

Dr Oliver Mathematics
GCSE Mathematics
2004 November Paper 5H: Non-Calculator
2 hours

The total number of marks available is 100.

You must write down all the stages in your working.

1. Rosa prepares the ingredients for pizzas. (3)
She uses cheese, topping, and dough in the ratio 2 : 3 : 5.
Rosa uses 70 grams of dough.
Work out the number of grams of cheese and the number of grams of topping Rosa uses.

Solution

Cheese:

$$\frac{2}{5} \times 70 = 2 \times 14 = \underline{\underline{28 \text{ g.}}}$$

Topping:

$$\frac{3}{5} \times 70 = 3 \times 14 = \underline{\underline{42 \text{ g.}}}$$

2. Work out (3)
 $12\frac{1}{2} \div \frac{5}{8}$.

Solution

$$\begin{aligned} 12\frac{1}{2} \div \frac{5}{8} &= \frac{25}{2} \times \frac{8}{5} \\ &= \frac{5}{1} \times \frac{4}{1} \\ &= \underline{\underline{20}}. \end{aligned}$$

3. (a) Expand the brackets (1)
 $p(q - p^2)$.

Solution

$$p(q - p^2) = \underline{pq - p^3}.$$

(b) Expand and simplify

$$5(3p + 2) - 2(5p - 3).$$

(2)

Solution

$$\begin{aligned} 5(3p + 2) - 2(5p - 3) &= 15p + 10 - 10p + 6 \\ &= \underline{5p + 16}. \end{aligned}$$

4. (a) (i) Write 40 000 000 in standard form.

(2)

Solution

$$40\,000\,000 = \underline{4 \times 10^7}.$$

(ii) Write 3×10^{-5} as an ordinary number

Solution

$$3 \times 10^{-5} = \underline{0.00003}.$$

(b) Work out the value of

$$3 \times 10^{-5} \times 40\,000\,000.$$

(2)

Give your answer in standard form.

Solution

$$\begin{aligned} 3 \times 10^{-5} \times 40\,000\,000 &= 3 \times 10^{-5} \times 4 \times 10^7 \\ &= 12 \times 10^2 \\ &= \underline{1.2 \times 10^3}. \end{aligned}$$

5. ABC is an isosceles triangle.

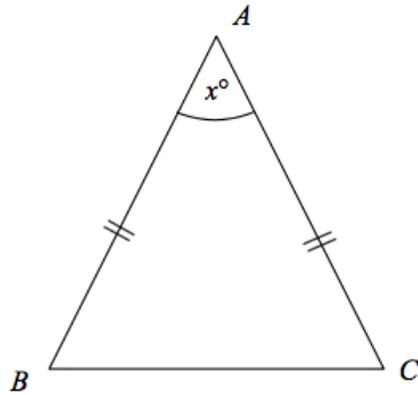


Diagram **NOT** accurately drawn

$AB = AC$.
Angle $A = x^\circ$.

- (a) Find an expression, in terms of x , for the size of angle B . (2)

Solution

$$\angle B = \frac{1}{2}(180 - x) = \underline{\underline{90 - \frac{1}{2}x}}$$

- (b) Solve the simultaneous equations (3)

$$\begin{aligned} 3p + q &= 11 \\ p + q &= 3. \end{aligned}$$

Solution

Subtract:

$$\begin{aligned} 2p &= 8 \Rightarrow \underline{\underline{p = 4}} \\ &\Rightarrow \underline{\underline{q = -1}}. \end{aligned}$$

6. Calculate the size of the exterior angle of a regular hexagon. (2)

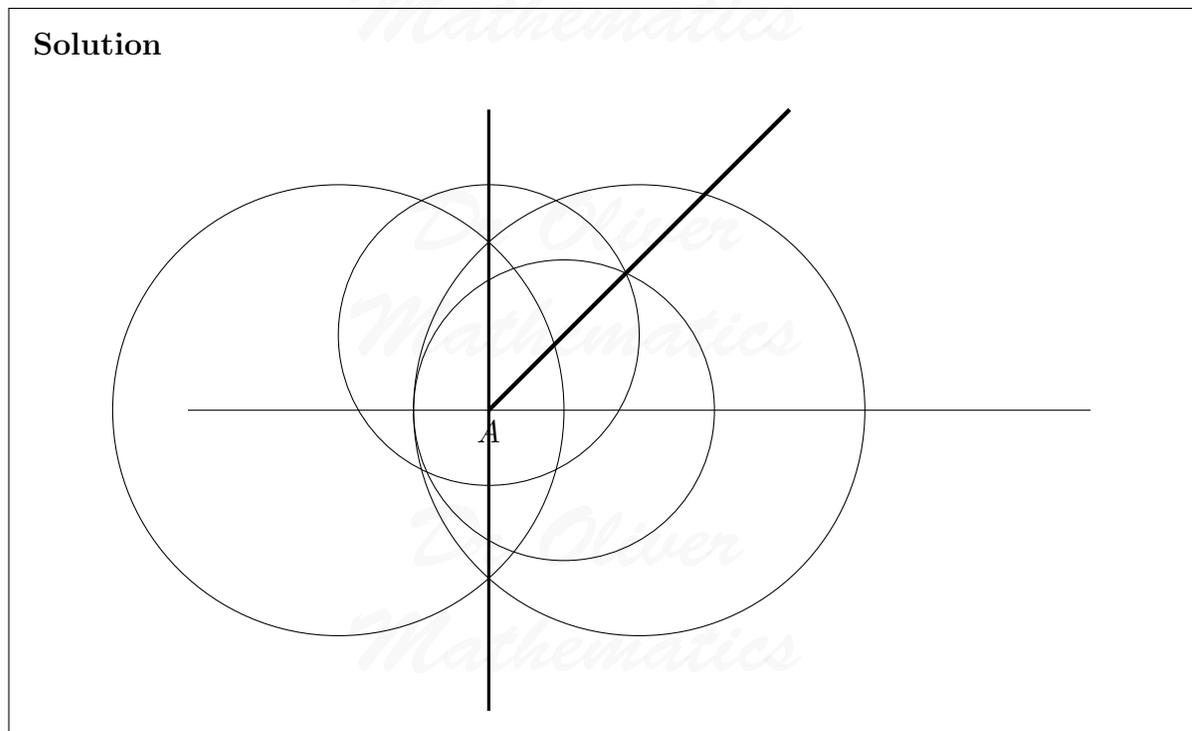
Solution

$$\frac{360}{6} = \underline{\underline{60^\circ}}$$

7. Use ruler and compasses to **construct** an angle of 45° at A .
You must show **all** construction lines.

(3)

A



8. Calculate the volume of the triangular prism.

(4)

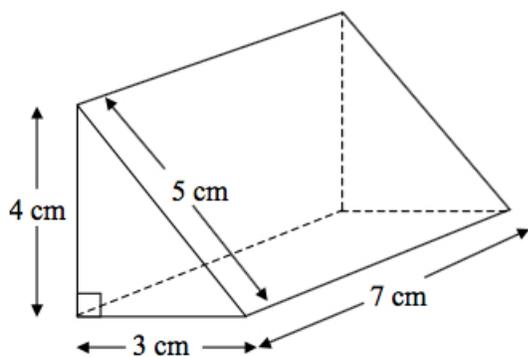


Diagram NOT accurately drawn

Solution

$$\begin{aligned} \text{Volume} &= \left(\frac{1}{2} \times 3 \times 4\right) \times 7 \\ &= 6 \times 7 \\ &= \underline{\underline{42 \text{ cm}^3}}. \end{aligned}$$

9. (a) Simplify

(2)

(i) $\frac{x^6}{x^2}$,

Solution

$$\frac{x^6}{x^2} = \underline{\underline{x^4}}.$$

(ii) $(y^4)^3$.

Solution

$$(y^4)^3 = \underline{\underline{y^{12}}}.$$

(b) Expand and simplify

(2)

$$(t + 4)(t - 2).$$

Solution

\times	t	$+4$
t	t^2	$+4t$
-2	$-2t$	-8

Hence,

$$(t + 4)(t - 2) = \underline{\underline{t^2 + 2t - 8.}}$$

- (c) Write down the integer values of x that satisfy the inequality (2)

$$-2 \leq x < 4.$$

Solution

$$\underline{\underline{-2, -1, 0, 1, 2, 3.}}$$

- (d) Find the value of (2)

(i) $36^{-\frac{1}{2}}$,

Solution

$$36^{-\frac{1}{2}} = \frac{1}{\sqrt{36}} = \underline{\underline{\frac{1}{6}}}.$$

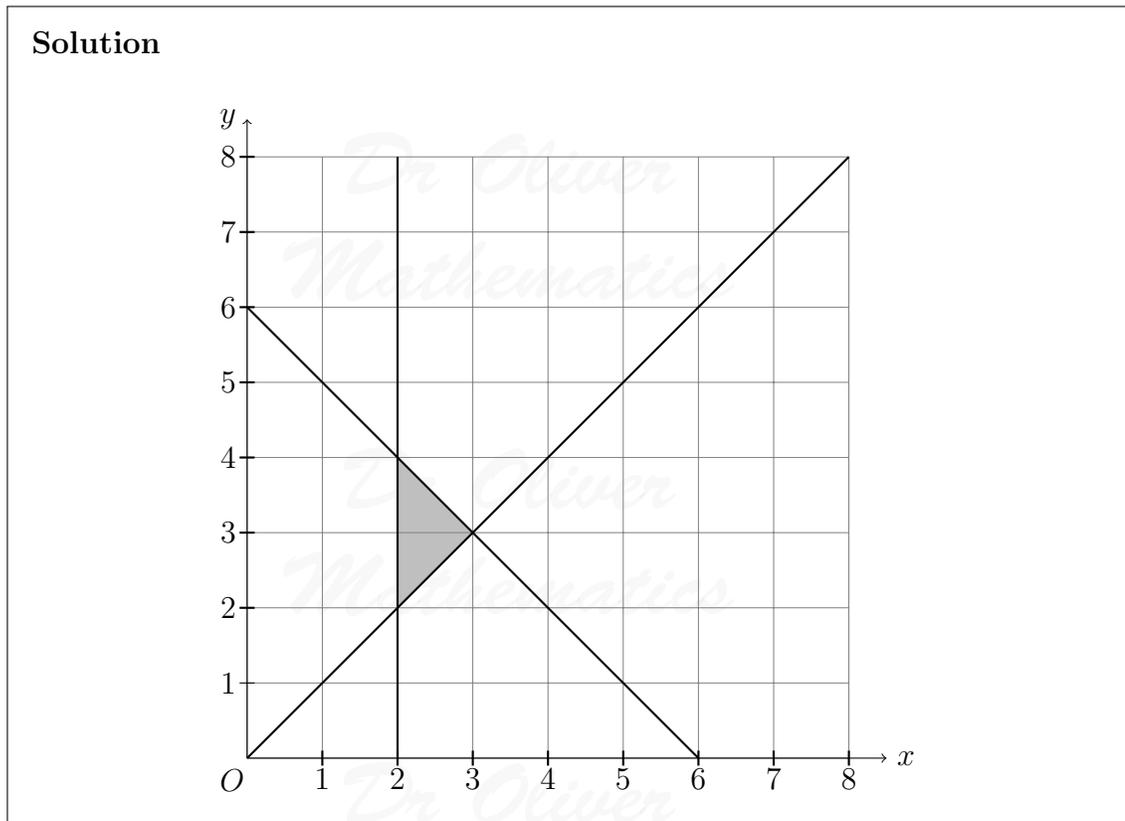
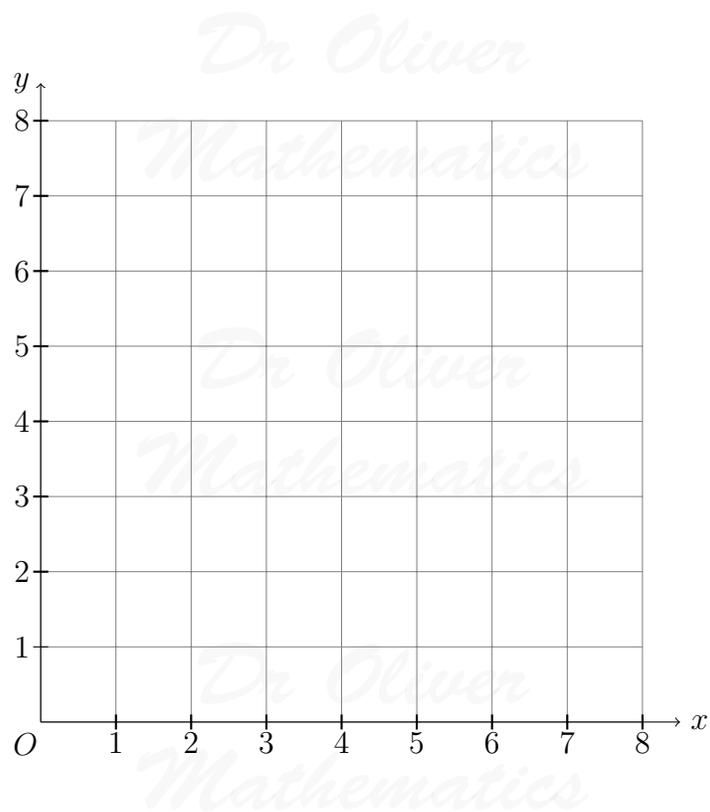
(ii) $27^{\frac{2}{3}}$.

Solution

$$\begin{aligned} 27^{\frac{2}{3}} &= (27^{\frac{1}{3}})^2 \\ &= 3^2 \\ &= \underline{\underline{9}}. \end{aligned}$$

10. (a) On the grid below, draw straight lines and use shading to show the region **R** that satisfies the inequalities: (3)

$$x \geq 2, y \geq x, \text{ and } x + y \leq 6.$$



The point P with coordinates (x, y) lies inside the region \mathbf{R} .
 x and y are **integers**.

- (b) Write down the coordinates of **all** points of \mathbf{R} whose coordinates are both integers. (2)

Solution

The points are (2, 2), (2, 3), (3, 3), and (2, 4).

11. Make u the subject of the formula (2)

$$D = ut + kt^2.$$

Solution

$$\begin{aligned} D &= ut + kt^2 \Rightarrow ut = kt^2 - D \\ &\Rightarrow u = \frac{kt^2 - D}{t}. \end{aligned}$$

12. The table gives information about the ages of 160 employees of an IT company.

Age (A) in years	Frequency
$15 < A \leq 25$	44
$25 < A \leq 35$	56
$35 < A \leq 45$	34
$45 < A \leq 55$	19
$55 < A \leq 65$	7

- (a) Complete the cumulative frequency table. (1)

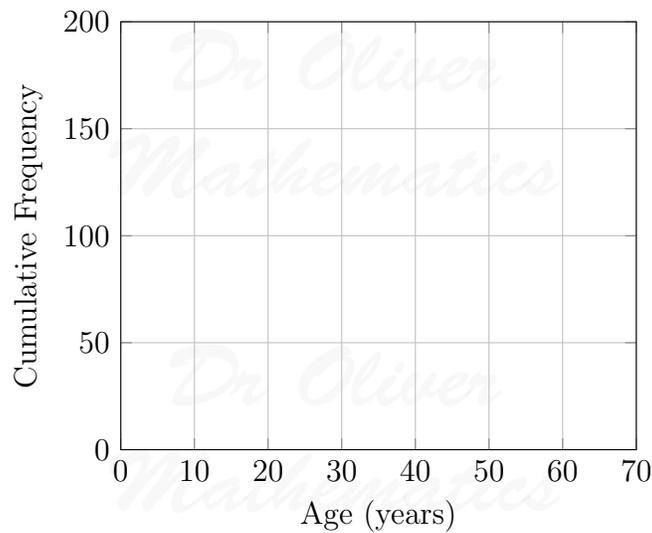
Age (A) in years	Cumulative Frequency
$15 < A \leq 25$	
$25 < A \leq 35$	
$35 < A \leq 45$	
$45 < A \leq 55$	
$55 < A \leq 65$	

Solution

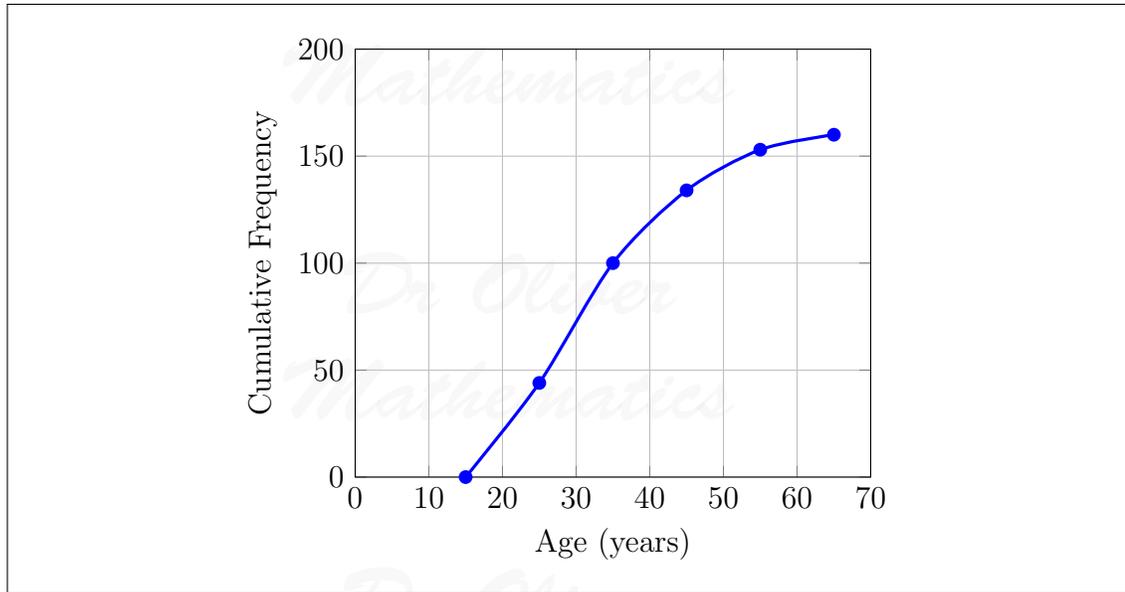
Age (A) in years	Cumulative Frequency
$15 < A \leq 25$	<u>44</u>
$25 < A \leq 35$	$44 + 56 = \underline{100}$
$35 < A \leq 45$	$100 + 34 = \underline{134}$
$45 < A \leq 55$	$134 + 19 = \underline{153}$
$55 < A \leq 65$	$153 + 7 = \underline{160}$

(b) Draw a cumulative frequency graph for your table.

(2)

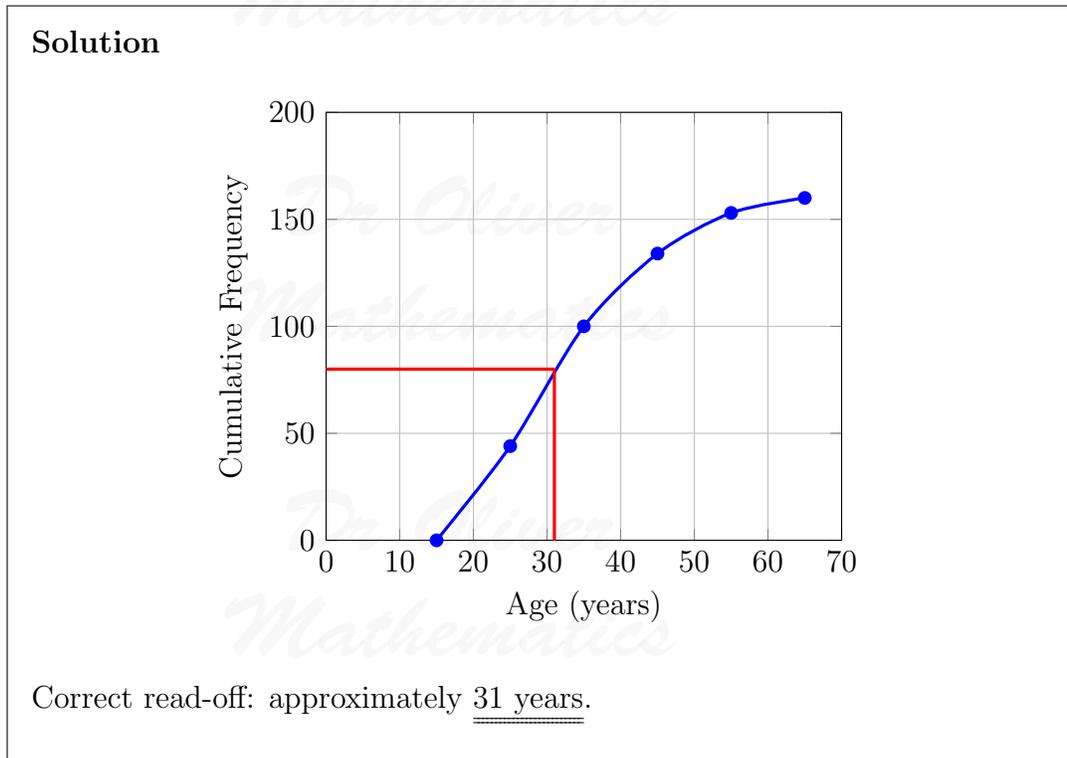


Solution



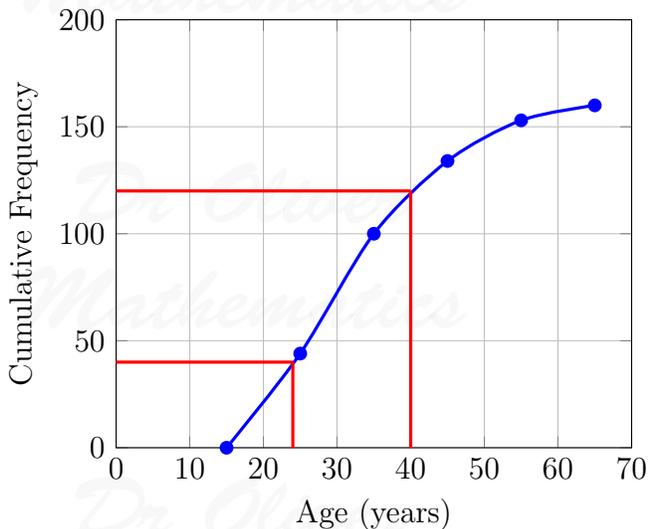
- (c) Use your graph to find an estimate for
 (i) the median age of the employees,

(3)



- (ii) the interquartile range of the ages of the employees.

Solution



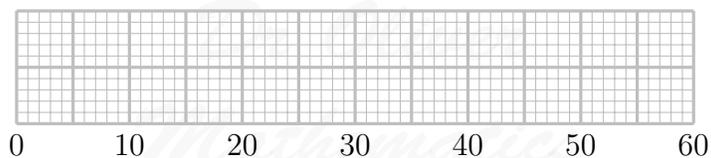
The IQR is approximately

$$40 - 24 = \underline{\underline{16 \text{ years}}}.$$

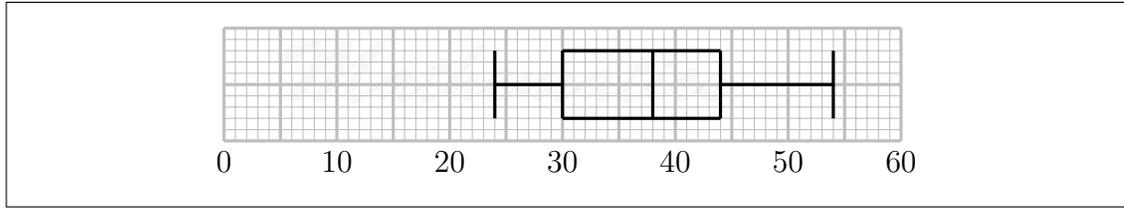
Another IT company has 80 employees.
The age of the youngest employee is 24 years.
The age of the oldest employee is 54 years.
The median age is 38 years.
The lower quartile is 30 years.
The lower quartile is 44 years.

(d) Draw a box plot to show information about the ages of the employees.

(2)



Solution



13. The diagram shows a circle, centre O .

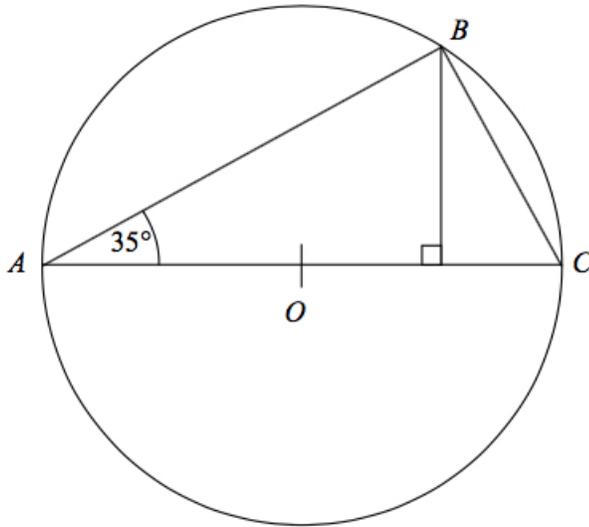


Diagram **NOT** accurately drawn

AC is a diameter.

Angle $BAC = 35^\circ$.

D is the point on AC such that angle BDA is a right angle.

- (a) Work out the size of angle BCA . (2)
Give reasons for your answer.

Solution

Angle $BCA = 180 - 90 - 35 = \underline{\underline{55^\circ}}$ (angles in a triangle).

- (b) Calculate the size of angle DBC . (1)

Solution

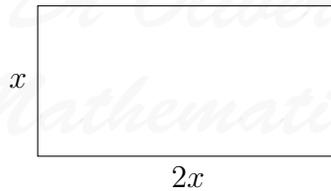
Angle $DBC = \underline{\underline{35^\circ}}$.

- (c) Calculate the size of angle BOA . (2)

Solution

$$\text{Angle } BOA = 180 - 35 - 35 = \underline{\underline{110^\circ}}.$$

14. The length of a rectangle is twice the width of the rectangle.
The length of a diagonal of the rectangle is 25 cm. (3)

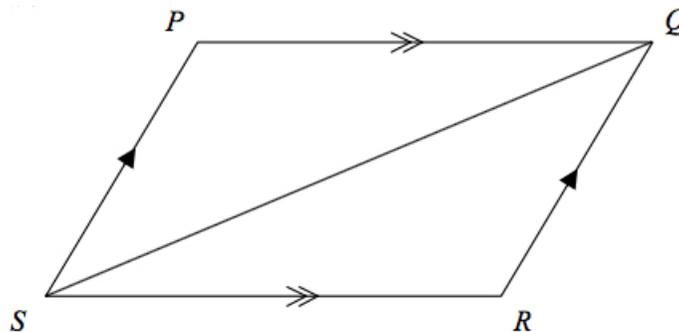


Work out the area of the rectangle.
Give your answer as an integer.

Solution

$$\begin{aligned}x^2 + (2x)^2 &= 625 \Rightarrow 5x^2 = 625 \\&\Rightarrow x^2 = 125 \\&\Rightarrow 2x^2 = \underline{\underline{250 \text{ cm}^2}}.\end{aligned}$$

15. $PQRS$ is a quadrilateral.



PQ is parallel to SR .
 SP is parallel to RQ .

- (a) Prove that triangle PQS is congruent to triangle RSQ . (3)

Solution

Angle $PSQ =$ angle RQS .

Angle $PQS =$ angle RSQ .

SQ is common.

Hence, $\triangle PQS$ is congruent to $\triangle RSQ$ (AAS).

- (b) In quadrilateral $PQRS$, angle SPQ is obtuse. (2)
Explain why $PQRS$ cannot be a cyclic quadrilateral.

Solution

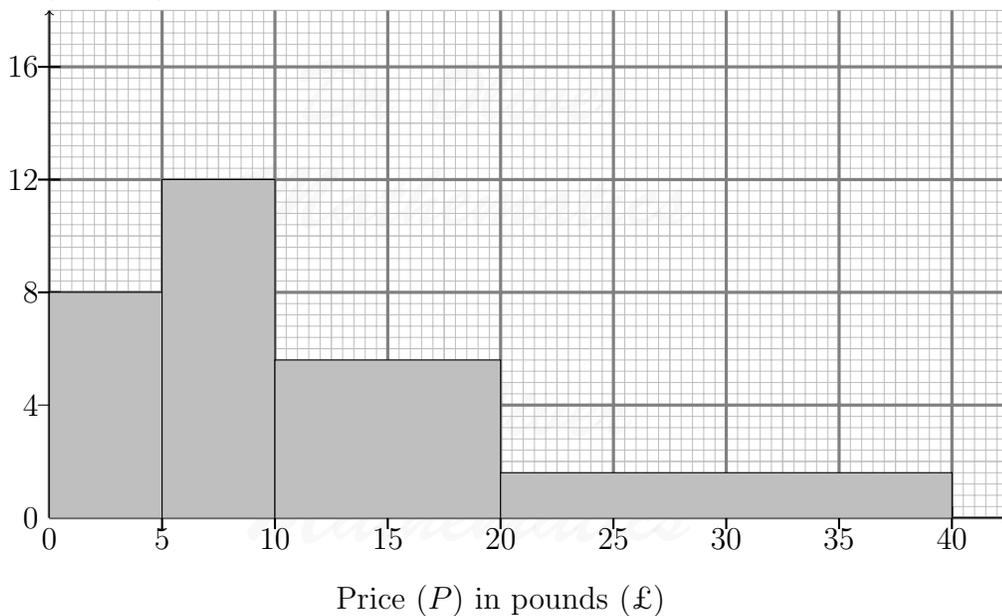
Angle $SPQ =$ angle SRQ and

$$\text{angle } SPQ + \text{angle } SRQ > 180^\circ$$

which means the shape cannot be cyclic.

16. This histogram gives information about the books sold in a bookshop one Saturday.

Frequency density



- (a) Use the histogram to complete the table. (2)

Price (P) in pounds (£)	Frequency
$0 < P \leq 5$	8
$5 < P \leq 10$	12
$10 < P \leq 20$	56
$20 < P \leq 40$	32

Solution

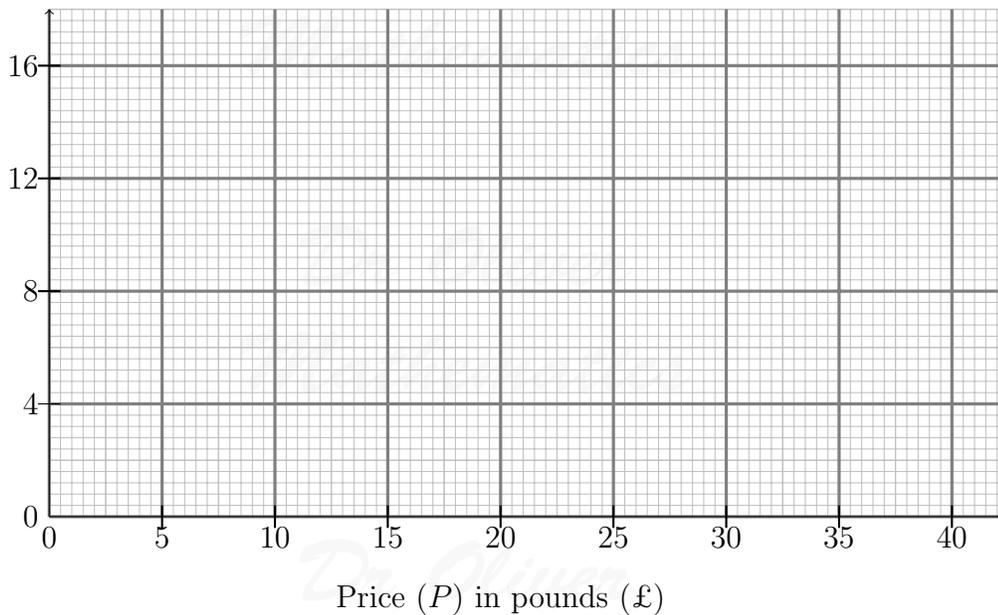
Price (P) in pounds (£)	Width	Frequency Density	Frequency
$0 < P \leq 5$	5	8	$5 \times 8 = \underline{40}$
$5 < P \leq 10$	5	12	$5 \times 12 = \underline{60}$
$10 < P \leq 20$	10	5.6	$10 \times 5.6 = \underline{56}$
$20 < P \leq 40$	20	1.6	$20 \times 1.6 = \underline{32}$

The frequency table below gives information about the books sold in a second bookshop on the same Saturday.

Price (P) in pounds (£)	Frequency
$0 < P \leq 5$	80
$5 < P \leq 10$	20
$10 < P \leq 20$	24
$20 < P \leq 40$	96

- (b) Draw a histogram to represent the information about the books sold in the second bookshop. (3)

Frequency density



Solution

Price (P) in pounds (£)	Frequency	Width	Frequency Density
$0 < P \leq 5$	80	5	$\frac{80}{5} = 16$
$5 < P \leq 10$	20	5	$\frac{20}{5} = 4$
$10 < P \leq 20$	24	10	$\frac{24}{10} = 2.4$
$20 < P \leq 40$	96	20	$\frac{96}{20} = 4.8$



17. (a) Express

$$\frac{6}{\sqrt{2}}$$

(2)

in the form $a\sqrt{b}$, where a and b are positive integers.

Solution

$$\begin{aligned} \frac{6}{\sqrt{2}} &= \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{6\sqrt{2}}{2} \\ &= \underline{\underline{3\sqrt{2}}}. \end{aligned}$$

The diagram shows a right-angled isosceles triangle.

The length of each of its equal sides is $\frac{6}{\sqrt{2}}$ cm.

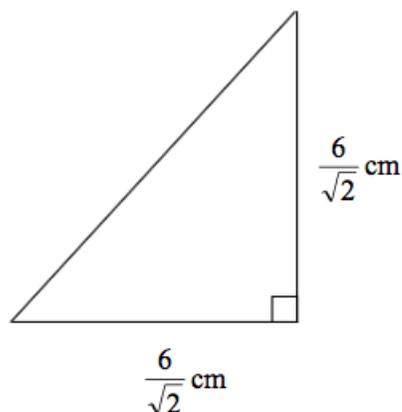


Diagram **NOT**
accurately drawn

- (b) Find the area of the triangle.
Give your answer as an integer.

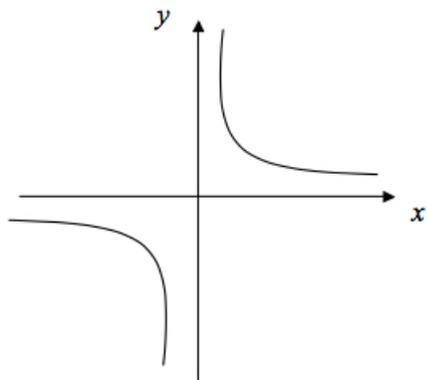
(2)

Solution

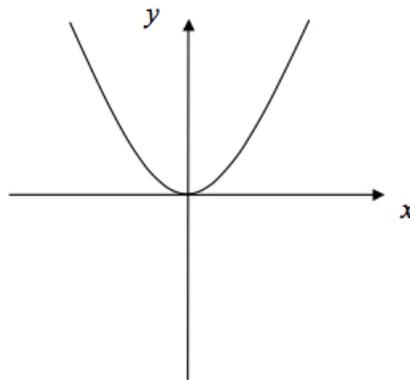
$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 3\sqrt{2} \times 3\sqrt{2} \\ &= \frac{1}{2} \times 18 \\ &= \underline{\underline{9 \text{ cm}^2}}.\end{aligned}$$

18. The graphs of y against x represent four different types of proportionality.

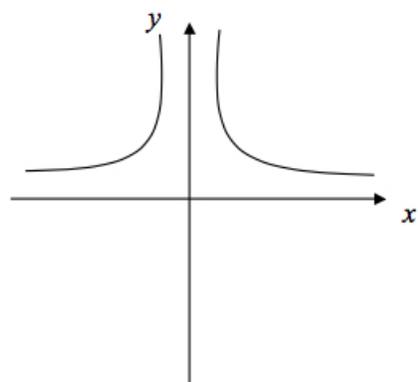
(2)



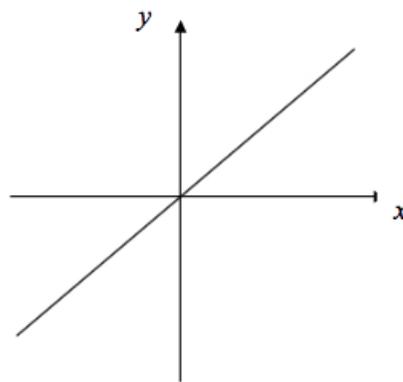
Graph A



Graph B



Graph C



Graph D

Write down the letter of the graph which represents the type of proportionality.

Type of proportionality	Graph letter
y is directly proportional to x	
y is inversely proportional to x	
y is proportional to the square of x	
y is inversely proportional to the square of x	

Solution

Type of proportionality	Graph letter
y is directly proportional to x	<u><u>D</u></u>
y is inversely proportional to x	<u><u>A</u></u>
y is proportional to the square of x	<u><u>B</u></u>
y is inversely proportional to the square of x	<u><u>C</u></u>

19. (a) Write down an expression, in terms of n , for the n th multiple of 5. (1)

Solution

$5n$.

- (b) Hence, or otherwise, (5)

- (i) prove that the sum of two consecutive multiples of 5 is always an odd number,

Solution

Let the numbers be $5n$ and $(5n + 5)$ for $n \in \mathbb{Z}$. Then the sum of them is

$$\begin{aligned} 5n + (5n + 5) &= 10n + 5 \\ &= 2(5n + 2) + 1 \end{aligned}$$

and this is an odd number.

- (ii) prove that the product of two consecutive multiples of 5 is always an even number.

Solution

Let the numbers be $5n$ and $(5n + 5)$ for $n \in \mathbb{Z}$. Then the product of them is

$$\begin{aligned} 5n(5n + 5) &= 25n^2 + 25n \\ &= 25n(n + 1); \end{aligned}$$

and this is an even number because we have pair of numbers n and $(n + 1)$ and *their* product is even.

20. The diagram shows part of two graphs.

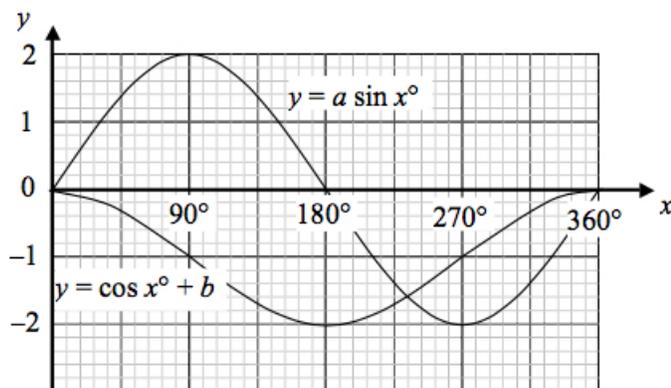


Diagram NOT accurately drawn

The equation of one graph is $y = a \sin x^\circ$.
 The equation of the other graph is $y = \cos x^\circ + b$.

- (a) Find the value of a and find the value of b . (2)

Solution

$a = 2$ and $b = -1$.

- (b) Use the graphs to find the values of x in the range $0 \leq x \leq 720^\circ$ when (2)

$$a \sin x^\circ = \cos x^\circ + b.$$

Solution

0, 234, 360, $360 + 234 =$ 594, and 720.

- (c) Use the graphs to find the value of (2)

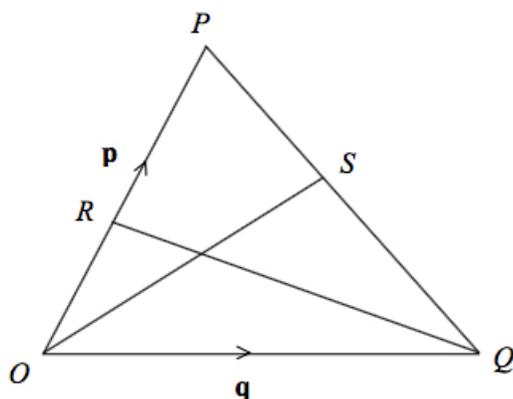
$$a \sin x^\circ - (\cos x^\circ + b)$$

when $x = 450^\circ$.

Solution

$$2 \sin 450^\circ - (\cos 450^\circ - 1) = 1 - (-1) = \underline{\underline{2}}.$$

21. OPQ is a triangle. (5)

Diagram NOT
accurately drawn

R is the midpoint of OP .

S is the midpoint of PQ .

$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$,

- (a) Find \overrightarrow{OS} in terms of \mathbf{p} and \mathbf{q} .

Solution

$$\begin{aligned}\overrightarrow{OS} &= \overrightarrow{OQ} + \overrightarrow{QS} \\ &= \overrightarrow{OQ} + \frac{1}{2}\overrightarrow{QP} \\ &= \mathbf{q} + \frac{1}{2}(\mathbf{q} - \mathbf{p}) \\ &= \underline{\underline{\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}}}.\end{aligned}$$

- (b) Show that RS is parallel to OQ .

Solution

$$\begin{aligned}\overrightarrow{RS} &= \overrightarrow{RP} + \overrightarrow{PS} \\ &= \frac{1}{2}\overrightarrow{OP} + \frac{1}{2}\overrightarrow{PQ} \\ &= \frac{1}{2}\mathbf{p} + \frac{1}{2}(\mathbf{q} - \mathbf{p}) \\ &= \frac{1}{2}\mathbf{q} \\ &= \frac{1}{2}\overrightarrow{OQ};\end{aligned}$$

hence, RS is parallel to OQ .

22. Solve

$$\frac{2}{x+1} + \frac{3}{x-1} = \frac{5}{x^2-1}.$$

(4)

Solution

$$\begin{aligned}\frac{2}{x+1} + \frac{3}{x-1} &= \frac{5}{x^2-1} \Rightarrow \frac{2}{x+1} + \frac{3}{x-1} = \frac{5}{(x-1)(x+1)} \\ &\Rightarrow 2(x-1) + 3(x+1) = 5 \\ &\Rightarrow 2x - 2 + 3x + 3 = 5 \\ &\Rightarrow 5x = 4 \\ &\Rightarrow x = \underline{\underline{\frac{4}{5}}}\end{aligned}$$

23. The diagram shows a sector of a circle with a radius of x cm and centre O .

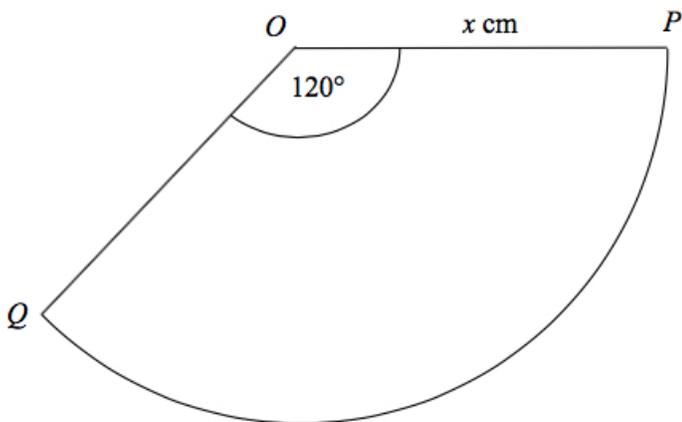


Diagram **NOT**
accurately drawn

PQ is an arc of the circle.

Angle $POQ = 120^\circ$.

- (a) Write down an expression in terms of π and x for

(2)

- (i) the area of this sector,

Solution

$$\text{Area} = \underline{\underline{\frac{1}{3}\pi x^2}}.$$

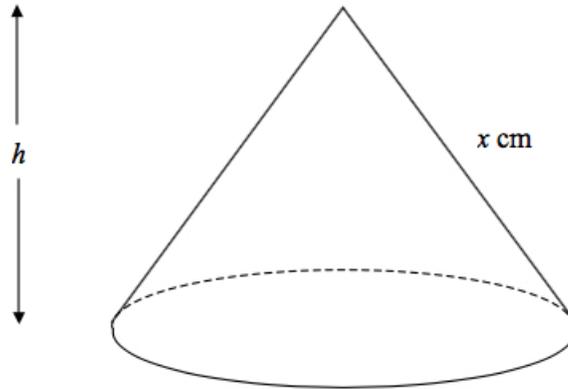
- (ii) the arc length of this sector.

Solution

$$\text{Arc length} = \underline{\underline{\frac{2}{3}\pi x}}$$

The sector is the net of the curved surface of this cone.

Arc PQ forms the circumference of the circle that makes the base of the cone.



The curved surface area of the cone is $A \text{ cm}^2$.

The volume of the cone is $V \text{ cm}^3$.

The height of the cone is $h \text{ cm}$.

Given that $V = 3A$,

(b) find the value of h .

(3)

Solution

$$\frac{1}{3}\pi \times \left(\frac{1}{3}x\right)^2 \times h = 3 \times \frac{1}{3}\pi x^2$$

$$\Rightarrow \frac{1}{27}x^2h = x^2$$

$$\Rightarrow \underline{\underline{h = 27 \text{ cm}}}$$