Dr Oliver Mathematics
OCR FMSQ Additional Mathematics
2022 Paper
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.
Final answers should be given correct to three significant figures where appropriate.

1. Solve the inequality (3)

$$-3 < 2(x+1) < 7.$$

Solution

$$\begin{aligned} -3 < 2(x+1) < 7 &\Rightarrow -1\tfrac{1}{2} < x + 1 < 3\tfrac{1}{2} \\ &\Rightarrow \underline{\underline{-2\tfrac{1}{2} < x < 2\tfrac{1}{2}}}. \end{aligned}$$

2. A passenger train is 175 m long and is stationary in a station.

   As it leaves the station it accelerates with uniform acceleration.

   When the front of the train reaches the end of the platform it is travelling at a speed of 3 ms^{-1} and when the rear of the train reaches the end of the platform it is travelling at a speed of 18 ms^{-1}.

   (a) Determine the uniform acceleration of the train. (2)

Solution

$s = 175$, $u = 3$, $v = 18$, $a = ?$, and $t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 18^2 = 3^2 + 2 \times a \times 175 \\ &\Rightarrow 324 = 9 + 350a \\ &\Rightarrow 350a = 315 \\ &\Rightarrow \underline{\underline{a = 0.9 \text{ ms}^{-2}}}. \end{aligned}$$

(b) Determine the time taken from when the train starts to move until it reaches a speed of 18 ms^{-1} (2)

Solution

$$v = u + at \Rightarrow 18 = 3 + 0.9t$$
$$\Rightarrow \underline{\underline{t = 16\tfrac{2}{3}\text{ s}}}.$$

3. A photograph is to be taken of 9 students who have represented their college in sailing this year. (3)

The students are to be arranged in a line.

The captain is to stand in the middle with the vice-captain standing beside the captain.

How many ways are there of arranging the students?

Solution

There are two places where the vice-captain can stand: either to the left or the right of the captain. Hence, there are

$$2 \times 7! = 2 \times 5\,040$$
$$= \underline{\underline{10\,080 \text{ ways}}}.$$

4. **In this question you must show detailed reasoning.**

You are given the cubic polynomial

$$\mathrm{f}(x) = x^3 + 6x^2 + 5x - 12.$$

(a) Show that (1)

$$\mathrm{f}(1) = 0.$$

Solution

We use synthetic division:

$$
\begin{array}{c|cccc}
1 & 1 & 6 & 5 & -12 \\
 & \downarrow & 1 & 7 & 12 \\
\hline
 & 1 & 7 & 12 & 0
\end{array}
$$

Hence,

$$f(x) = (x-1)(x^2+7x+12)$$

and so

$$\underline{\underline{f(1) = 0}}.$$

(b) Hence solve the equation (4)

$$f(x) = 0.$$

Solution

$$f(x) = 0 \Rightarrow (x-1)(x^2+7x+12) = 0$$

$$\left.\begin{array}{ll} \text{add to:} & +7 \\ \text{multiply to:} & +12 \end{array}\right\} +3,\ +4$$

$$\begin{aligned} &\Rightarrow (x-1)(x+3)(x+4) = 0 \\ &\Rightarrow x-1=0,\ x+3=0 \text{ or } x+4=0 \\ &\Rightarrow \underline{\underline{x=-4,\ x=-3, \text{ or } x=1}}. \end{aligned}$$

5. **In this question you must show detailed reasoning.**

Find all the values of θ in the range $0^\circ < \theta < 360^\circ$ that satisfy the following equations, giving your answers correct to **1 decimal place**.

(a) $\cos 2\theta = 0.6$. (3)

Solution

$$\begin{aligned}\cos 2\theta = 0.6 \Rightarrow 2\theta &= 53.130\,102\,235\,306.869\,897\,6, \\ &\quad 413.130\,102\,235,\ 666.869\,897\,6 \text{ (FCD)} \\ \Rightarrow \theta &= 26.565\,051\,18,\ 153.434\,948\,8, \\ &\quad 206.565\,051\,18,\ 333.434\,948\,8 \text{ (FCD)} \\ \Rightarrow &\underline{\underline{\theta = 26.6,\ 153.4,\ 206.6,\ 333.4 \text{ (1 dp)}}}.\end{aligned}$$

(b) $12\cos^2\theta + \sin\theta = 11$. (5)

Solution

$$\begin{aligned}12\cos^2\theta + \sin\theta = 11 &\Rightarrow 12(1 - \sin^2\theta) + \sin\theta = 11 \\ &\Rightarrow 12 - 12\sin^2\theta + \sin\theta = 11 \\ &\Rightarrow 12\sin^2\theta - \sin\theta - 1 = 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -1 \\ \text{multiply to:} & (+12)\times(-1) = -12\end{array}\right\} -4,\ +3$$

$$\begin{aligned}&\Rightarrow 12\sin^2\theta - 4\sin\theta + 3\sin\theta - 1 = 0 \\ &\Rightarrow 4\sin\theta(3\sin\theta - 1) + 1(3\sin\theta - 1) = 0 \\ &\Rightarrow (4\sin\theta + 1)(3\sin\theta - 1) = 0 \\ &\Rightarrow 4\sin\theta + 1 = 0 \text{ or } 3\sin\theta - 1 = 0 \\ &\Rightarrow \sin\theta = -\tfrac{1}{4} \text{ or } \sin\theta = \tfrac{1}{3}.\end{aligned}$$

$\underline{\sin\theta = -\tfrac{1}{4}}$:

$$\begin{aligned}\sin\theta = -\tfrac{1}{4} &\Rightarrow \theta = 194.477\,512\,2,\ 345.522\,487\,8 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\theta = 194.4,\ 345.5 \text{ (1 dp)}}}.\end{aligned}$$

$\underline{\sin\theta = \tfrac{1}{3}}$:

$$\begin{aligned}\sin\theta = \tfrac{1}{3} &\Rightarrow \theta = 19.471\,220\,63,\ 160.528\,779\,4 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\theta = 19.4,\ 160.5 \text{ (1 dp)}}}.\end{aligned}$$

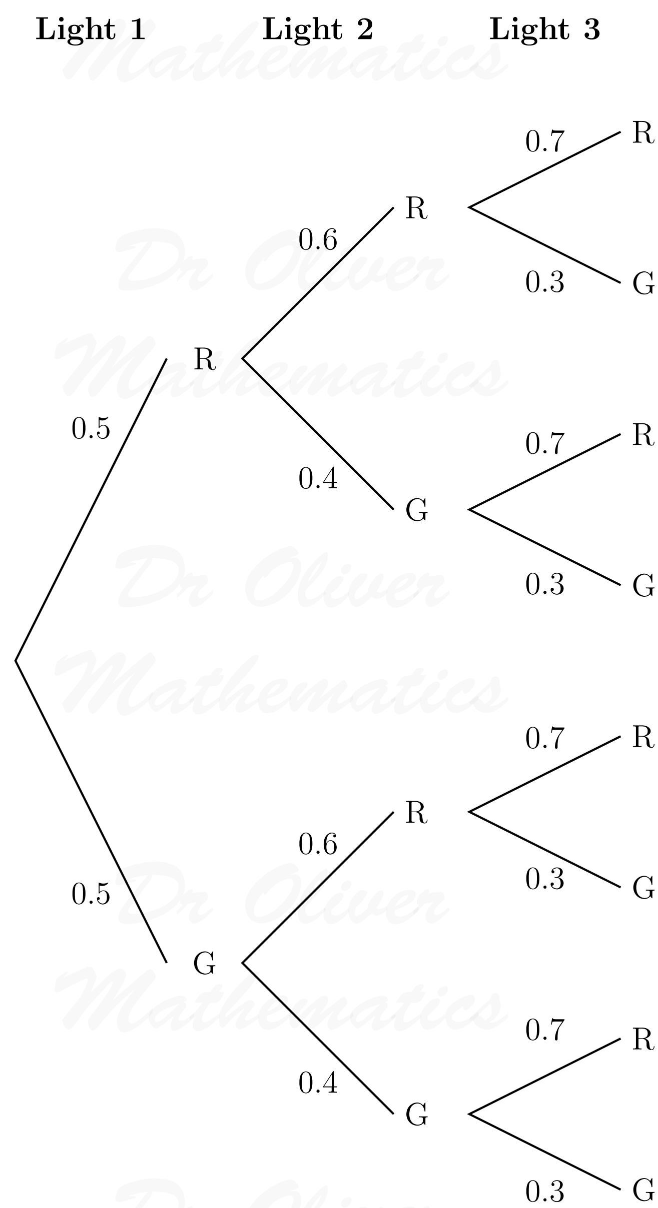
6. Layla drives to work along a road which has three sets of traffic lights.

The lights work independently of each other.

Experience indicates that the probability that Layla has to stop at each set of lights is 0.5, 0.6, and 0.7 respectively.

(a) Draw a tree diagram to illustrate the probabilities of Layla having to stop at each set of lights on a particular journey to work. (3)

Solution



(b) Calculate the probability that Layla has to stop exactly once on a particular journey (3) to work.

Solution

$$\begin{aligned} \text{P(exactly once)} &= \text{P(G,G,R)} + \text{P(G,R,G)} + \text{P(R,G,G)} \\ &= (0.5 \times 0.4 \times 0.7) + (0.5 \times 0.6 \times 0.3) + (0.5 \times 0.4 \times 0.3) \\ &= 0.14 + 0.09 + 0.06 \\ &= \underline{\underline{0.29}}. \end{aligned}$$

7. A local council investigated the pattern of travel to a nearby town in one month. Residents were asked whether they drove, cycled or walked into the town. They had an opportunity to state if they used more than one mode of transport during the course of the month.

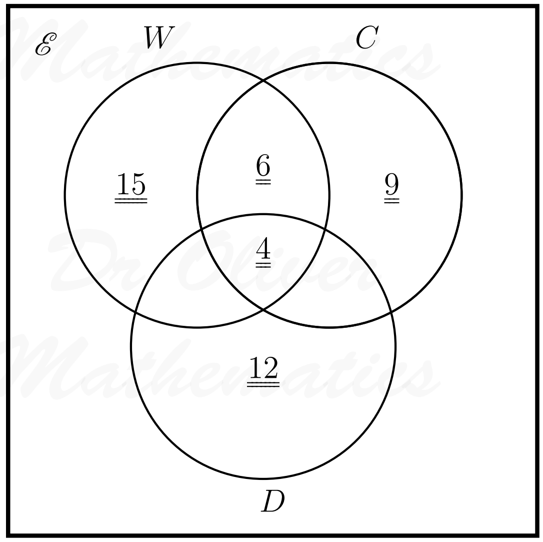
   Some of the results are summarised in the table below.

Mode of travel	Frequency
Only walked	15
Only cycled	9
Only drove	12
Used all three modes of transport	4
Walked and cycled but did not drive	6

The total number of residents surveyed was 60.

(a) Draw a Venn diagram to illustrate this incomplete set of data. (3)

Solution



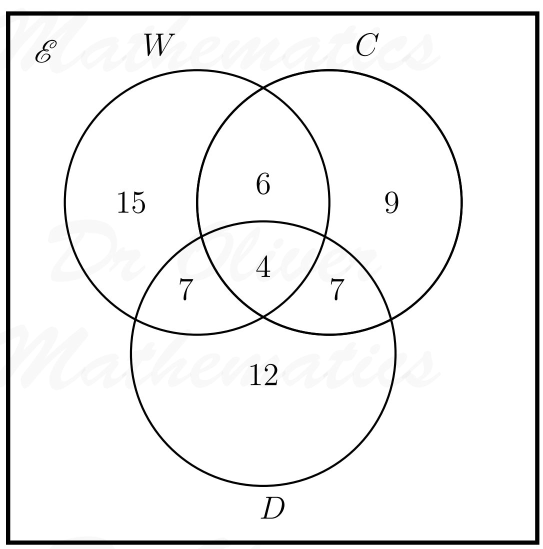
The number that said they drove and walked but did not cycle was the same as the number that said they drove and cycled but did not walk.

All those surveyed said that they had travelled to the nearby town by one of these three modes of transport at least once.

(b) Determine how many of these residents drove and walked but did not cycle. (3)

Solution

There were 14 people not counted:



Hence, 7 of these residents drove and walked but did not cycle

8. The gradient function of a curve is given by (4)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2 + 4x - 3x^2$$

and the curve passes through the point $(1, 2)$.

Determine the equation of the curve.

Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2 + 4x - 3x^2 \Rightarrow y = 2x + 2x^2 - x^3 + c,$$

for some constant c. Now,

$$\begin{aligned} x = 1, y = 2 &\Rightarrow 2 = 2 + 2 - 1 + c \\ &\Rightarrow c = -1; \end{aligned}$$

hence, the equation of the curve is

$$\underline{\underline{y = 2x + 2x^2 - x^3 - 1}}.$$

9. **In this question you must show detailed reasoning.**

(a) You are given that (2)

$$y = 4\log_3 x.$$

Rewrite this equation with x as the subject.

Solution

$$\begin{aligned} y = 4\log_3 x &\Rightarrow \log_3 x = \tfrac{1}{4}y \\ &\Rightarrow \underline{\underline{x = 3^{\left(\frac{1}{4}y\right)}}}. \end{aligned}$$

(b) Write (3)

$$2\log_{10} 5 + \tfrac{1}{2}\log_{10} 16$$

as a single number.

Solution

$$\begin{aligned}2\log_{10}5+\tfrac{1}{2}\log_{10}16 &= \log_{10}5^2+\log_{10}16^{\frac{1}{2}}\\ &= \log_{10}25+\log_{10}4\\ &= \log_{10}(25\times 4)\\ &= \log_{10}100\\ &= \log_{10}10^2\\ &= 2\log_{10}10\\ &= \underline{\underline{2}}.\end{aligned}$$

(c) The equation (2)

$$a^x = 17$$

has the solution $x = 2.58$, correct to 3 significant figures.

Given that a is an integer, determine the value of a.

Solution

$$\begin{aligned}a^x = 17 &\Rightarrow \log_{10}a^x = \log_{10}17\\ &\Rightarrow x\log_{10}a = \log_{10}17\\ &\Rightarrow \log_{10}a = \frac{\log_{10}17}{x}\\ &\Rightarrow a = 10^{\frac{\log_{10}17}{x}}\\ &\Rightarrow a = 2.998\,597\,583\ \text{(FCD)};\end{aligned}$$

hence, $\underline{\underline{a = 3}}$.

10. **In this question you must show detailed reasoning.**

Two curves have the following equations:

$$\begin{aligned}C_1:&\quad y = x^2-4x+4,\\ C_2:&\quad y = -x^2+8x-6.\end{aligned}$$

(a) Find the coordinates of the **two** points of intersection of these curves. (4)

Solution

$$\begin{aligned} x^2 - 4x + 4 = -x^2 + 8x - 6 &\Rightarrow 2x^2 - 12x + 10 = 0 \\ &\Rightarrow 2(x^2 - 6x + 5) = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -6 \\ \text{multiply to:} & +5 \end{array}\right\} -5,\ -1$$

$$\begin{aligned} &\Rightarrow 2(x-5)(x-1) = 0 \\ &\Rightarrow x - 5 = 0 \text{ or } x - 1 = 0 \\ &\Rightarrow x = 5 \text{ or } x = 1. \end{aligned}$$

Now,

$$x = 1 \Rightarrow y = 1 - 4 + 4 = 0$$

and

$$x = 5 \Rightarrow y = 25 - 20 + 4 = 9;$$

hence, the coordinates of the two points are $\underline{\underline{(1,1)}}$ and $\underline{\underline{(5,9)}}$.

(b) Find the area of the region enclosed by these two curves. (5)

Solution

Which is on top? Easy: we will take the modulus of the area!

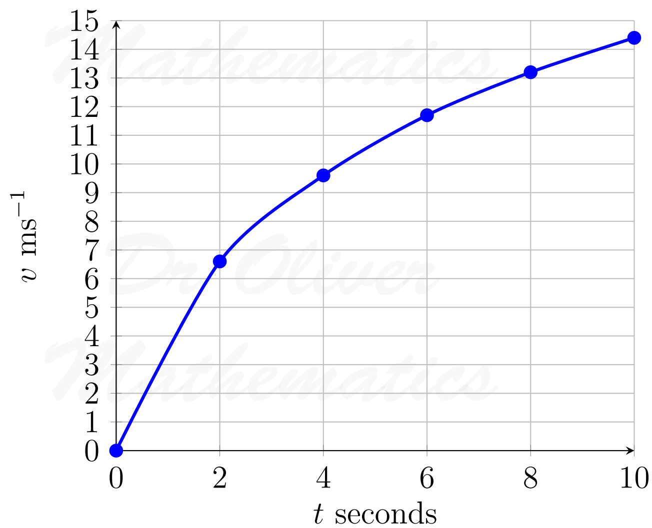
$$\begin{aligned} \text{Area} &= \left| \int_1^5 \left\{ (x^2 - 4x + 4) - (-x^2 + 8x - 6) \right\} \, \mathrm{d}x \right| \\ &= \left| \int_1^5 (2x^2 - 12x + 10) \, \mathrm{d}x \right| \\ &= \left| \left[\tfrac{2}{3}x^3 - 6x^2 + 10x \right]_{x=1}^{5} \right| \\ &= \left| \left(83\tfrac{1}{3} - 150 + 50\right) - \left(\tfrac{2}{3} - 6 + 10\right) \right| \\ &= \left| -21\tfrac{1}{3} \right| \\ &= \underline{\underline{21\tfrac{1}{3}}}. \end{aligned}$$

11. Nina records the speed, v ms^{-1}, at which she is travelling in her car t seconds after accelerating from rest.
The results are shown in the table below.

Time (t seconds)	0	2	4	6	8	10
Speed (v ms^{-1})	0	6.6	9.6	11.7	13.2	14.4

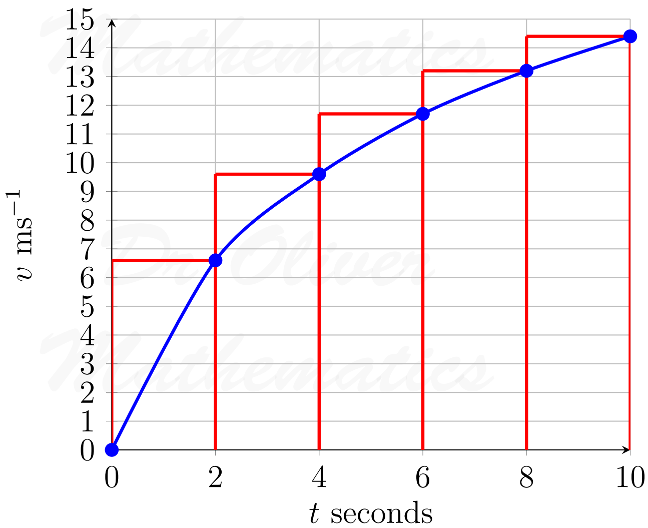
(a) Use these results and the axes below to draw a curve to show how her speed varies with time during these 10 seconds. (2)

Solution



(b) By constructing 5 rectangles of equal width above your curve, estimate the distance she has travelled during the first 10 seconds. (4)

Solution



Now,

$$\begin{aligned} \text{distance} &\approx 2(6.6 + 9.6 + 11.7 + 13.2 + 14.4) \\ &= \underline{\underline{111 \text{ m}}}. \end{aligned}$$

(c) Without doing any further calculations, explain how she could obtain a better estimate of the distance she has travelled. (1)

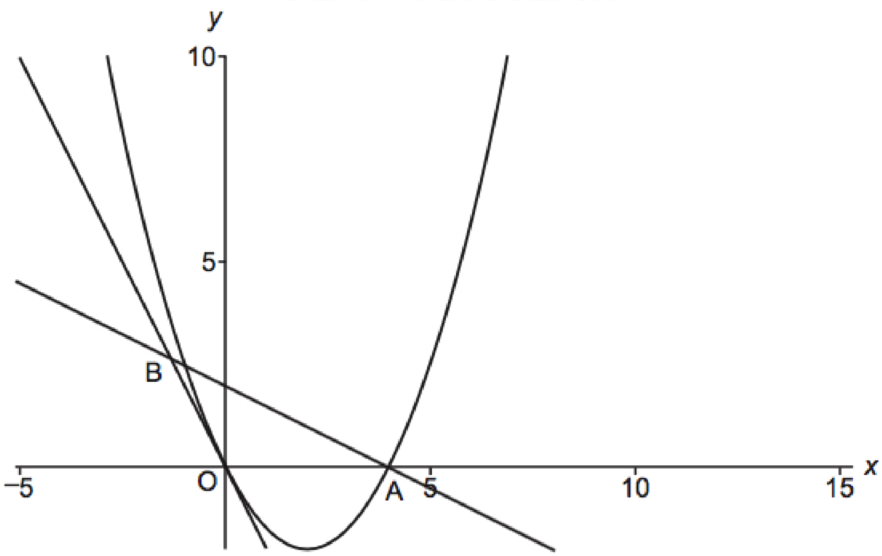
Solution

E.g., make the interval smaller, more rectangles.

12. In the diagram the curve with equation

$$y = \tfrac{1}{2}x^2 - 2x$$

crosses the x-axis at the origin, O, and the point A.



The tangent to this curve at O and the normal to this curve at A intersect at the point B.

(a) Determine the equation of the line OB. (3)

Solution

$$y = \tfrac{1}{2}x^2 - 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = x - 2$$

and

$$x = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -2.$$

Hence, the equation of the line OB is

$$\underline{\underline{y = -2x}}.$$

(b) Determine the equation of the line AB. (6)

Solution

Well,

$$\begin{aligned} \tfrac{1}{2}x^2 - 2x = 0 &\Rightarrow \tfrac{1}{2}x(x-4) = 0 \\ &\Rightarrow x = \text{ or } x = 4; \end{aligned}$$

so, $A(4, 0)$. Now,

$$x = 4 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2$$

and so

$$m_{\text{normal}} = -\tfrac{1}{2}.$$

Finally, the equation of the line OB is

$$\underline{\underline{y = -\tfrac{1}{2}x + 2.}}$$

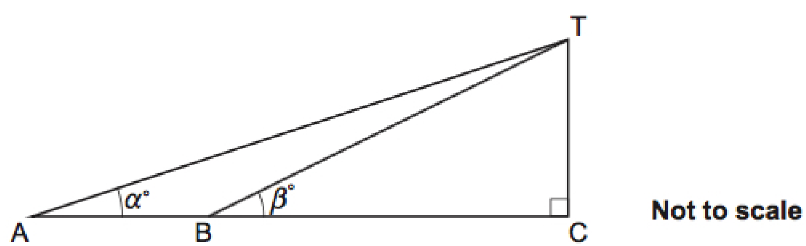
(c) Hence determine the coordinates of the point B. (3)

Solution

$$\begin{aligned} -\tfrac{1}{2}x + 2 = -2x &\Rightarrow \tfrac{3}{2}x = -2 \\ &\Rightarrow x = -\tfrac{4}{3} \\ &\Rightarrow y = \tfrac{8}{3}; \end{aligned}$$

hence, the coordinates of the point $\underline{\underline{B(-1\tfrac{1}{3}, 2\tfrac{2}{3})}}$.

13. A vertical tower CT stands with its base, C, on horizontal ground.
Amir stands at a point A and observes that the angle of elevation of the top of the tower, T, is $\alpha°$.
He then walks directly towards the base of the tower to a point B where he observes that the angle of elevation of the top of the tower is $\beta°$.



(a) Show that (3)

$$BC = \frac{AB \tan \alpha°}{\tan \beta° - \tan \alpha°}.$$

Solution

Well,

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \text{opp} = \text{adj} \times \tan.$$

In particular,

$$CT = BC \tan \beta^\circ \text{ and } CT = AC \tan \alpha^\circ.$$

Now,

$$\begin{aligned} CT = AC \tan \alpha^\circ &\Rightarrow CT = (AB + BC) \tan \alpha^\circ \\ &\Rightarrow CT = AB \tan \alpha^\circ + BC \tan \alpha^\circ. \end{aligned}$$

Equating the two terms:

$$\begin{aligned} & BC \tan \beta^\circ = AB \tan \alpha^\circ + BC \tan \alpha^\circ \\ \Rightarrow\ & BC \tan \beta^\circ - BC \tan \alpha^\circ = AB \tan \alpha^\circ \\ \Rightarrow\ & BC(\tan \beta^\circ - \tan \alpha^\circ) = AB \tan \alpha^\circ \\ \Rightarrow\ & \underline{\underline{BC = \frac{AB \tan \alpha^\circ}{\tan \beta^\circ - \tan \alpha^\circ}}}, \end{aligned}$$

as required.

You are given that

- $AB = 25$ m,
- $\alpha^\circ = 15^\circ$, and
- $\beta^\circ = 20^\circ$.

(b) Find

(i) BC, (2)

Solution
Now,

$$\begin{aligned} BC &= \frac{25 \tan 15^\circ}{\tan 20^\circ - \tan 15^\circ} \\ &= 69.763\,144\,44 \text{ (FCD)} \\ &= \underline{\underline{69.8 \text{ m (3 sf)}}}. \end{aligned}$$

(ii) the height of the tower. (2)

Solution

$$\begin{aligned} \text{opp} = \text{adj} \times \tan \Rightarrow CT &= 69.763\ldots \tan 20^\circ \\ \Rightarrow CT &= 25.391\,708\,02 \text{ (FCD)} \\ \Rightarrow CT &= 25.4 \text{ m (3 sf)}. \end{aligned}$$

14. Ben wishes to estimate the gradient of a curve at the point where $x = 1.4$. He calculates points on the curve which are given in the table below.

x	1.2	1.4	1.8
y	1.073 2	1.535 8	3.144 7

(a) (i) Explain why he should **not** use the coordinates at $x = 1.2$ and $x = 1.8$ to obtain a reasonable estimate for the gradient of the curve at $x = 1.4$. (1)

Solution
E.g., $x = 1.2$ and $x = 1.4$ are 0.2 apart but $x = 1.4$ and $x = 1.8$ are 0.4 apart.

(ii) Calculate an estimate for the gradient of the curve at $x = 1.4$ by using the coordinates at $x = 1.4$ and $x = 1.8$. (2)

Solution

$$\begin{aligned} \text{Estimate} &= \frac{3.144\,7 - 1.535\,8}{1.8 - 1.4} \\ &= 4.022\,25 \text{ (exact)} \\ &= 4.022 \text{ (3 dp)}. \end{aligned}$$

(iii) Calculate an estimate for the gradient of the curve at $x = 1.4$ by using the coordinates at $x = 1.2$ and $x = 1.4$. (1)

Solution

$$\begin{aligned} \text{Estimate} &= \frac{1.535\,8 - 1.073\,2}{1.4 - 1.2} \\ &= 2.313 \text{ (3 dp)}. \end{aligned}$$

Mia wishes to estimate the gradient of another curve at the point where $x = 1.4$. She calculates points on this curve which are given in the table below.

x	1.2	1.4	1.6
y	0.6899	0.9518	1.3132

(b) Calculate a reasonable estimate of the gradient of this curve when $x = 1.4$. (3)

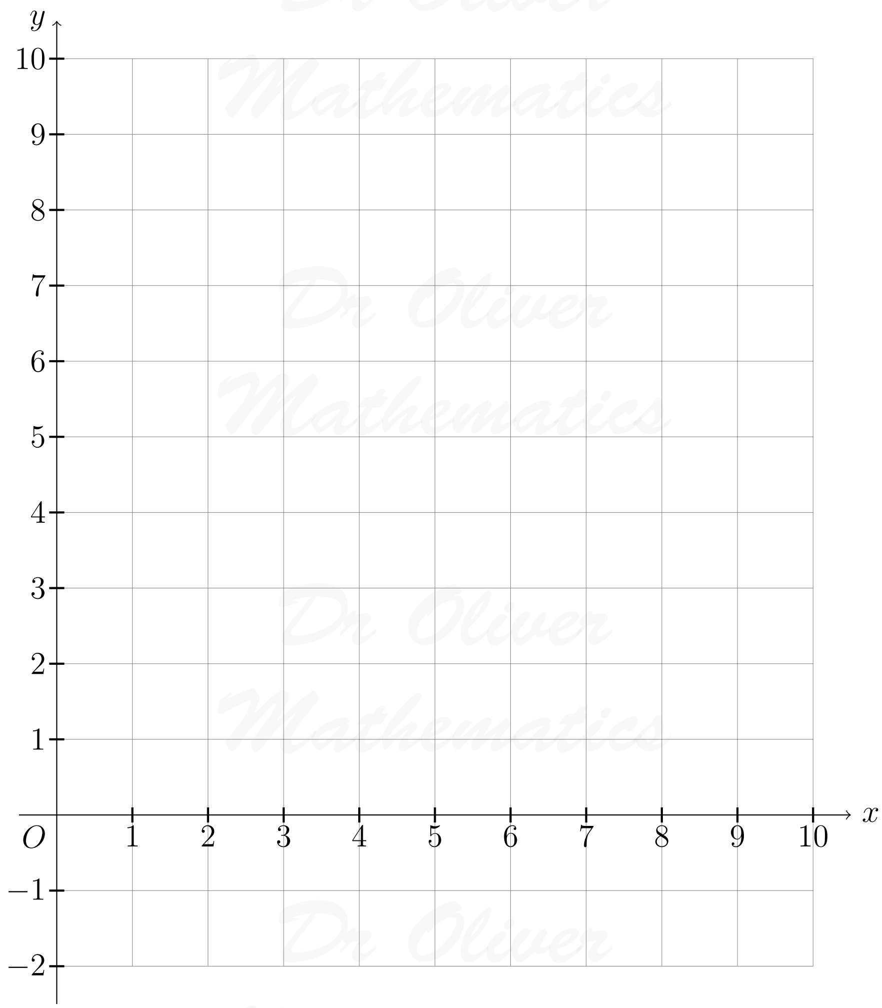
Solution

$$\begin{aligned}\text{Estimate} &= \frac{1.3132 - 0.6899}{1.6 - 1.2} \\ &= 1.55825 \text{ (exact)} \\ &= \underline{\underline{1.558 \text{ (3 dp)}}}.\end{aligned}$$

15. The points A and B have coordinates $(3, 3)$ and $(5, 7)$ respectively.

(a) On the grid below plot

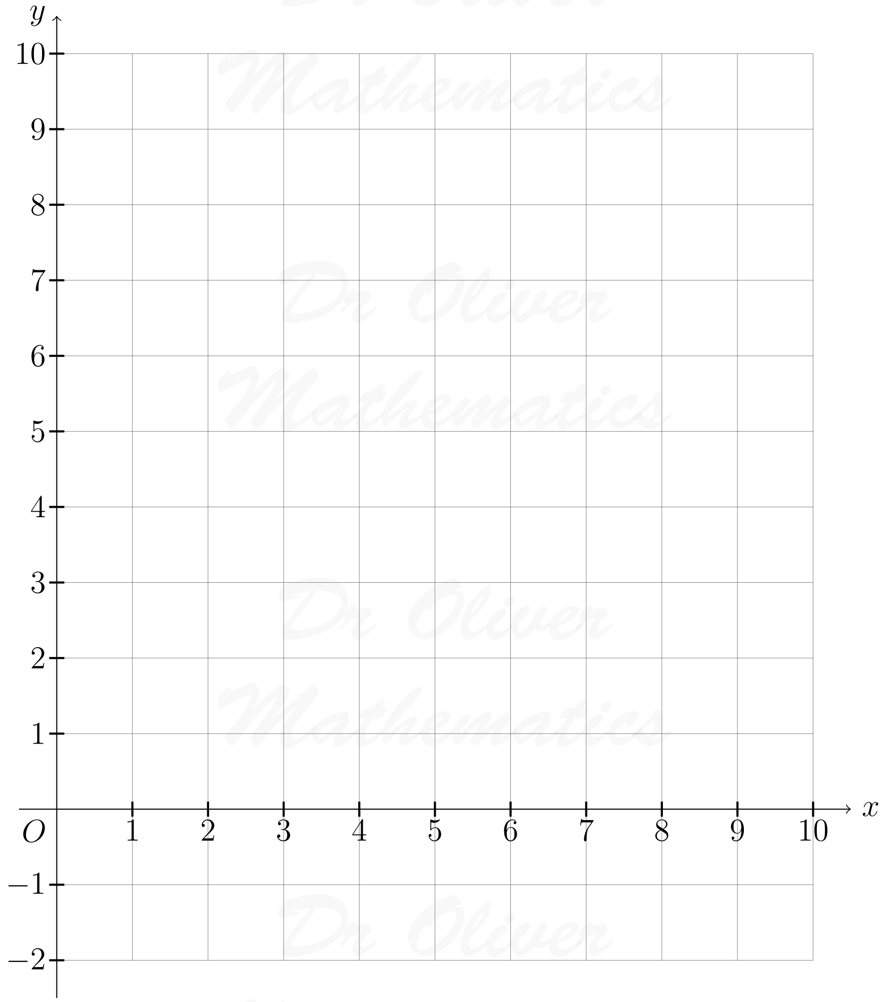
(i) the points A and B, (1)



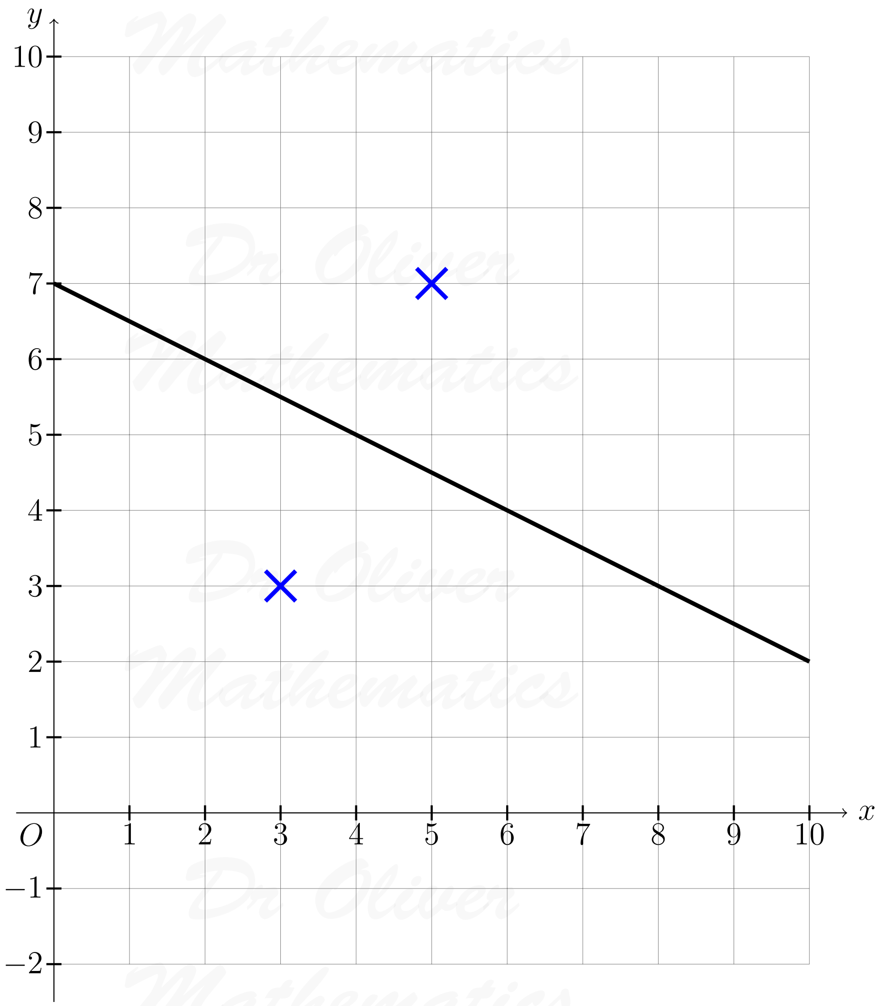
Solution
See below.

(ii) the line l with equation (2)

$$x + 2y = 14.$$



Solution



(b) Verify that the line l is the perpendicular bisector of AB. (4)

Solution

The midpoint of line joining A and B is

$$\left(\frac{3+5}{2}, \frac{3+7}{2}\right) = (4, 5).$$

Now,

$$\begin{aligned} m_{AB} &= \frac{7-3}{5-2} \\ &= 2 \end{aligned}$$

and, hence,

$$m_{\text{normal}} = -\tfrac{1}{2}.$$

Finally, the equation of the line is

$$\begin{aligned} y - 5 = -\tfrac{1}{2}(x-4) &\Rightarrow 2(y-5) = -(x-4) \\ &\Rightarrow 2y - 10 = -x + 4 \\ &\Rightarrow \underline{\underline{x + 2y = 14}}, \end{aligned}$$

as required.

AB is a diameter of the circle C.

(c) Find the equation of the circle C. (2)

Solution

$$\begin{aligned} (x-4)^2 + (y-5)^2 &= (5-4)^2 + (7-5)^2 \\ &= 1 + 4 \\ &= 5; \end{aligned}$$

hence, the equation of the circle C is

$$\underline{\underline{(x-4)^2 + (y-5)^2 = 5}}.$$

The line l cuts the circle C in two points, P and Q.

(d) Determine the coordinates of P and Q. (3)

Solution

$$x + 2y = 14 \Rightarrow x = -2y + 14$$

and we will substitute into the circle:

$$\begin{aligned} (x-4)^2 + (y-5)^2 = 5 &\Rightarrow (-2y+10)^2 + (y-5)^2 = 5 \\ &\Rightarrow [-2(y-5)]^2 + (y-5)^2 = 5 \\ &\Rightarrow 4(y-5)^2 + (y-5)^2 = 5 \\ &\Rightarrow 5(y-5)^2 = 5 \\ &\Rightarrow (y-5)^2 = 1 \\ &\Rightarrow y - 5 = 1 \text{ or } y - 5 = -1 \\ &\Rightarrow y = 6 \text{ or } y = 4 \\ &\Rightarrow x = 2 \text{ or } x = 6; \end{aligned}$$

hence, the coordinates of P and Q are

$$\underline{\underline{(2, 6)}} \text{ and } \underline{\underline{(6, 4)}}.$$