

Dr Oliver Mathematics

Mathematical Writing

In this note, we will explore mathematical writing.

You must always remember that a good proof should also include words and “Correct English” is as important as the symbols and numbers when writing mathematics. Since it is important to present proofs clearly, it is good to add the explanation of what is happening at each step using full sentences. The whole page with just numbers and symbols, without a single word, will nearly always be an example of a bad proof.

1 Guidelines

- (a) Start each sentence with a word, not a mathematical symbol (*not* $\exists x \in \mathbb{R}$, etc).
- (b) End each sentence with a full stop.
- (c) Separate mathematical symbols and expressions with words.
- (d) Symbols (such as \neq , Σ , μ , 2, etc) are not words. While it is appropriate to use them in mathematical expressions, they are out of place in other contexts.
- (e) Use the first person plural, i.e., “we” and “us”.
- (f) Use the active voice; this is just a suggestion, but the active voice makes your writing more full of life and energy.
- (g) You must explain each new symbol you introduce.
- (h) If there is *any* possibility of confusion, you should avoid the word “it.”
- (i) In proofs, it is common to use these words as conjunctions joining two statements, and meaning that one statement is valid and, as a result, the other is valid.
- (j) *Thus, hence, therefore, consequently*, etc, precede a statement that follows logically from previous sentences or clauses. Be sure that a statement follows them.
- (k) Clarity is the gold standard of mathematical writing. If you believe breaking a rule makes your writing clearer, then break the rule.

2 Advice to students for writing good proofs

- (a) At the start of your proof-work, you will need to do some rough work to figure out how your proof should go. In particular, you will be able to get a good idea of how a proof

should go by noticing that it can be similar to the proof of something else.

- (b) You should understand every line that you write, and *do not* make untrue claims. In particular, understanding the proofs in the class can take you a long way in understanding the type of reasoning that is involved in a proof.
- (c) How you lay out a proof is at least as important as the content ('It's all in there somewhere. I think.').
- (d) Try to make your proofs easier to follow by including brief phrases where appropriate. For example rather than writing "(statement A) implies (statement B)" it may be more informative to write "(statement B) follows from (statement A) because..."
- (e) Try to write out many of them, understanding every step.
- (f) Try to follow proofs in class or in books and ask others (your class mates or your teacher) about unclear points.

3 A proof is *not* ...

- (a) *not* a test of readers' mind-reading abilities,
- (b) *not* a collection of unexplained mathematical symbols,
- (c) *not* a handwave-y, rambling act of prolixity, and
- (d) *not* an appeal to "common sense."

4 A proof is ...

- (a) precisely explained, leaving no ambiguities,
- (b) concise and to the point,
- (c) a sequence of impeccable logic, leading to the inevitable conclusion "Because of Theorem X , ...", "A and B, therefore C", etc,
- (d) a correct but badly written proof will not receive full marks, and
- (e) each of your proofs will be thoroughly examined, with perhaps with an annoying amount of attention to details, and
- (f) say what you mean and mean what you say.