

Dr Oliver Mathematics
GCSE Mathematics
2009 June Paper 4H: Calculator
1 hour 45 minutes

The total number of marks available is 100.

You must write down all the stages in your working.

1. Tania went to Italy.

She changed £325 into euros (€).

The exchange rate was £1 = €1.68.

- (a) Change £325 into euros (€).

(2)

Solution

$$325 \times 1.68 = \underline{\underline{€546.}}$$

When she came home she changed €117 into pounds.

The new exchange rate was £1 = €1.50.

- (b) Change €117 into pounds.

(2)

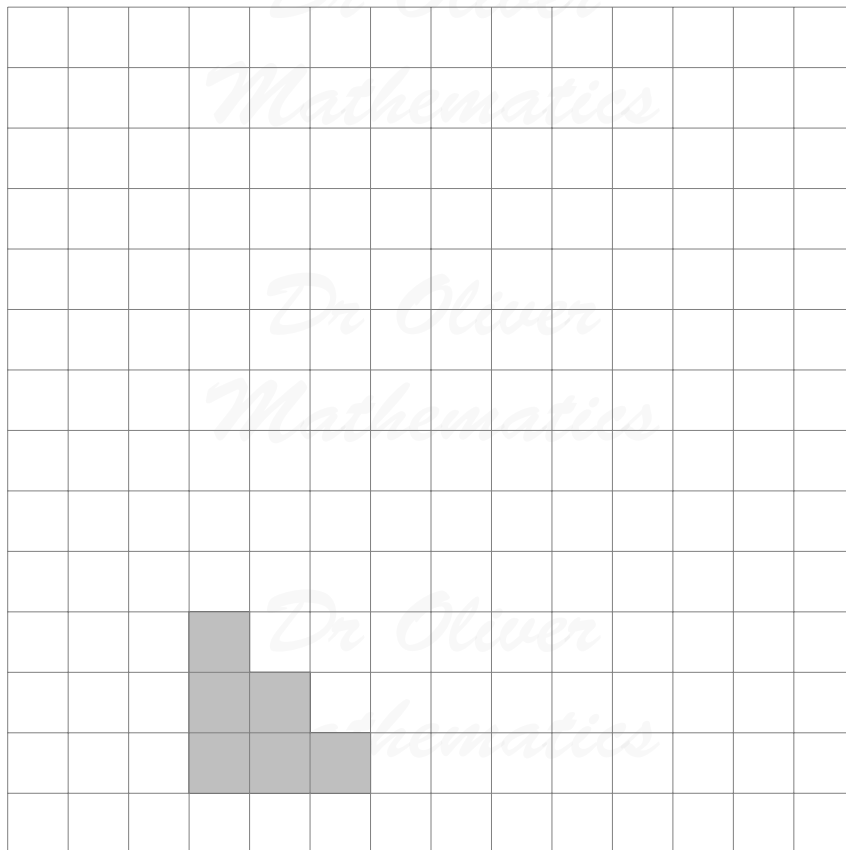
Solution

$$\frac{117}{1.50} = \underline{\underline{£78.}}$$

2. (a) On the grid, draw an enlargement, scale factor 2, of the shaded shape.

(2)

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Solution

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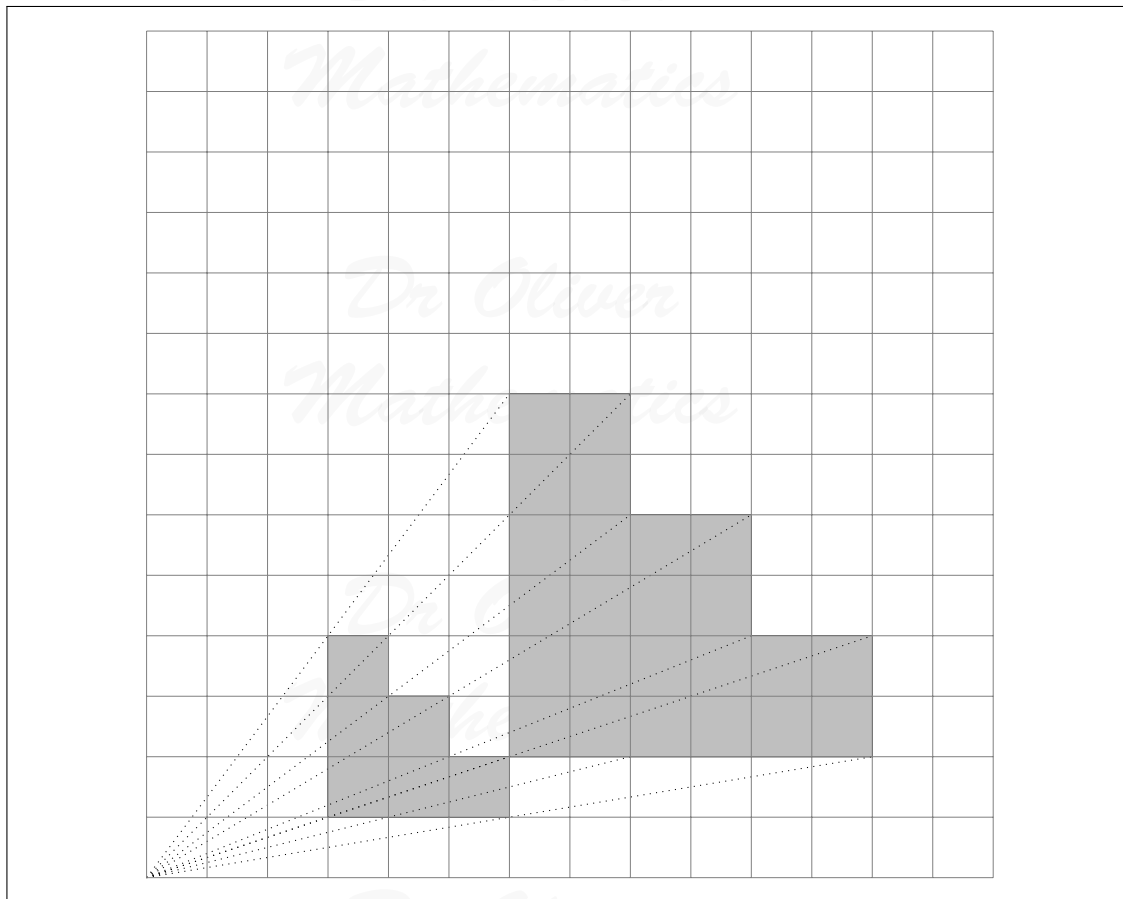
Mathematics

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Mathematics

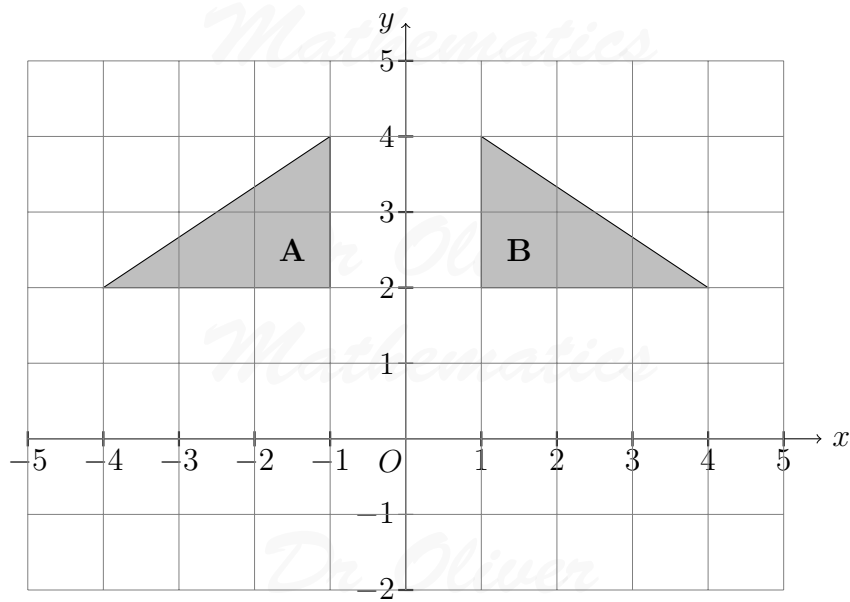
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(b) Describe fully the single transformation that maps triangle **A** onto triangle **B**.

(2)



Solution

It is a reflection in the y-axis.

3. The n th term of a number sequence is $n^2 + 1$.

(2)

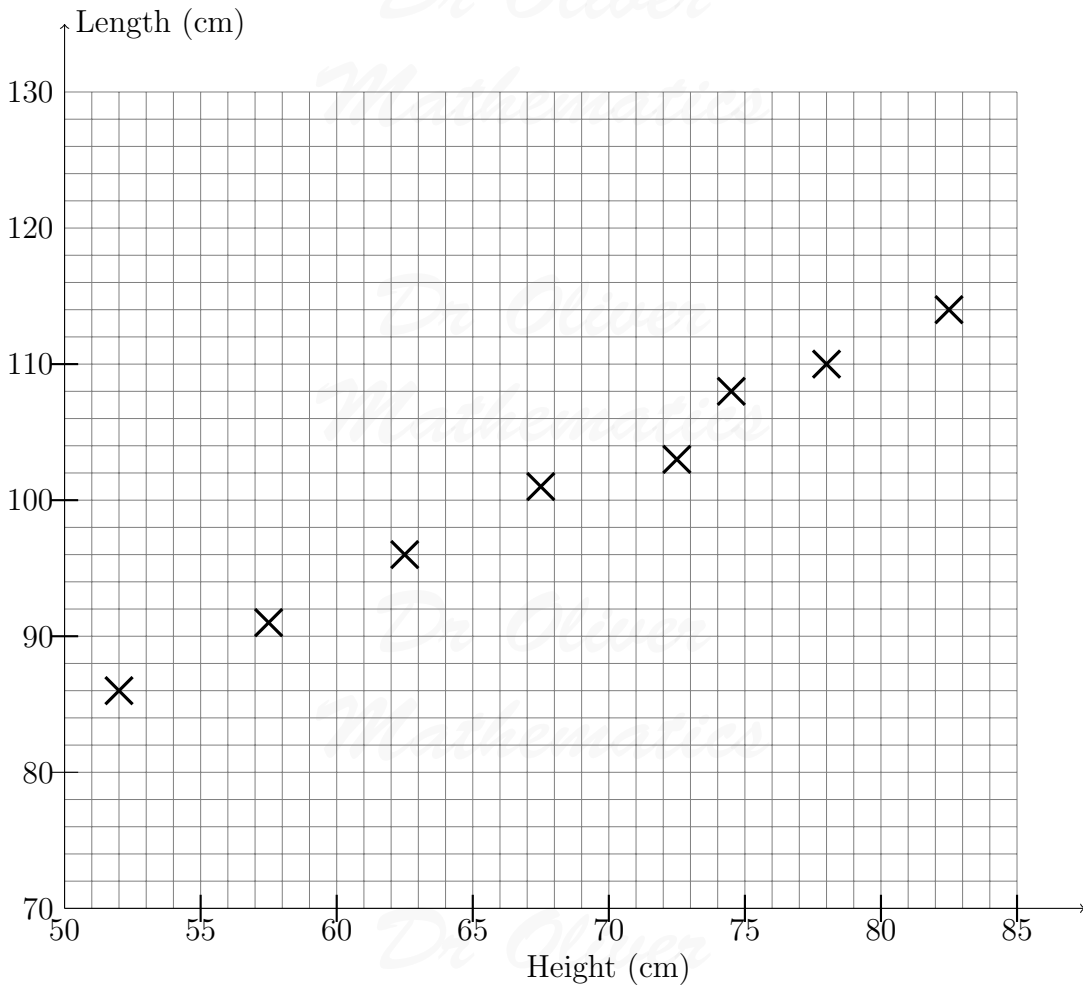
Write down the first three terms of the sequence.

Solution

2, 5, 10.

4. The scatter graph shows information about eight sheep.

It shows the height and the length of each sheep.

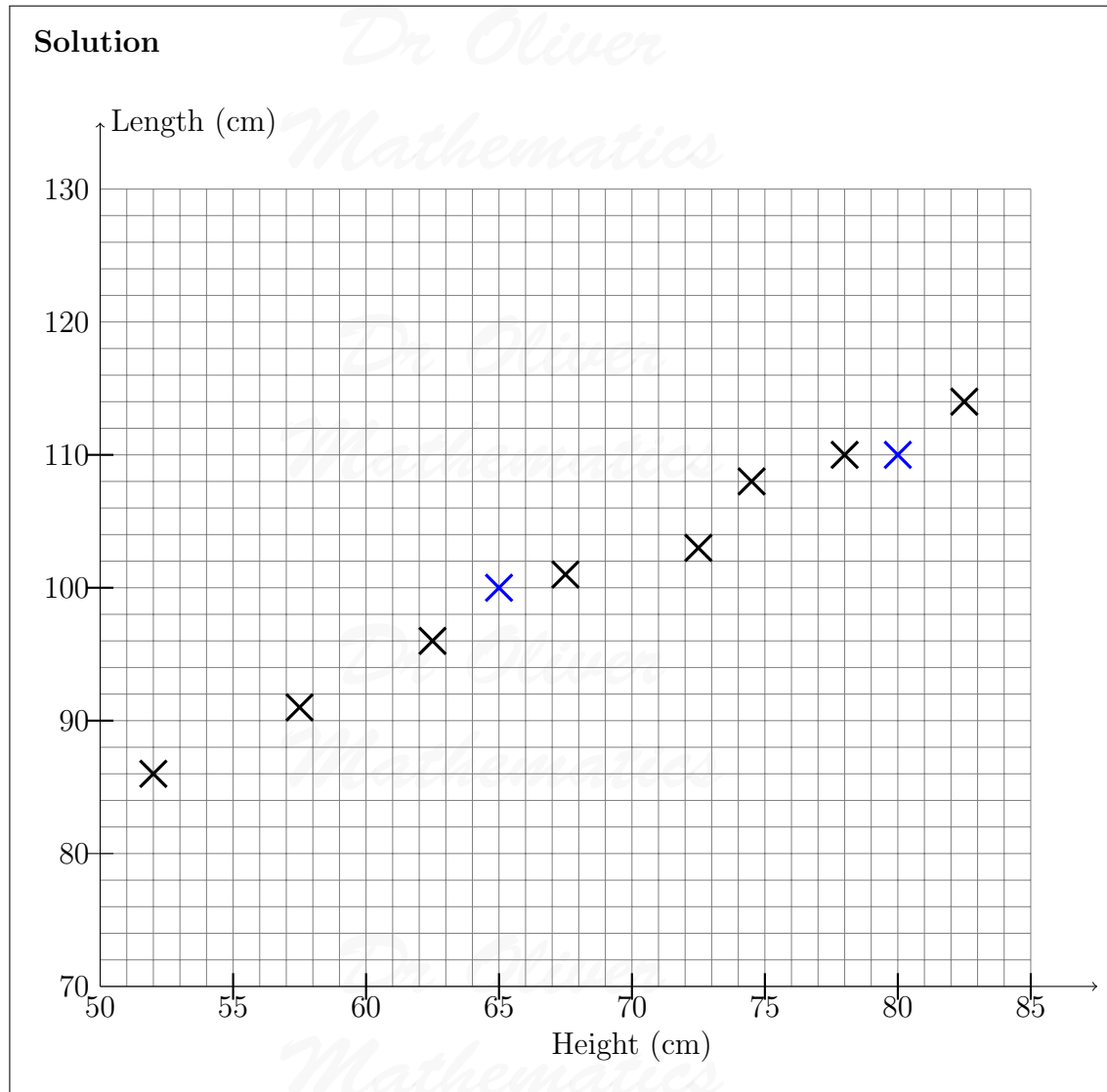


The table gives the height and the length of two more sheep.

Height (cm)	65	80
Length (cm)	100	110

(a) On the scatter graph, plot the information from the table.

(1)



(b) Describe the relationship between the height and the length of these sheep.

(1)

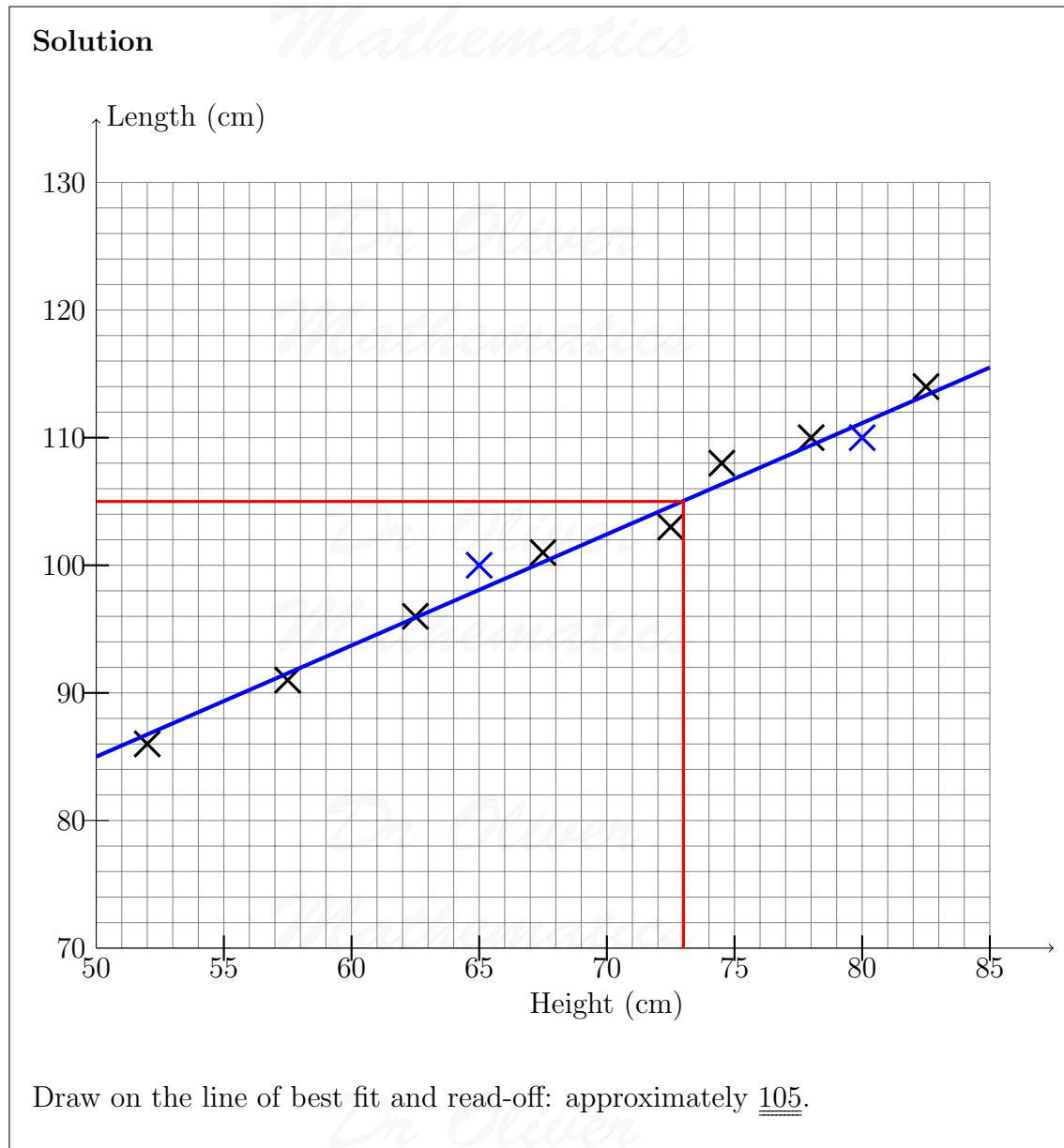
Solution

E.g., as the height of the sheep increases, the length of the sheep increases.

The height of a sheep is 76 cm.

(c) Estimate the length of this sheep.

(2)



5. Julie buys 19 identical calculators.

The total cost is £143.64.

Work out the total cost of 31 of these calculators.

(3)

Solution

$$\frac{143.64 \times 31}{19} = \underline{\underline{£234.36}}$$

6. $F = 1.8C + 32$.

(a) Work out the value of F when $C = -8$.

(2)

Solution

$$F = 1.8(-8) + 32 = \underline{\underline{17.6}}.$$

(b) Work out the value of C when $F = 68$.

(2)

Solution

$$\begin{aligned} 68 &= 1.8C + 32 \Rightarrow 1.8C = 36 \\ &\Rightarrow \underline{\underline{C = 20}}. \end{aligned}$$

7. The diagram shows the position of two boats, P and Q .

(3)

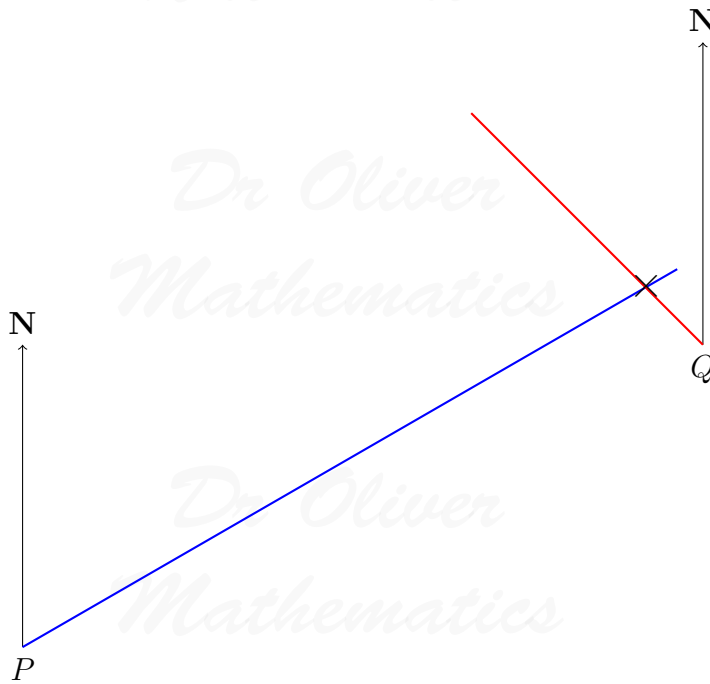


The bearing of a boat R from boat P is 060° .

The bearing of boat R from boat Q is 310° .

In the space above, draw an accurate diagram to show the position of boat R . Mark the position of boat R with a cross (\times). Label it R .

Solution



8. There are some sweets in a bag.
18 of the sweets are toffees.
12 of the sweets are mints.

(a) Write down the ratio of the number of toffees to the number of mints.
Give your ratio in its simplest form.

(2)

Solution

$$18 : 12 = 6 \times 3 : 6 \times 2 = \underline{\underline{3 : 2}}.$$

There are some oranges and apples in a box.
The total number of oranges and apples is 54.
The ratio of the number of oranges to the number of apples is 1 : 5.

(b) Work out the number of apples in the box.

(2)

Solution

$1 + 5 = 6$ and so the number of apples in the box is

$$\frac{5}{6} \times 54 = \underline{\underline{45}}.$$

9. The equation

$$x^3 + 20x = 71$$

(4)

has a solution between 2 and 3.

Use a trial and improvement method to find this solution.

Give your answer correct to one decimal place.

You must show **all** your working.

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.

x	$f(x)$	Comment
2.6	69.576	too low
2.65	71.609	too high

Clearly,

$$2.6 < x < 2.65$$

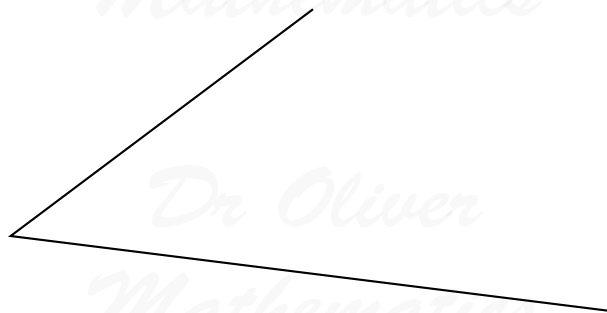
and the answer is

$$\underline{\underline{x = 2.6 \text{ (1 dp)}}}$$

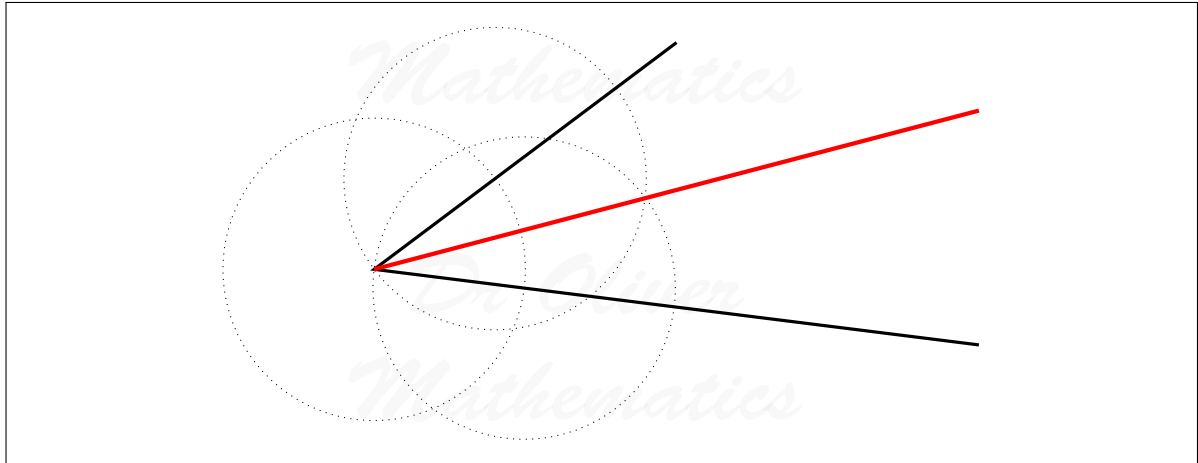
10. Use ruler and compasses to **construct** the bisector of this angle.

(2)

You must show all your construction lines.



Solution



11. Tarish says, ‘The sum of two prime numbers is always an even number.’ (2)
 He is **wrong**.
 Explain why.

Solution

E.g., $2 + 3 = 5$ and it is odd.

12. Sethina recorded the times, in minutes, taken to repair 80 car tyres. (4)
 Information about these times is shown in the table.

Time (t minutes)	Frequency
$0 < t \leq 6$	15
$6 < t \leq 12$	25
$12 < t \leq 18$	20
$18 < t \leq 24$	12
$24 < t \leq 30$	8

Calculate an estimate for the mean time taken to repair each car tyre.

Solution

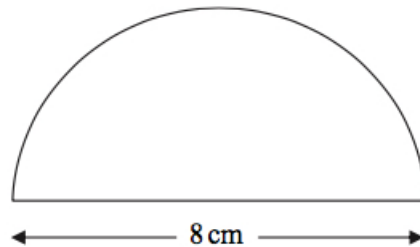
Time (t minutes)	Frequency	Midpoint	Time \times Frequency
$0 < t \leq 6$	15	3	45
$6 < t \leq 12$	25	9	225
$12 < t \leq 18$	20	15	300
$18 < t \leq 24$	12	21	252
$24 < t \leq 30$	8	27	216
Total			1 038

An estimate for the mean time taken to repair each car tyre is

$$\frac{1\,038}{80} = 12\frac{39}{40}.$$

13. Here is a tile in the shape of a semicircle.

(3)



**Diagram NOT
accurately drawn**

The diameter of the semicircle is 8 cm.

Work out the perimeter of the tile.

Give your answer correct to 2 decimal places.

Solution

The radius is 4 cm and so the perimeter of the tile is

$$\begin{aligned} 8 + \frac{1}{2} \times 2 \times \pi \times 4 &= 8 + 4\pi \\ &= 20.566\,370\,61 \text{ (FCD)} \\ &= \underline{\underline{20.57 \text{ cm (2 dp)}}}. \end{aligned}$$

14. (a) Simplify $a \times a \times a$. (1)

Solution

$$a \times a \times a = \underline{\underline{a^3}}.$$

- (b) Expand $5(3x - 2)$. (1)

Solution

$$5(3x - 2) = \underline{\underline{15x - 10}}.$$

- (c) Expand $3y(y + 4)$. (2)

Solution

$$3y(y + 4) = \underline{\underline{3y^2 + 12y}}.$$

- (d) Expand and simplify $2(x - 4) + 3(x + 2)$. (2)

Solution

$$\begin{aligned} 2(x - 4) + 3(x + 2) &= 2x - 8 + 3x + 6 \\ &= \underline{\underline{5x - 2}}. \end{aligned}$$

- (e) Expand and simplify $(x + 4)(x - 3)$. (2)

Solution

$$\begin{array}{r|rr} \times & x & +4 \\ \hline x & x^2 & +4x \\ -3 & -3x & -12 \\ \hline \end{array}$$

$$(x + 4)(x - 3) = \underline{\underline{x^2 + x - 12}}.$$

15. Work out (2)

$$\frac{4.6 + 3.85}{3.2^2 - 6.51}$$

Write down all the numbers on your calculator display.

Solution

$$\begin{aligned}\frac{4.6 + 3.85}{3.2^2 - 6.51} &= \frac{8.45}{10.24 - 6.51} \\ &= \frac{8.45}{3.73} \\ &= 2\frac{99}{373} \\ &= \underline{\underline{2.265\ 415\ 55}} \text{ (FCD).}\end{aligned}$$

16. (a) Simplify $t^6 \times t^2$. (1)

Solution

$$t^6 \times t^2 = \underline{\underline{t^8}}.$$

- (b) Simplify $\frac{m^8}{m^3}$. (1)

Solution

$$\frac{m^8}{m^3} = m^{8-3} = \underline{\underline{m^5}}.$$

- (c) Simplify $(2x)^3$. (2)

Solution

$$(2x)^3 = \underline{\underline{8x^3}}.$$

- (d) Simplify $3a^2h \times 4a^5h^4$. (2)

Solution

$$3a^2h \times 4a^5h^4 = \underline{\underline{12a^7h^5}}.$$

17. ABC is a right-angled triangle. (3)

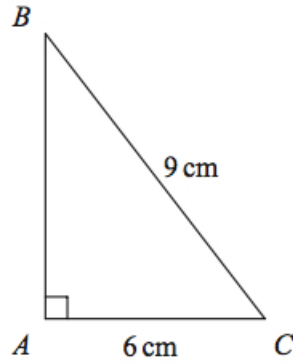


Diagram **NOT**
accurately drawn

$AC = 6$ cm.

$BC = 9$ cm.

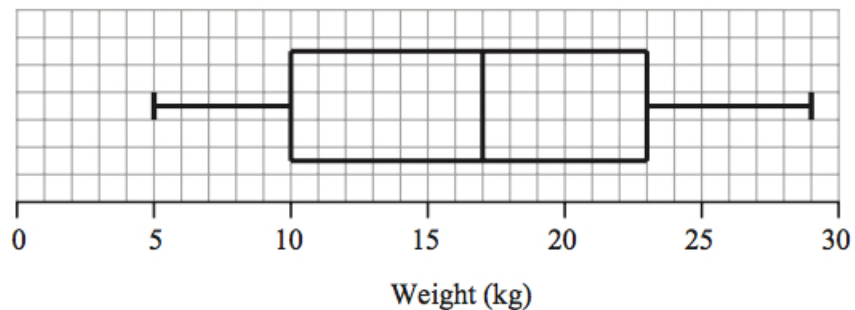
Work out the length of AB .

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}
 BC^2 &= AB^2 + AC^2 \Rightarrow AB^2 = BC^2 - AC^2 \\
 &\Rightarrow AB^2 = 9^2 - 6^2 \\
 &\Rightarrow AB^2 = 45 \\
 &\Rightarrow AB = 6.708\ 203\ 932 \text{ (FCD)} \\
 &\Rightarrow \underline{\underline{AB = 6.71 \text{ cm (3 sf)}}}
 \end{aligned}$$

18. The box plot gives information about the distribution of the weights of bags on a plane.



- (a) Jean says the heaviest bag weighs 23 kg.

She is **wrong**.

Explain why.

(1)

Solution

The is at least one bag who weighs 29 kg.

- (b) Write down the median weight. (1)

Solution

The median weight is 17 kg.

- (c) Work out the interquartile range of the weights. (1)

Solution

LQ = 10, UQ = 23, and so

$$\text{IQR} = 23 - 10 = \underline{\underline{13 \text{ kg}}}.$$

There are 240 bags on the plane.

- (d) Work out the number of bags with a weight of 10 kg or less. (2)

Solution

The number of bags with a weight of 10 kg or less is

$$\frac{1}{4} \times 240 = \underline{\underline{60}}.$$

19. Toby invested £4 500 for 2 years in a savings account.
He was paid 4% per annum compound interest.

- (a) How much did Toby have in his savings account after 2 years? (3)

Solution

$$4\,500 \times 1.04^2 = \underline{\underline{£4\,867.20}}.$$

Jaspir invested £2 400 for n years in a savings account.
He was paid 7.5% per annum compound interest.
At the end of the n years he had £3 445.51 in the savings account.

- (b) Work out the value of n . (2)

Solution

$$3445.51 = 2400 \times 1.075^n \Rightarrow 1.075^n = 1.435629167 \text{ (FCD).}$$

Now,

n	New amount
1	2580
2	2773.50
3	2981.51
4	3205.13
5	3445.51

It must have been 5 years.

20. Here is a right-angled triangle.

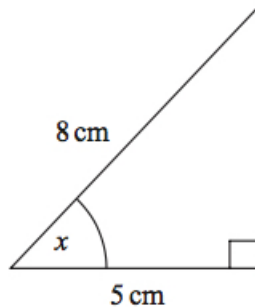


Diagram **NOT**
accurately drawn

- (a) Calculate the size of the angle marked x .
Give your answer correct to 1 decimal place.

(3)

Solution

$$\begin{aligned} \cos &= \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos x^\circ = \frac{5}{8} \\ &\Rightarrow x^\circ = 51.31781255 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x^\circ = 51.3^\circ \text{ (1 dp)}}}. \end{aligned}$$

Here is another right-angled triangle.

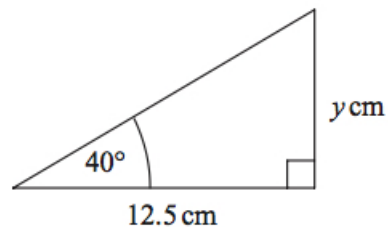


Diagram **NOT**
accurately drawn

- (b) Calculate the size of the angle marked y . (3)
Give your answer correct to 1 decimal place.

Solution

$$\begin{aligned} \text{opp} &= \text{adj} \times \tan \Rightarrow y = 12.5 \tan 40^\circ \\ &\Rightarrow y^\circ = 10.488\ 745\ 39 \text{ (FCD)} \\ &\Rightarrow \underline{y^\circ = 10.5^\circ \text{ (1 dp)}}. \end{aligned}$$

21. 258 students each study one of three languages.
The table shows information about these students.

	German	French	Spanish
Male	45	52	26
Female	25	48	62

A sample, stratified by the language studied and by gender, of 50 of the 258 students is taken.

- (a) Work out the number of male students studying Spanish in the sample. (2)

Solution

The number of male students studying Spanish in the sample is

$$\begin{aligned} \frac{26}{258} \times 50 &= 5.03\dots \\ &= \underline{5}. \end{aligned}$$

- (b) Work out the number of female students in the sample. (2)

Solution

$$\begin{aligned}\left(\frac{25 + 48 + 62}{258}\right) \times 50 &= \frac{135}{258} \times 50 \\ &= 26.16\dots \\ &= \underline{\underline{26}}.\end{aligned}$$

22. Prove that

$$(3n + 1)^2 - (3n - 1)^2$$

(3)

is a multiple of 4, for all positive integer values of n .

Solution

$$\begin{aligned}(3n + 1)^2 - (3n - 1)^2 &= (9n^2 + 6n + 1) - (9n^2 - 6n + 1) \\ &= 12n \\ &= 4 \times 3n;\end{aligned}$$

hence, the expression is a multiple of 4.

23. OAB is a triangle.

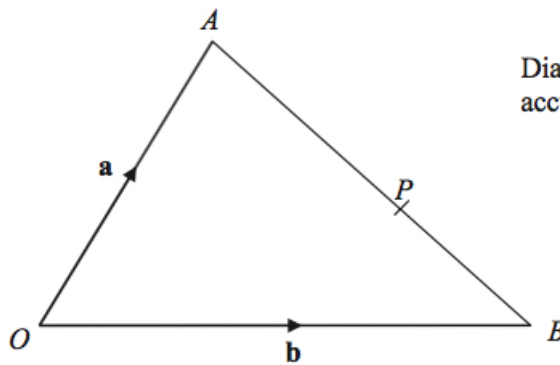


Diagram **NOT**
accurately drawn

$$\begin{aligned}\overrightarrow{OA} &= \mathbf{a}. \\ \overrightarrow{OB} &= \mathbf{b}.\end{aligned}$$

(a) Find the vector \overrightarrow{AB} in terms of \mathbf{a} and \mathbf{b} .

(1)

Solution

$$\overrightarrow{AB} = \underline{\underline{\mathbf{b} - \mathbf{a}}}.$$

P is the point on AB such that $AP : PB = 3 : 2$.

(b) Show that $\overrightarrow{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$.

(3)

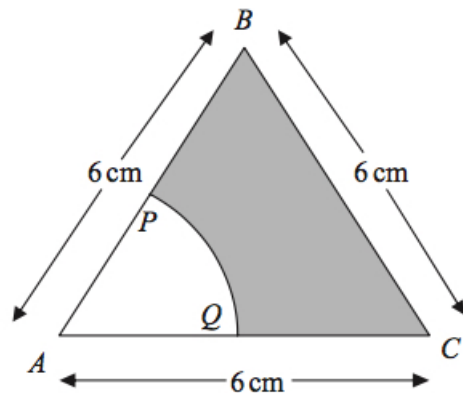
Solution

$$\begin{aligned}\overrightarrow{OP} &= \overrightarrow{OA} + \overrightarrow{AP} \\ &= \overrightarrow{OA} + \frac{3}{5}\overrightarrow{AB} \\ &= \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a}) \\ &= \mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{3}{5}\mathbf{a} \\ &= \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} \\ &= \underline{\underline{\frac{1}{5}(2\mathbf{a} + 3\mathbf{b})}},\end{aligned}$$

as required.

24. The diagram shows an equilateral triangle ABC with sides of length 6 cm.

(4)



**Diagram NOT
accurately drawn**

P is the midpoint of AB .

Q is the midpoint of AC .

APQ is a sector of a circle, centre A .

Calculate the area of the shaded region.

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\text{Area of the shaded region} &= \frac{1}{2} \times 6 \times \sqrt{6^2 - 3^2} - \frac{1}{6} \times \pi \times 3^2 \\ &= 9\sqrt{3} - \frac{3}{2}\pi \\ &= 10.876\,068\,29 \text{ (FCD)} \\ &= \underline{\underline{10.9 \text{ cm}^2}} \text{ (3 sf).}\end{aligned}$$

25. Simplify fully

$$\frac{x^2 - 8x + 15}{2x^2 - 7x - 15}.$$

(3)

Solution

$$\left. \begin{array}{l} \text{add to:} \quad -8 \\ \text{multiply to:} \quad +15 \end{array} \right\} -3, -5$$

So

$$x^2 - 8x + 15 = (x - 3)(x - 5).$$

$$\left. \begin{array}{l} \text{add to:} \quad -7 \\ \text{multiply to:} \quad (+2) \times (-15) = -30 \end{array} \right\} +3, -10$$

So

$$\begin{aligned}2x^2 - 7x - 15 &= 2x^2 - 10x + 3x - 15 \\ &= 2x(x - 5) + 3(x - 5) \\ &= (2x + 3)(x - 5).\end{aligned}$$

Hence,

$$\begin{aligned}\frac{x^2 - 8x + 15}{2x^2 - 7x - 15} &\equiv \frac{(x - 3)(x - 5)}{(2x + 3)(x - 5)} \\ &\equiv \frac{x - 3}{\underline{\underline{2x + 3}}}.\end{aligned}$$

26. Phil has 20 sweets in a bag.
5 of the sweets are orange.
7 of the sweets are red.

(4)

8 of the sweets are yellow.

Phil takes at random **two** sweets from the bag.

Work out the probability that the sweets will **not** be the same colour.

Solution

$$\begin{aligned} P(\text{the sweets will be the same colour}) &= \left(\frac{5}{20} \times \frac{4}{19}\right) + \left(\frac{7}{20} \times \frac{6}{19}\right) + \left(\frac{8}{20} \times \frac{7}{19}\right) \\ &= \frac{59}{190} \end{aligned}$$

and

$$P(\text{the sweets will not be the same colour}) = 1 - \frac{59}{190} = \frac{131}{190}.$$

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