Dr Oliver Mathematics
Implicit Differentiation: Part 1

1. If
$$\sin x = e^y, \ 0 < x < \pi,$$

what is $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x?

Solution

We use implicit differentiation:

$$\sin x = e^y \Rightarrow \frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \frac{\mathrm{d}}{\mathrm{d}x}(e^y)$$
$$\Rightarrow \cos x = \frac{\mathrm{d}}{\mathrm{d}y}(e^y) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}$$
$$\Rightarrow \cos x = e^y \cdot \frac{\mathrm{d}y}{\mathrm{d}x}$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos x}{e^y}$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos x}{\sin x}$$
$$\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = \cot x.}}$$