7	11	15	19	
Second line of differences:	4	4	4		

We compare terms:

$$2a = 4 \Rightarrow a = 2,$$

$$3a + b = 7 \Rightarrow 3 \times 2 + b = 7$$
$$\Rightarrow b = 1,$$

and

$$a + b + c = 9 \Rightarrow 2 + 1 + c = 4$$
$$\Rightarrow c = 1;$$

hence,

$$n\text{th term} = \underline{\underline{2n^2 + n + 1}}.$$

23. **L** is the circle with equation $x^2 + y^2 = 4$. (3)

$P\left(\frac{3}{2}, \frac{\sqrt{7}}{2}\right)$ is a point on **L**.

Find an equation of the tangent to **L** at the point P.

Solution

The gradient of OP is

$$\frac{\frac{\sqrt{7}}{2}}{\frac{3}{2}} = \frac{\sqrt{7}}{3}$$

and the tangent is

$$-\frac{3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}.$$

Finally,

$$y - \tfrac{\sqrt{7}}{2} = -\tfrac{3\sqrt{7}}{7}(x - \tfrac{3}{2}) \Rightarrow y - \tfrac{\sqrt{7}}{2} = -\tfrac{3\sqrt{7}}{7}x + \tfrac{9\sqrt{7}}{14}$$
$$\Rightarrow \underline{\underline{y = -\tfrac{3\sqrt{7}}{7}x + \tfrac{8\sqrt{7}}{7}}}.$$