

Dr Oliver Mathematics
Worked Examples
Indices 1

From: Edexcel GCSE Mathematics (9-1) Practice Tests Set 14: Paper 1H (Non-calculator)

1. Given that

$$M = \frac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2},$$

(6)

find the values of n for which $M = 2$

Solution

Well,

$$\begin{aligned} 18^{4n} &= (2 \times 3^2)^{4n} \\ &= 2^{4n} \times 3^{2(4n)} \\ &= 2^{4n} \times 3^{8n} \end{aligned}$$

and

$$\begin{aligned} 12^2 &= (2^2 \times 3)^2 \\ &= 2^{2(2)} \times 3^2 \\ &= 2^4 \times 3^2. \end{aligned}$$

Now,

$$\begin{aligned} 2 &= \frac{18^{4n} \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{12^2} \\ \Rightarrow 2 &= \frac{(2^{4n} \times 3^{8n}) \times 2^{3(n^2-6n)} \times 3^{2(1-4n)}}{2^4 \times 3^2} \\ \Rightarrow 2^5 \times 3^2 &= (2^{4n} \times 2^{3(n^2-6n)}) \times (3^{8n} \times 3^{2(1-4n)}) \\ \Rightarrow 2^5 \times 3^2 &= 2^{4n+3(n^2-6n)} \times 3^{8n+2(1-4n)} \\ \Rightarrow 2^5 \times 3^2 &= 2^{4n+3n^2-18n} \times 3^{8n+2-8n} \\ \Rightarrow 2^5 \times 3^2 &= 2^{3n^2-14n} \times 3^2 \\ \Rightarrow 2^5 &= 2^{3n^2-14n} \end{aligned}$$

so

$$5 = 3n^2 - 14n.$$

Finally,

$$5 = 3n^2 - 14n \Rightarrow 3n^2 - 14n - 5 = 0$$

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array} \right\} \begin{array}{l} -14 \\ (+3) \times (-5) = -15 \end{array} \quad -15, +1$$

$$\Rightarrow 3n^2 - 15n + n - 5 = 0$$

$$\Rightarrow 3n(n - 5) + 1(n - 5) = 0$$

$$\Rightarrow (3n + 1)(n - 5) = 0$$

$$\Rightarrow 3n + 1 = 0 \text{ or } n - 5 = 0$$

$$\Rightarrow \underline{\underline{n = -\frac{1}{3} \text{ or } n = 5.}}$$