

Dr Oliver Mathematics

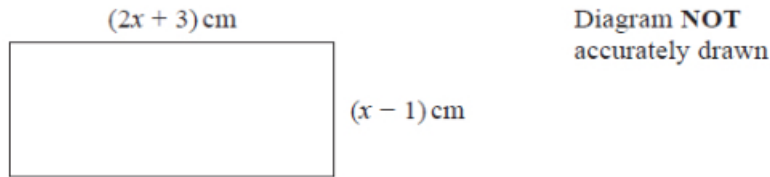
Worked Examples

Quadratic Inequality 2

From: Edexcel GCSE Mathematics (9-1) Practice Tests Set 19: Paper 2H/3H (Calculator)

1. Here is a rectangle.

(5)



Given that the area of the rectangle is less than 75 cm^2 , find the range of possible values of x .

Solution

Well, we want the product of the linear dimensions to be less than 75:

$$\begin{array}{r|l}
 \times & 2x \quad +3 \\
 \hline
 x & 2x^2 \quad +3x \\
 -1 & -2x \quad -3 \\
 \hline
 \end{array}$$

So,

$$\begin{aligned}
 (2x + 3)(x - 1) < 75 &\Rightarrow 2x^2 + x - 3 < 75 \\
 &\Rightarrow 2x^2 + x - 78 < 0
 \end{aligned}$$

$$\left. \begin{array}{l}
 \text{add to:} \\
 \text{multiply to: } (+2) \times (-78) = -156
 \end{array} \right\} -12, +13$$

$$\begin{aligned}
 &\Rightarrow 2x^2 - 12x + 13x - 78 < 0 \\
 &\Rightarrow 2x(x - 6) + 13(x - 6) < 0 \\
 &\Rightarrow (2x + 13)(x - 6) < 0.
 \end{aligned}$$

Now, if $(2x + 13)(x - 6) = 0$ — it isn't, but trust me,

$$(2x + 13)(x - 6) = 0 \Rightarrow 2x + 13 = 0 \text{ or } x - 6 = 0 \\ \Rightarrow x = -6\frac{1}{2} \text{ or } x = 6.$$

We now do a 'table of signs':

	$x < -6\frac{1}{2}$	$x = -6\frac{1}{2}$	$-6\frac{1}{2} < x < 6$	$x = 6$	$x > 6$
$2x + 3$	-	0	+	+	+
$x - 1$	-	-	-	0	+
$(2x + 1)(x - 3)$	+	0	-	0	+

Clearly,

$$-6\frac{1}{2} < x < 6.$$

Well, hold on! We need

$$2x + 3 > 0 \text{ and } x - 1 > 0.$$

There is no problem with the former one but the latter one?

$$x - 1 > 0 \Rightarrow x > 1.$$

Hence,

$$\underline{\underline{1 < x < 6.}}$$