

Dr Oliver Mathematics
AQA Further Maths Level 2
June 2024 Paper 2
1 hour 45 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

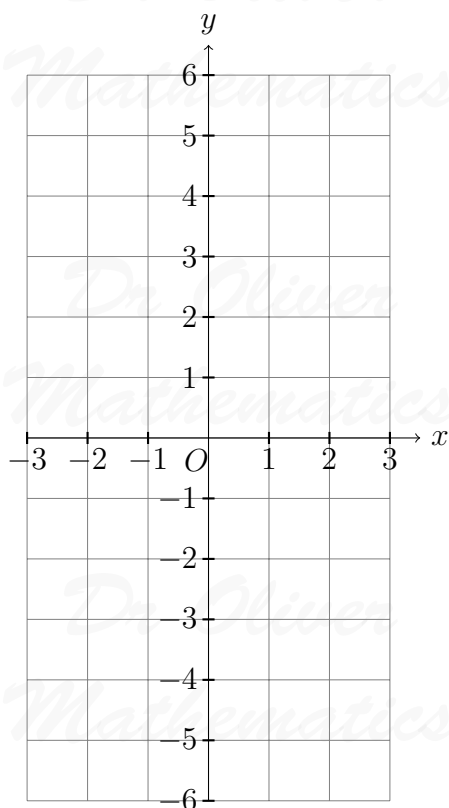
You are permitted to use a scientific or graphical calculator in this paper.

1. A straight line

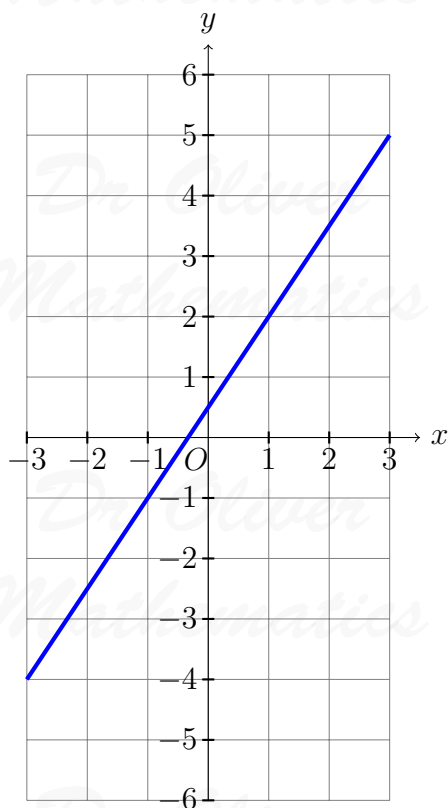
(3)

- passes through the point $(-1, -1)$ and
- has gradient $\frac{3}{2}$.

Draw the line for values of x from -3 to 3 .



Solution



2.

$$5n^2 + 2 < 38.$$

(2)

Work out all the possible **integer** values of n .

Solution

Well,

$$\begin{aligned} 5n^2 + 2 < 38 &\Rightarrow 5n^2 < 36 \\ &\Rightarrow n^2 < \frac{36}{5} \\ &\Rightarrow -\frac{6\sqrt{5}}{5} < n < \frac{6\sqrt{5}}{5} \\ &\Rightarrow -2.68\dots < n < 2.68\dots; \end{aligned}$$

hence, the possible integer values of n are

$$\underline{\underline{-2, -1, 0, 1, \text{ and } 2.}}$$

3. The equation of a curve is

$$y = x^4 - 3kx^2,$$

where k is a constant.

When $x = 2$, the rate of change of y with respect to x is 23.

Work out the value of k .

(3)

Solution

Well,

$$y = x^4 - 3kx^2 \Rightarrow \frac{dy}{dx} = 4x^3 - 6kx.$$

Now,

$$\begin{aligned} x = 2, \frac{dy}{dx} = 23 &\Rightarrow 32 - 12k = 23 \\ &\Rightarrow -12k = -9 \\ &\Rightarrow \underline{\underline{k = \frac{3}{4}}}. \end{aligned}$$

4. Here is some information about three linear sequences, A , B , and C .

(4)

$$\boxed{n\text{th term of } C = n\text{th term of } A + n\text{th term of } B.}$$

The n th term of C is

$$42 - 3n.$$

The first four terms of B are

$$14 \quad 22 \quad 30 \quad 38.$$

Work out the 20th term of A .

Solution

Let the

$$n\text{th term of } B = an + b.$$

Well,

$$\begin{array}{cccc}
 14 & 22 & 30 & 38 \\
 8 & 8 & 8 & \\
 a+b & 2a+b & 3a+b & 4a+b \\
 a & a & a &
 \end{array}$$

We compare terms:

$$a = 8$$

and

$$\begin{aligned}
 a + b = 14 &\Rightarrow 8 + b = 14 \\
 &\Rightarrow b = 6.
 \end{aligned}$$

Now,

$$n\text{th term of } B = 8n + 6.$$

So,

$$\begin{aligned}
 n\text{th term of } A &= (42 - 3n) - (8n + 6) \\
 &= 36 - 11n.
 \end{aligned}$$

Finally,

$$\begin{aligned}
 20\text{th term of } A &= 36 - 11(20) \\
 &= \underline{\underline{-184}}.
 \end{aligned}$$

5. (a)

(3)

$$\begin{pmatrix} c & 1 \\ 5d & 3 \end{pmatrix} \begin{pmatrix} -2 & 0 \\ 7 & 3 \end{pmatrix} = \begin{pmatrix} -5 & 3 \\ 0 & 9 \end{pmatrix}.$$

Work out the values of c and d .

Solution

Well,

$$\begin{aligned}
 &\begin{pmatrix} c & 1 \\ 5d & 3 \end{pmatrix} \begin{pmatrix} -2 & 0 \\ 7 & 3 \end{pmatrix} = \begin{pmatrix} -5 & 3 \\ 0 & 9 \end{pmatrix} \\
 \Rightarrow &\begin{pmatrix} -2c + 7 & 3 \\ -10d + 21 & 9 \end{pmatrix} = \begin{pmatrix} -5 & 3 \\ 0 & 9 \end{pmatrix}.
 \end{aligned}$$

Now,

$$\begin{aligned} -2c + 7 &= -5 \Rightarrow 2c = 12 \\ &\Rightarrow \underline{\underline{c = 6}} \end{aligned}$$

and

$$\begin{aligned} -10d + 21 &= 0 \Rightarrow 10d = 21 \\ &\Rightarrow \underline{\underline{d = 2.1}}. \end{aligned}$$

(b)

(1)

$$\begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix} \mathbf{M} = \begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix}.$$

Write down matrix $\mathbf{M}$.

Solution

$$\underline{\underline{\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}}.$$

6. The function f is given by

(2)

$$f(x) = 3x^2 + 2,$$

with domain $-1 \leq x \leq 4$.

Work out the range of the function.

Solution

Well, the vertex is clearly at $x = 0$:

$$f(-1) = 3[(-1)^2] + 2 = 5$$

$$f(0) = 3[0^2] + 2 = 2$$

$$f(4) = 3[4^2] + 2 = 50;$$

hence, the range of the function is

$$\underline{\underline{2 \leq f(x) \leq 50}}.$$

7. The equation of a curve is

$$y = 4 - (x - 3)^2.$$

(a) Circle the coordinates of the point where the curve crosses the y -axis.

(1)

$$(-5, 0) \quad (0, -5) \quad (-13, 0) \quad (0, -13)$$

Solution

Well,

$$x = 0 \Rightarrow y = 4 - (0 - 3)^2$$

$$\Rightarrow y = 4 - 9$$

$$\Rightarrow y = -5;$$

hence, $(0, -5)$.

(b) Write down the coordinates of the maximum point of the curve.

(1)

Solution

Now,

$$x = 3 \Rightarrow y = 4$$

and so the coordinates of the maximum point of the curve are $(3, 4)$.

8.

(3)

$$y = \frac{1}{2}x^2 + \frac{3}{4x^4}.$$

Work out $\frac{d^2y}{dx^2}$.

Give your answer in the form

$$a + bx^n,$$

where a , b , and n are integers.

Solution

Well,

$$\begin{aligned}y &= \frac{1}{2}x^2 + \frac{3}{4x^4} \Rightarrow y = \frac{1}{2}x^2 + \frac{3}{4}x^{-4} \\&\Rightarrow \frac{dy}{dx} = x - 3x^{-5} \\&\Rightarrow \underline{\underline{\frac{d^2y}{dx^2} = 1 + 15x^{-6};}}\end{aligned}$$

hence, $a = 1$, $b = 15$, and $n = -6$,

9. A set of 4-digit integers each have

(3)

- a first digit **greater than** 6 and
- a second digit **less than** 8.

What is the greatest possible number of integers in the set that are multiples of 5?

Solution

Well,

- the first digit is 7, 8 or 9 (3 choices),
- the second digit 0, 1, 2, 3, 4, 5, 6, or 7 (8 choices);
- the third digit is anything (10 choices); and
- the fourth digit is 0 or 5 (2 choices).

Finally,

$$\begin{aligned}\text{greatest possible number of integers} &= 3 \times 8 \times 10 \times 2 \\&= \underline{\underline{480}}.\end{aligned}$$

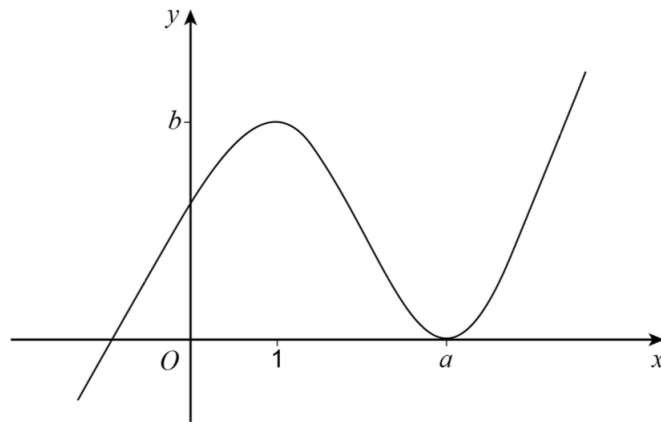
10. f is a cubic function.

(2)

The curve $y = f(x)$ has

- a minimum point at $(a, 0)$ and

- a maximum point at $(1, b)$.



Not drawn
accurately

Tick one box for each statement.

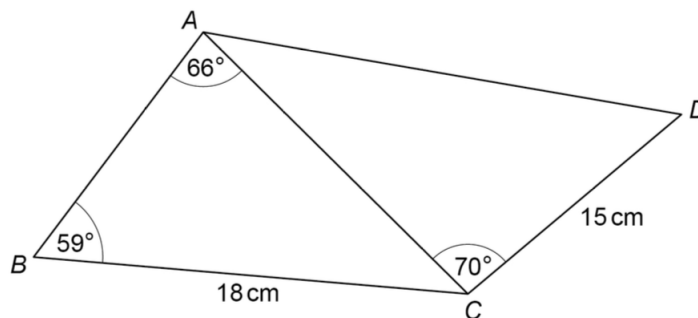
	True	False
The tangent to the curve at $(1, b)$ is parallel to the x -axis		
There are three different values of x for which $y = 0$		
The function is increasing for $0 < x < 1$		

Solution

	True	False
The tangent to the curve at $(1, b)$ is parallel to the x -axis	✓	
There are three different values of x for which $y = 0$		✓
The function is increasing for $0 < x < 1$	✓	

11. ABC and ACD are triangles.

(4)



Not drawn
accurately

Work out the area of triangle ACD .

Solution

Sine rule:

$$\begin{aligned}\frac{AC}{\sin ABC} &= \frac{BC}{\sin BAC} \Rightarrow \frac{AC}{\sin 59^\circ} = \frac{18}{\sin 66^\circ} \\ \Rightarrow AC &= \frac{18 \sin 59^\circ}{\sin 66^\circ} \\ \Rightarrow AC &= 16.889\,155\,63 \text{ (FCD)}.\end{aligned}$$

The area of a triangle:

$$\begin{aligned}\text{area of } ACD &= \frac{1}{2} \times AC \times CD \times \sin ACD \\ &= \frac{1}{2} \times 16.889\dots \times 15 \times \sin 70^\circ \\ &= 119.029\,611\,9 \text{ (FCD)} \\ &= \underline{\underline{119 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

12. The equation of a circle is

$$(x - 2)^2 + (y + 3)^2 = 16.$$

The equation of a line is

$$y = 4 - x.$$

The circle and line intersect at two points, A and B .

(a) Show that the x -coordinates of A and B satisfy the equation

(3)

$$2x^2 - 18x + 37 = 0.$$

Solution

Well,

$$\begin{aligned}(x - 2)^2 + (y + 3)^2 &= 16 \\ \Rightarrow (x - 2)^2 + (7 - x)^2 &= 16\end{aligned}$$

$\times$	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

$\times$	7	$-x$
7	49	$-7x$
$-x$	$-7x$	$+x^2$

$$\Rightarrow (x^2 - 4x + 4) + (49 - 14x + x^2) = 16$$

$$\Rightarrow \underline{\underline{2x^2 - 18x + 37 = 0}},$$

as required.

For A , the x -coordinate and y -coordinate are both **positive**.

(b) Work out the coordinates of A .

Give each coordinate to 2 decimal places.

(3)

Solution

Quadratic formula:

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{18 \pm \sqrt{18^2 - 4 \times 2 \times 37}}{2 \times 2} \\
 &= \frac{18 \pm \sqrt{28}}{4} \\
 &= 3.177\,124\,344, \, 5.822\,856\,656 \text{ (FCD);} \\
 y &= 0.822\,875\,655\,5, \, -1.822\,875\,656 \text{ (FCD)}.
 \end{aligned}$$

As both $x > 0$ and $y > 0$, the coordinates of A are $(3.18, 0.82)$ (2 dp).

13. Expand and simplify fully

$$(3x - 1)(4 - x)(2x + 5).$$

(3)

Solution

Well,

×		$3x$	-1
$2x$		$6x^2$	$-2x$
$+5$		$+15x$	-5

so

$$(3x - 1)(2x + 5) = 6x^2 + 13x - 5.$$

Now,

×		$6x^2$	$+13x$	-5
4		$24x^2$	$+52x$	-20
$-x$		$-6x^3$	$-13x^2$	$+5x$

so

$$(3x - 1)(2x + 5)(4 - x) = \underline{\underline{-6x^3 + 11x^2 + 57x - 20.}}$$

14. P is the point on the graph

$$y = 3^{-x}$$

(3)

that has x -coordinate -2 .

Q is a point on the graph

$$y = 1152 \times \left(\frac{1}{2}\right)^x.$$

y -coordinate of $P = y$ -coordinate of Q

Work out the x -coordinate of Q .

Solution

Well,

$$x = -2 \Rightarrow y = 9,$$

and $P(-2, 9)$.

Now, y -coordinate of P equals y -coordinate of Q so

$$\begin{aligned} 1152 \times \left(\frac{1}{2}\right)^x &= 9 \Rightarrow \left(\frac{1}{2}\right)^x = \frac{1}{128} \\ &\Rightarrow \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^7 \\ &\Rightarrow \underline{\underline{x = 7.}} \end{aligned}$$

15. Simplify fully

(3)

$$\frac{2x^2 + 9x - 18}{12x^2 - 8x - 15}.$$

Solution

Well,

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to: } (+2) \times (-18) = -36 \end{array} \right\} +12, -3$$

e.g.,

$$\begin{aligned} 2x^2 + 9x - 18 &= 2x^2 + 12x - 3x - 18 \\ &= 2x(x + 6) - 3(x + 6) \\ &= (2x - 3)(x + 6). \end{aligned}$$

Now,

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to: } (+12) \times (-15) = -180 \end{array} \right\} +10, -18$$

e.g.,

$$\begin{aligned} 12x^2 - 8x - 15 &= 12x^2 + 10x - 18x - 15 \\ &= 2x(6x + 5) - 3(6x + 5) \\ &= (2x - 3)(6x + 5). \end{aligned}$$

Finally,

$$\begin{aligned}\frac{2x^2 + 9x - 18}{12x^2 - 8x - 15} &= \frac{(2x - 3)(x + 6)}{(2x - 3)(6x + 5)} \\ &= \frac{x + 6}{\underline{\underline{6x + 5}}}.\end{aligned}$$

16. Simplify

(3)

$$\frac{\sqrt{x^3}(\sqrt{x^3} + x^3)}{\sqrt{x}}.$$

Give your answer in the form

$$x^{\frac{a}{b}} + x^c,$$

where a , b , and c are integers.

Solution

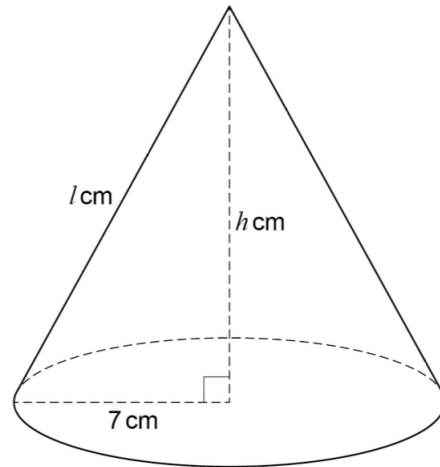
Well,

$$\begin{aligned}\frac{\sqrt{x^3}(\sqrt{x^3} + x^3)}{\sqrt{x}} &= \frac{x^{\frac{3}{2}}(x^{\frac{3}{2}} + x^3)}{x^{\frac{1}{2}}} \\ &= \frac{x^3 + x^{\frac{9}{2}}}{x^{\frac{1}{2}}} \\ &= \underline{\underline{x^{\frac{5}{2}} + x^4}};\end{aligned}$$

hence, $\underline{\underline{a = 5}}$, $\underline{\underline{b = 2}}$, and $\underline{\underline{c = 4}}$.

17. (a) A solid cone has base radius 7 cm, perpendicular height h cm and slant height l cm.

(4)



Curved surface area of a cone = $\pi r l$

The total surface area of the cone is $224\pi \text{ cm}^2$.

Work out the size of the angle between the slant height and the base.

Solution

Well,

$$\begin{aligned} \text{total surface area} &= \text{surface area} + \text{base} \Rightarrow 224\pi = (\pi \times 7 \times l) + (\pi \times 7^2) \\ &\Rightarrow 224\pi = 7\pi l + 49\pi \\ &\Rightarrow 7\pi l = 175\pi \end{aligned}$$

divide by 7π :

$$\Rightarrow l = 25.$$

Cosine:

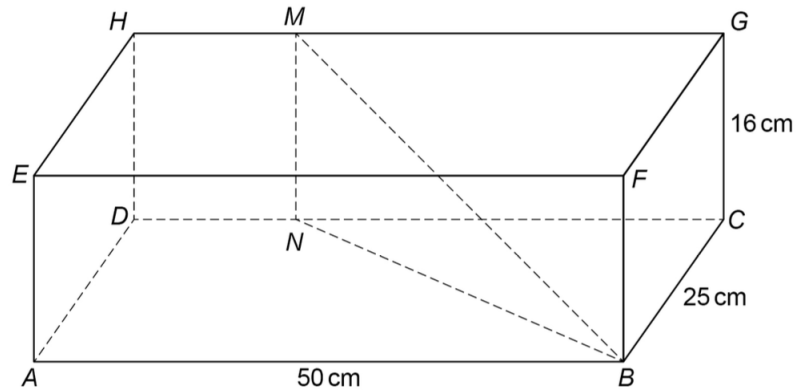
$$\begin{aligned} \cos &= \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos \text{ angle} = \frac{7}{25} \\ &\Rightarrow \text{angle} = 73.739\,795\,29 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\text{angle} = 73.7^\circ \text{ (3 sf)}}}. \end{aligned}$$

(b) $ABCDEFGH$ is a cuboid.

- $ABCD$ is horizontal.
- MN is vertical.

(2)

- The acute angle between the planes MNB and $HDCG$ is 36° .



Work out the length CN .

Solution

Well,

$$\begin{aligned}\tan &= \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 36^\circ = \frac{25}{CN} \\ \Rightarrow CN &= \frac{25}{\tan 36^\circ} \\ \Rightarrow CN &= 34.409\,548\,01 \text{ (FCD)} \\ \Rightarrow CN &= \underline{\underline{34.4 \text{ cm (3 sf)}}}.\end{aligned}$$

18. Rearrange

(4)

$$m = \frac{2k^3 - 7}{5m - 3k^3}$$

to make k the subject.

Solution

Now,

$$\begin{aligned}
 m &= \frac{2k^3 - 7}{5m - 3k^3} \Rightarrow m(5m - 3k^3) = 2k^3 - 7 \\
 &\Rightarrow 5m^2 - 3k^3m = 2k^3 - 7 \\
 &\Rightarrow 5m^2 + 7 = 2k^3 + 3k^3m \\
 &\Rightarrow 5m^2 + 7 = k^3(2 + 3m) \\
 &\Rightarrow k^3 = \frac{5m^2 + 7}{2 + 3m} \\
 &\Rightarrow k = \sqrt[3]{\frac{5m^2 + 7}{2 + 3m}}.
 \end{aligned}$$

19. In the expansion of

$$(3 + ax)^5,$$

(4)

where a is a non-zero constant,

$$8 \times \text{coefficient of } x^2 = \text{coefficient of } x^4.$$

Work out the possible values of a .

Solution

Well, the coefficient of x^2 is

$$\binom{5}{3}(3)^3(a)^2 = 270a^2$$

and the coefficient of x^4 is

$$\binom{5}{4}(3)^1(a)^4 = 15a^4.$$

Now,

$$\begin{aligned}
 8 \times \text{coefficient of } x^2 &= \text{coefficient of } x^4 \Rightarrow 8 \times 270a^2 = 15a^4 \\
 &\Rightarrow 15a^4 = 2160a^2 \\
 &\Rightarrow 15a^4 - 2160a^2 = 0 \\
 &\Rightarrow 15a^2(a^2 - 144) = 0 \\
 &\Rightarrow a^2 = 0, a^2 - 144 = 0 \\
 &\Rightarrow a = 0, a = -12, \text{ or } a = 12.
 \end{aligned}$$

Clearly, $a \neq 0$ (why?) so

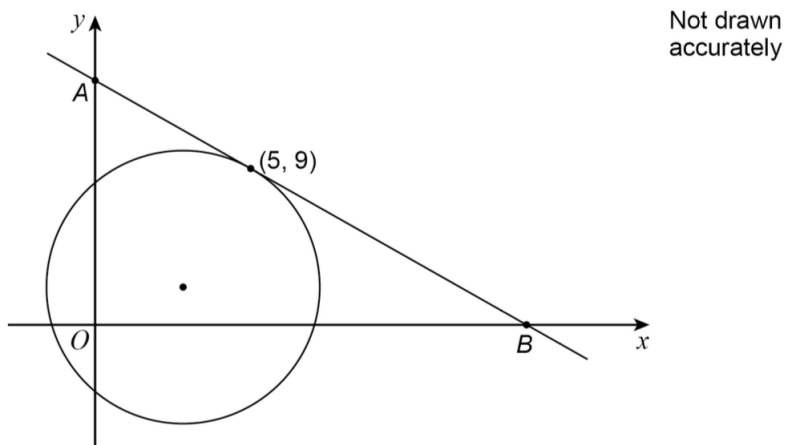
$$\underline{\underline{a = -12 \text{ or } a = 12.}}$$

20. AB is the tangent to the circle

(5)

$$(x - 3)^2 + (y - 4)^2 = 29$$

at the point $(5, 9)$.



Work out the area of triangle AOB .

Solution

Let $C(3, 4)$ be the centre of the circle and $D(5, 9)$ be the point of contact. Now,

$$\begin{aligned} m_{CD} &= \frac{9 - 4}{5 - 3} \\ &= \frac{5}{2}, \end{aligned}$$

and so

$$m_{\text{normal}} = -\frac{2}{5}.$$

Next, the normal through D is

$$y - 9 = -\frac{2}{5}(x - 5).$$

So,

$$\begin{aligned}x = 0 &\Rightarrow y - 9 = -\frac{2}{5}(0 - 5) \\&\Rightarrow y - 9 = 2 \\&\Rightarrow y = 11\end{aligned}$$

and

$$\begin{aligned}y = 0 &\Rightarrow 0 - 9 = -\frac{2}{5}(x - 5) \\&\Rightarrow x - 5 = 22\frac{1}{2} \\&\Rightarrow x = 27\frac{1}{2};\end{aligned}$$

hence, $A(0, 11)$ and $B(27\frac{1}{2}, 0)$.

Finally,

$$\begin{aligned}\text{area of triangle } OAB &= \frac{1}{2} \times 27\frac{1}{2} \times 11 \\&= \underline{\underline{151\frac{1}{4}}}.\end{aligned}$$

21.

(6)

$$f(x) = \frac{2-x}{3}$$

$$g(x) = 18x^2 + 15x.$$

$f^{-1}(x) + g f(x)$ simplifies to $ax^2 + bx + c$.
--

Work out the values of a , b , and c .

Solution

Well,

$$\begin{aligned}y &= \frac{2-x}{3} \Rightarrow 3y = 2-x \\&\Rightarrow x = 2-3y\end{aligned}$$

so

$$f^{-1}(x) = 2 - 3x.$$

Now,

$$\begin{aligned}
 gf(x) &= g(f(x)) \\
 &= g\left(\frac{2-x}{3}\right) \\
 &= 18\left(\frac{2-x}{3}\right)^2 + 15\left(\frac{2-x}{3}\right) \\
 &= 2(2-x)^2 + 5(2-x)
 \end{aligned}$$

$\times$	2	$-x$
2	4	$-2x$
$-x$	$-2x$	$+x^2$

$$\begin{aligned}
 &= 2(4 - 4x + x^2) + 5(2 - x) \\
 &= 8 - 8x + 2x^2 + 10 - 5x \\
 &= 2x^2 - 13x + 18.
 \end{aligned}$$

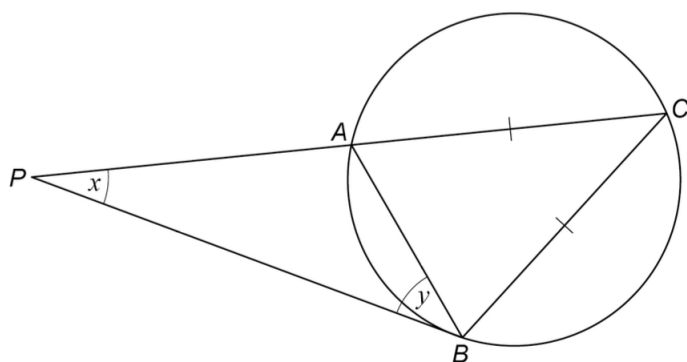
Finally,

$$\begin{aligned}
 f^{-1}(x) + gf(x) &= (2 - 3x) + (2x^2 - 13x + 18) \\
 &= \underline{\underline{2x^2 - 16x + 20}};
 \end{aligned}$$

hence, $a = 2$, $b = -16$, and $c = 20$.

22. A , B , and C are points on a circle.

(5)



Not drawn
accurately

- PAC is a straight line.
- PB is a tangent to the circle.
- $AC = BC$.

Prove that

$$x = 90 - \frac{3}{2}y.$$

Solution

$\angle ACB = y$ (alternate segment theorem)

$\angle PAC = 180 - (x + y)$ (completing the triangle)

$\angle BAC = x + y$ (supplementary angles)

$\angle BCA = x + y$ (base angles).

Now,

$$\begin{aligned} x + [y + (x + y)] + y &= 180 \Rightarrow 2x + 3y = 180 \\ &\Rightarrow 2x = 180 - 3y \\ &\Rightarrow \underline{\underline{x = 90 - \frac{3}{2}y}}, \end{aligned}$$

as required.