

Dr Oliver Mathematics

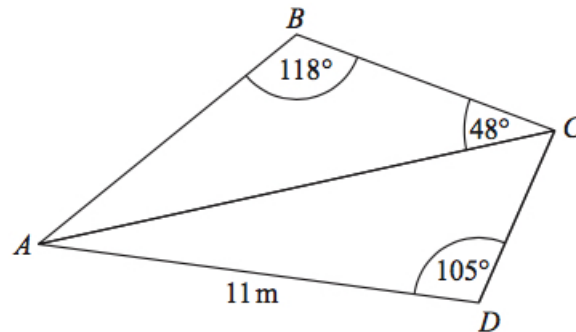
Worked Examples

Super Trigonometry 1

From: Edexcel 2017 November Paper 3H (Calculator)

1. ABC and ADC are triangles.

(5)



The area of triangle ADC is 56 m^2 .

Work out the length of AB .

Give your answer correct to 1 decimal place.

Solution

This is an excellent question! We need

- the sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$$

- the cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A,$$

- the area of a triangle:

$$\text{area} = \frac{1}{2}ab \sin C,$$

though not necessarily in that order ...

Step 1: area of a triangle

$$\begin{aligned}\text{Area of } \triangle ADC &= 56 \Rightarrow \frac{1}{2} \times 11 \times CD \times \sin 105^\circ = 56 \\ &\Rightarrow 5.5 CD \sin 105^\circ = 56 \\ &\Rightarrow CD = \frac{56}{5.5 \sin 105^\circ} \\ &\Rightarrow CD = 10.540\,993\,84 \text{ (FCD)}.\end{aligned}$$

Step 2: the cosine rule

Now, we apply the cosine rule:

$$\begin{aligned}AC^2 &= AD^2 + CD^2 - 2 \times AD \times CD \times \cos ADC \\ \Rightarrow AC^2 &= 11^2 + 10.540\dots^2 - 2 \times 11 \times 10.540\dots \times \cos 105^\circ \\ \Rightarrow AC^2 &= 292.133\,170\,2 \text{ (FCD)} \\ \Rightarrow AC &= 17.091\,903\,64 \text{ (FCD)}.\end{aligned}$$

Step 3: the sine rule

Finally, we apply the sine rule:

$$\begin{aligned}\frac{AB}{\sin ACB} &= \frac{AC}{\sin ABC} \Rightarrow \frac{AB}{\sin 48^\circ} = \frac{17.091\dots}{\sin 118^\circ} \\ \Rightarrow AB &= \frac{17.091\dots \sin 48^\circ}{\sin 118^\circ} \\ \Rightarrow AB &= 14.385\,632\,68 \text{ (FCD)} \\ \Rightarrow AB &= \underline{\underline{14.4 \text{ m (1 dp)}}}.\end{aligned}$$

If I have the *teeniest* qualification about this question, I think that 5 marks is *not* enough: after all, you are required to get the side CD and present your answer, you are required to get the side AC and present your answer, and you are required to get the side AB and present your answer. Still, this is a very good question!