

Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2012 November Paper 2 Variant 2: Calculator
2 hours

The total number of marks available is 80.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You must write down all the stages in your working.

1. Solve the equation

$$|5x + 7| = 13.$$

(3)

2. (a) Given that

$$\mathbf{A} = \begin{pmatrix} 7 & 8 \\ 4 & 6 \end{pmatrix},$$

(2)

find the inverse matrix, $\mathbf{A}^{-1}$.

- (b) Use your answer to part (a) to solve the simultaneous equations

(2)

$$7x + 8y = 39,$$

$$4x + 6y = 23.$$

3. Without using a calculator, simplify

(4)

$$\frac{(3\sqrt{3} - 1)^2}{2\sqrt{3} - 3},$$

giving your answer in the form

$$\frac{a\sqrt{3} + b}{3},$$

where a and b are integers.

4. The points X , Y , and Z are such that $\overrightarrow{XY} = 3\overrightarrow{YZ}$.

(5)

The position vectors of X and Z , relative to an origin O , are

$$\begin{pmatrix} 4 \\ -27 \end{pmatrix} \text{ and } \begin{pmatrix} 20 \\ -7 \end{pmatrix}$$

respectively.

Find the unit vector in the direction $\overrightarrow{OY}$.

5. Find the set of values of m for which the line (6)

$$y = mx + 2$$

does not meet the curve

$$y = mx^2 + 7x + 11.$$

6. (a) Given that (3)

$$\cos x = p,$$

find an expression, in terms of p , for $\tan^2 x$.

- (b) Prove that (3)

$$(\cot \theta + \tan \theta)^2 \equiv \sec^2 \theta + \operatorname{cosec}^2 \theta.$$

7. (a) Find (3)

$$\int (x + 3)\sqrt{x} \, dx.$$

- (b) Find (4)

$$\int \frac{20}{(2x + 5)^2} \, dx$$

and hence evaluate

$$\int_0^{10} \frac{20}{(2x + 5)^2} \, dx.$$

8. **Solutions to this question by accurate drawing will not be accepted.** (8)

The points $A(4, 5)$, $B(-2, 3)$, $C(1, 9)$, and D are the vertices of a trapezium in which BC is parallel to AD and angle BCD is 90° .

Find the area of the trapezium.

9. The table shows experimental values of two variables x and y .

x	1	2	3	4
y	9.41	1.29	-0.69	-1.77

It is known that x and y are related by the equation

$$y = \frac{a}{x^2} + bx,$$

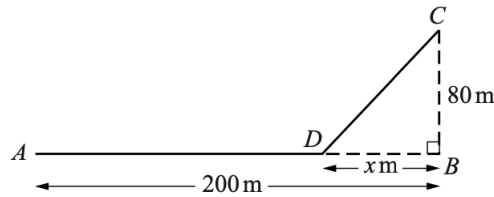
where a and b are constants.

- (a) A straight line graph is to be drawn to represent this information. (1)

Given that x^2y is plotted on the vertical axis, state the variable to be plotted on the horizontal axis.

- (b) Draw this straight line graph. (3)
 (c) Use your graph to estimate the value of a and of b . (3)
 (d) Estimate the value of y when x is 3.7. (2)

10. A track runs due east from A to B , a distance of 200 m.



- The point C is 80 m due north of B .
 - A cyclist travels on the track from A to D , where D is x m due west of B .
 - The cyclist then travels in a straight line across rough ground from D to C .
 - The cyclist travels at 10 ms^{-1} on the track and at 6 ms^{-1} across rough ground.
- (a) Show that the time taken, T s, for the cyclist to travel from A to C is given by (2)

$$T = \frac{200 - x}{10} + \frac{\sqrt{x^2 + 6400}}{6}.$$

- (b) Given that x can vary, find the value of x for which T has a stationary value and the corresponding value of T . (6)

11. (a) Solve (3)

$$(2^{x-2})^{\frac{1}{2}} = 100,$$

giving your answer to 1 decimal place.

- (b) Solve (3)

$$\log_y 2 = 3 - \log_y 256.$$

- (c) Solve (4)

$$\frac{6^{5z-2}}{36^z} = \frac{216^{z-1}}{36^{3-z}}.$$

EITHER

12. (a) Express (3)

$$4x^2 + 32x + 55$$

in the form

$$(ax + b)^2 + c,$$

where a , b , and c are constants and a is positive.

The functions f and g are defined by

$$f : x \mapsto 4x^2 + 32x + 55 \text{ for } x > -4,$$

$$g : x \mapsto \frac{1}{x} \text{ for } x > 0.$$

- (b) Find $f^{-1}(x)$. (3)

- (c) Solve the equation (4)

$$f g(x) = 135.$$

OR

13. The functions h and k are defined by

$$h : x \mapsto \sqrt{2x - 7} \text{ for } x \geq c,$$

$$k : x \mapsto \frac{3x - 4}{x - 2} \text{ for } x > 2.$$

- (a) State the least possible value of c . (1)

- (b) Find $h^{-1}(x)$. (2)

- (c) Solve the equation (3)

$$k(x) = x.$$

- (d) Find an expression for the function k^2 , in the form (4)

$$k^2 : x \mapsto a + \frac{b}{x},$$

where a and b are constants.