Dr Oliver Mathematics
AQA Further Maths Level 2
June 2019 Paper 1
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.
You are **not** permitted to use a scientific or graphical calculator in this paper.

1. A straight line passes through the points $(-2, 11)$ and $(1, 2)$. (3)

   Work out the equation of the line.

   Give your answer in the form $y = mx + c$.

   Solution

   $$\begin{aligned}\text{Gradient} &= \frac{11-2}{-2-1}\\ &= \frac{9}{-3}\\ &= -3\end{aligned}$$

   and the equation of the line is

   $$\begin{aligned}y - 11 = -3(x+2) &\Rightarrow y - 11 = -3x - 6\\ &\Rightarrow \underline{\underline{y = -3x + 5}}.\end{aligned}$$

2. Write (2)

   $$\frac{5}{6a} + \frac{a}{4}$$

   as a single fraction.

   Give your answer in its simplest form.

   Solution

   Well,

   $$\text{LCM}(6a, 4) = 12a$$

and

$$\frac{5}{6a} + \frac{a}{4} = \frac{10}{12a} + \frac{3a^2}{12a} = \underline{\underline{\frac{10 + 3a^2}{12a}}}.$$

3. Work out the **smallest** integer value of x that satisfies the inequality (2)

$$8 - 5x < 26.$$

Solution

$$8 - 5x < 26 \Rightarrow -5x < 18 \Rightarrow x > -3\tfrac{3}{5};$$

so, the smallest integer value is $\underline{\underline{-3}}$.

4. (4)

$$p(x-1) + 2(3x+k) \equiv 4(x+2),$$

where p and k are integers.

Work out the values of p and k.

Solution

$$p(x-1) + 2(3x+k) \equiv 4(x+2) \Rightarrow px - p + 6x + 2k \equiv 4x + 8 \Rightarrow (p+6)x + (-p+2k) \equiv 4x + 8$$

so

$$p + 6 = 4 \quad (1)$$
$$-p + 2k = 8 \quad (2).$$

From (1),

$$p + 6 = 4 \Rightarrow \underline{\underline{p = -2}},$$

and, from (2),

$$\begin{aligned} 2 + 2k = 8 &\Rightarrow 2k = 6 \\ &\Rightarrow \underline{\underline{k = 3}}. \end{aligned}$$

5. Solve (3)

$$\sqrt[3]{(2\sqrt{x} - 10)} = 2.$$

Solution

$$\begin{aligned} \sqrt[3]{(2\sqrt{x} - 10)} = 2 &\Rightarrow 2\sqrt{x} - 10 = 2^3 \\ &\Rightarrow 2\sqrt{x} - 10 = 8 \\ &\Rightarrow 2\sqrt{x} = 18 \\ &\Rightarrow \sqrt{x} = 9 \\ &\Rightarrow x = 9^2 \\ &\Rightarrow \underline{\underline{x = 81}}. \end{aligned}$$

6. The transformation matrix (5)

$$\begin{pmatrix} 2a & b \\ -b & -a \end{pmatrix}$$

maps the point $(3, 4)$ onto the point $(8, -7)$.

Work out the values of a and b.

Solution

Well,

$$\begin{pmatrix} 2a & b \\ -b & -a \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ -7 \end{pmatrix} \Rightarrow \begin{pmatrix} 6a + 4b \\ -3b - 4a \end{pmatrix} = \begin{pmatrix} 8 \\ -7 \end{pmatrix}$$

and

$$6a + 4b = 8 \quad (1)$$
$$-4a - 3b = -7 \quad (2).$$

E.g., $3 \times (1)$ and $4 \times (2)$:

$$18a + 12b = 24 \quad (3)$$
$$-16a - 12b = -28 \quad (4)$$

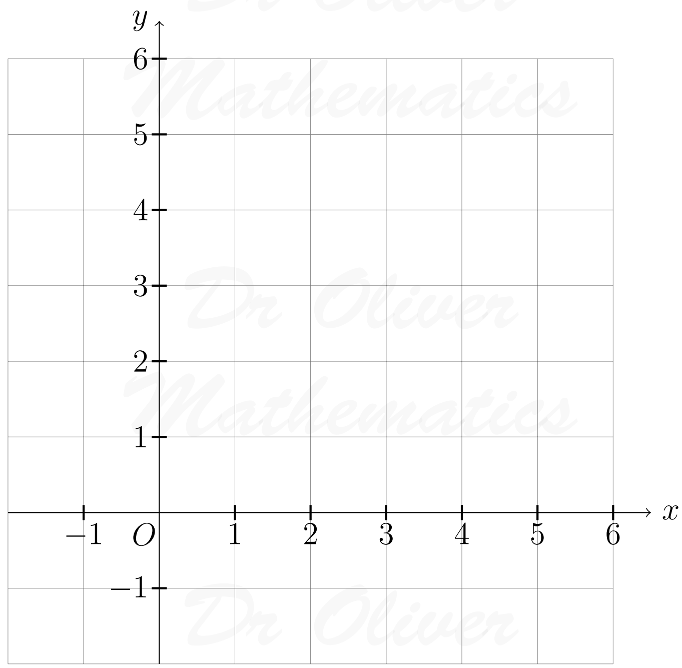
and do $(3) + (4)$:

$$2a = -4 \Rightarrow \underline{\underline{a = -2}}$$
$$\Rightarrow 6(-2) + 4b = 8$$
$$\Rightarrow -12 + 4b = 8$$
$$\Rightarrow 4b = 20$$
$$\Rightarrow \underline{\underline{b = 5}}.$$

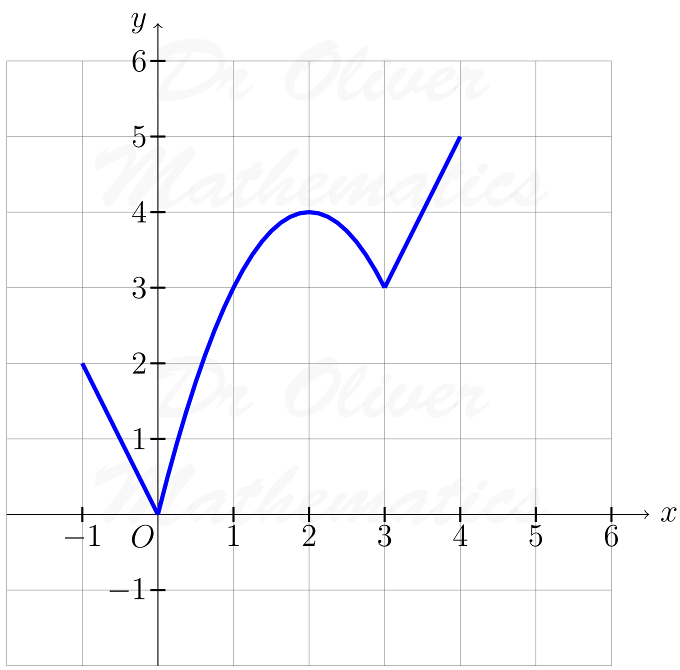
7. A function is given by (4)

$$\mathrm{f}(x) = \begin{cases} -2x & \text{for } -1 \leqslant x < 0, \\ x(4-x) & \text{for } 0 \leqslant x < 3, \\ 2x - 3 & \text{for } 3 \leqslant x \leqslant 4, \end{cases}$$

Draw the graph of $y = \mathrm{f}(x)$ on the grid.

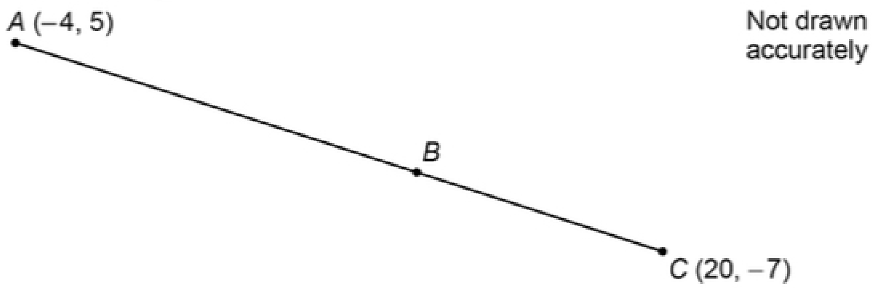


Solution



8. ABC is a straight line. (4)

- A is the point $(-4, 5)$.
- C is the point $(20, -7)$.
- $AB : BC = 5 : 3$.



Work out the coordinates of B.

Solution

Well,

$$5 + 3 = 8$$

and

$$\begin{aligned}\overrightarrow{OB} &= \overrightarrow{OA} + \overrightarrow{AB} \\ &= \overrightarrow{OA} + \tfrac{5}{8}\overrightarrow{AC} \\ &= \begin{pmatrix} -4 \\ 5 \end{pmatrix} + \tfrac{5}{8}\begin{pmatrix} 20-(-4) \\ -7-5 \end{pmatrix} \\ &= \begin{pmatrix} -4 \\ 5 \end{pmatrix} + \tfrac{5}{8}\begin{pmatrix} 24 \\ -12 \end{pmatrix} \\ &= \begin{pmatrix} -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 15 \\ -7\frac{1}{2} \end{pmatrix} \\ &= \begin{pmatrix} 11 \\ -2\frac{1}{2} \end{pmatrix};\end{aligned}$$

hence, $\underline{\underline{B(11, -2\tfrac{1}{2})}}$.

9. (1)

$$y = 2x(x^2 - 5x)$$

Circle the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$$2(2x-5) \qquad 6x^2-20 \qquad 3x^2-10x \qquad 6x^2-20x$$

Solution

$$\begin{aligned} y = 2x(x^2-5x) \Rightarrow y &= 2x^3-10x^2 \\ \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 6x^2-20x. \end{aligned}$$

Hence,

$$2(2x-5) \qquad 6x^2-20 \qquad 3x^2-10x \qquad \underline{\underline{6x^2-20x}}$$

10. Factorise fully (3)

$$6x^2+26xy-20y^2.$$

Solution

E.g.,

$$6x^2+26xy-20y^2 = 2(3x^2+13xy-10y^2)$$

$$\left.\begin{array}{ll} \text{add to:} & +13 \\ \text{multiply to:} & (+3)\times(-10) = -30 \end{array}\right\} -2,\ +15$$

$$\begin{aligned} &= 2[3x^2+15xy-2xy-10y^2] \\ &= 2[3x(x+5y)-2y(x+5y)] \\ &= \underline{\underline{2(3x-2y)(x+5y)}}. \end{aligned}$$

11. A cone has base radius r cm, perpendicular height h cm, and slant height l cm. (5)

- The curved surface area is 60π cm^2.
- $l = 3r$.

Work out the value of h.

Give your answer in the form $a\sqrt{10}$, where a is an integer greater than 1.

You **must** show your working.

Solution

$$\begin{aligned}\pi r l = 60\pi &\Rightarrow rl = 60 \\ &\Rightarrow r(3r) = 60 \\ &\Rightarrow 3r^2 = 60 \\ &\Rightarrow r^2 = 20 \\ &\Rightarrow r = 2\sqrt{5} \\ &\Rightarrow l = 6\sqrt{5}.\end{aligned}$$

Now,

$$\begin{aligned}h^2 + r^2 = l^2 &\Rightarrow h^2 + 20 = (6\sqrt{5})^2 \\ &\Rightarrow h^2 + 20 = 180 \\ &\Rightarrow h^2 = 160 \\ &\Rightarrow \underline{\underline{h = 4\sqrt{10}}};\end{aligned}$$

hence, $\underline{\underline{a = 4}}$.

12. A curve has the equation (5)

$$y = x^3 + ax^2 - 7,$$

where a is a constant.

The gradient of the curve when $x = 4$ is twice the gradient of the curve when $x = -1$.

Work out the value of a.

You **must** show your working.

Solution

Well,

$$y = x^3 + ax^2 - 7 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2ax$$

and

$$x = 4 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 48 + 8a$$
$$x = -1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3 - 2a.$$

Now, if the gradient of the curve when $x = 4$ is twice the gradient of the curve when $x = -1$,

$$\begin{aligned} 48 + 8a = 2(3 - 2a) &\Rightarrow 48 + 8a = 6 - 4a \\ &\Rightarrow 12a = -42 \\ &\Rightarrow \underline{\underline{a = -3\tfrac{1}{2}}}. \end{aligned}$$

13. Prove that (4)

$$(3x + 5)^2 - 5x(x + 10) \geqslant 0$$

for all values of x.

Solution

$\times$	$3x$	$+5$
$3x$	$9x^2$	$+15x$
$+5$	$+15x$	$+25$

Now,

$$\begin{aligned} (3x + 5)^2 - 5x(x + 10) &= 9x^2 + 30x + 25 - 5x^2 - 50x \\ &= 4x^2 - 20x + 25 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -20 \\ \text{multiply to:} & (+4) \times (-25) = -100 \end{array}\right\} -10,\ -10$$

$$\begin{aligned} &= 4x^2 - 10x - 10x + 25 \\ &= 2x(2x - 5) - 5(2x - 5) \\ &= (2x - 5)^2 \\ &\geqslant \underline{\underline{0}}, \end{aligned}$$

for all values of x.

14. Here are two transformations. (4)

- A Rotation 90° clockwise about the origin.
- B Reflection in the line $y = x$.

Use matrix multiplication to work out the single matrix which represents the combined transformation A followed by B.

Solution

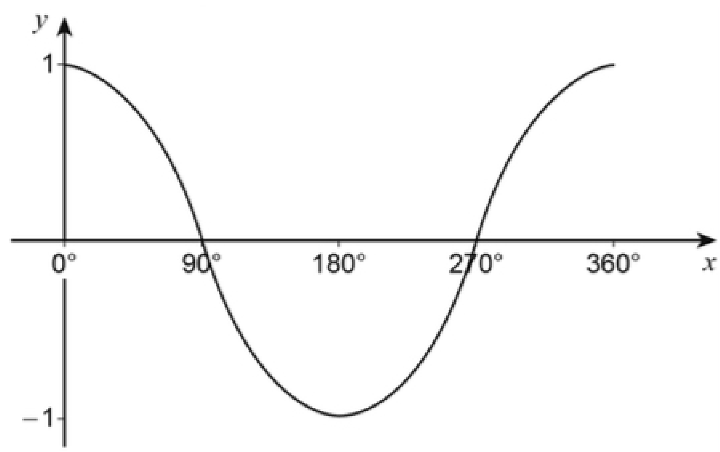
Now,

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Finally,

$$\begin{aligned} BA &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}}. \end{aligned}$$

15. Here is a sketch graph of $y = \cos x$ for $0^\circ \leqslant x \leqslant 360^\circ$. (2)



You are given that

$$\cos 36^\circ = 0.809\,0.$$

Solve

$$\cos x = -0.809\,0$$

for $0^\circ \leqslant x \leqslant 360^\circ$.

Solution

Well,

$$180 - 36 = \underline{\underline{144}}$$

and

$$180 + 36 = \underline{\underline{216}}.$$

16. Rationalise the denominator and simplify fully (4)

$$\frac{21 - 11\sqrt{5}}{3 - \sqrt{5}}.$$

Solution

$$\frac{21 - 11\sqrt{5}}{3 - \sqrt{5}} = \frac{21 - 11\sqrt{5}}{3 - \sqrt{5}} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}}$$

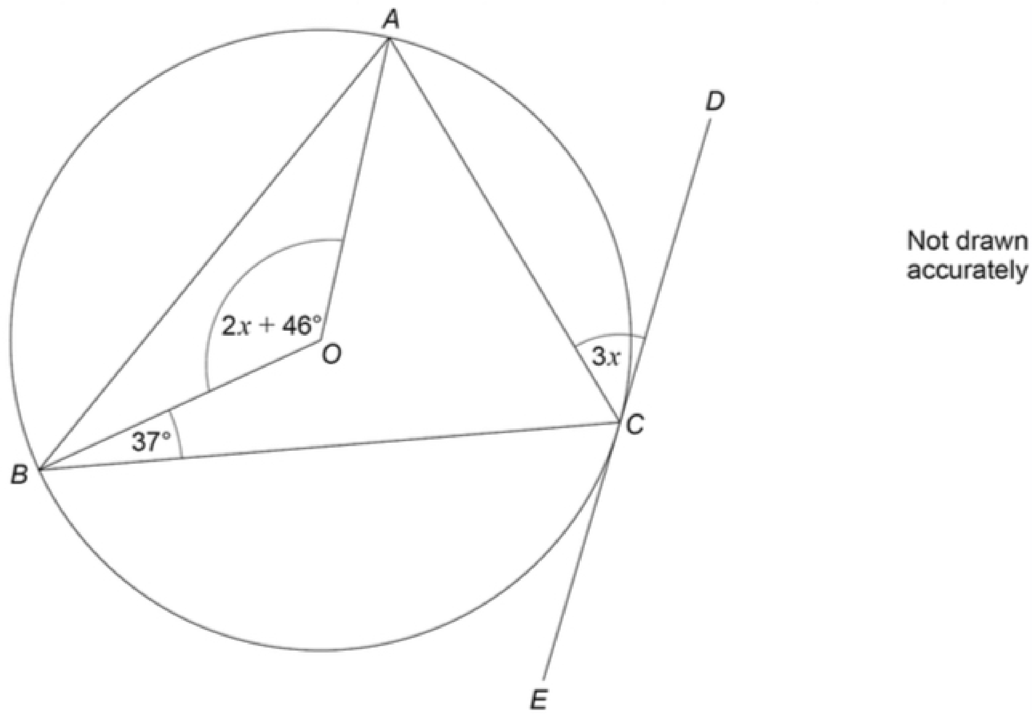
$\times$	21	$-11\sqrt{5}$
3	63	$-33\sqrt{5}$
$+\sqrt{5}$	$+21\sqrt{5}$	-55

$$= \frac{8 - 12\sqrt{5}}{3^2 - 5}$$
$$= \frac{8 - 12\sqrt{5}}{4}$$
$$= \underline{\underline{2 - 3\sqrt{5}}}.$$

17. A, B, and C are points on the circumference of a circle, centre O. (4)

- ECD is a tangent to the circle at C.
- Angle $AOB = (2x + 46)^\circ$.

- Angle $OBC = 37°$.
- Angle $ACD = (3x)°$.



Work out the value of x.

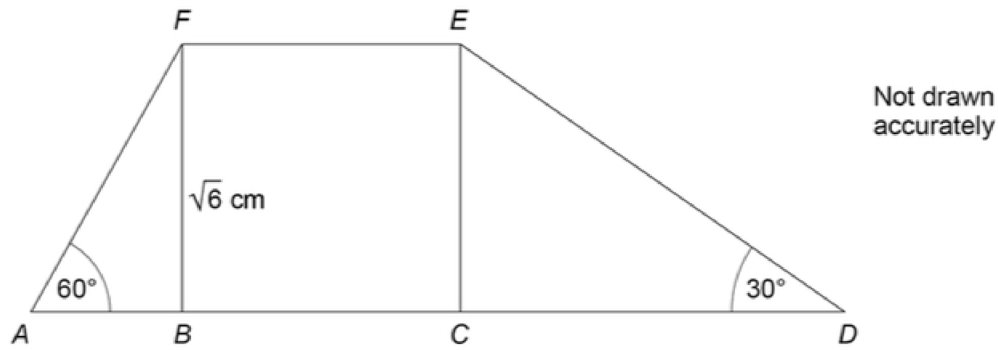
Solution

Well, $\angle ABC = 3x$ (alternate segment theorem)
$\angle ABO = 3x - 37$ (splitting up $\angle ABO$ and $\angle OBC$)
$\angle BAO = 3x - 37$ (completing the circle).
Now,

$$\begin{aligned}(3x-37)+(3x-37)+(2x+46) = 180 &\Rightarrow 8x - 28 = 180 \\ &\Rightarrow 8x = 208 \\ &\Rightarrow \underline{\underline{x = 26}}.\end{aligned}$$

18. - $ADEF$ is a trapezium.
    - $ABCD$ is a straight line.

- $BCEF$ is a square of side $\sqrt{6}$ cm.



(a) Show that $AB = \sqrt{2}$ cm. (1)

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 60^\circ = \frac{\sqrt{6}}{AB}$$
$$\Rightarrow AB = \frac{\sqrt{6}}{\tan 60^\circ}$$
$$\Rightarrow AB = \frac{\sqrt{6}}{\sqrt{3}}$$
$$\Rightarrow \underline{\underline{AB = \sqrt{2}}},$$

as required.

(b) Show that $DE = 2\sqrt{6}$ cm. (1)

Solution

$$\sin = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin 30^\circ = \frac{\sqrt{6}}{DE}$$
$$\Rightarrow DE = \frac{\sqrt{6}}{\sin 30^\circ}$$
$$\Rightarrow DE = \frac{\sqrt{6}}{\frac{1}{2}}$$
$$\Rightarrow \underline{\underline{DE = 2\sqrt{6}}},$$

as required.

(c) Work out the perimeter of the trapezium $ADEF$. (3)

Give your answer in the form $t\sqrt{2} + w\sqrt{6}$, where t and w are integers.

You **must** show your working.

Solution

Well,

$$\begin{aligned} AF^2 = AB^2 + BF^2 &\Rightarrow AF^2 = (\sqrt{2})^2 + (\sqrt{6})^2 \\ &\Rightarrow AF^2 = 2 + 6 \\ &\Rightarrow AF^2 = 8 \\ &\Rightarrow AF = 2\sqrt{2} \end{aligned}$$

and

$$\begin{aligned} DE^2 = CD^2 + CE^2 &\Rightarrow (2\sqrt{6})^2 = CD^2 + (\sqrt{6})^2 \\ &\Rightarrow 24 = CD^2 + 6 \\ &\Rightarrow CD^2 = 18 \\ &\Rightarrow CD = 3\sqrt{2}. \end{aligned}$$

Finally,

$$\begin{aligned} \text{perimeter} &= AF + FE + ED + CD + BC + AB \\ &= 2\sqrt{2} + \sqrt{6} + 2\sqrt{6} + 3\sqrt{2} + \sqrt{6} + \sqrt{2} \\ &= \underline{\underline{6\sqrt{2} + 4\sqrt{6}}}; \end{aligned}$$

hence,

$$\underline{\underline{t = 6}} \text{ and } \underline{\underline{w = 4}}.$$

19. (6)

$$\mathrm{f}(x) = \frac{x-3}{2x}.$$

Solve

$$\mathrm{f}(x+1) - \mathrm{f}(2x) = 0.5.$$

You **must** show your working.

Solution

Now,

$$\begin{aligned}\mathrm{f}(x+1) &= \frac{(x+1)-3}{2(x+1)}\\ &= \frac{x-2}{2(x+1)}\end{aligned}$$

and

$$\begin{aligned}\mathrm{f}(2x) &= \frac{(2x)-3}{2(2x)}\\ &= \frac{2x-3}{4x}.\end{aligned}$$

Next,

$$\mathrm{f}(x+1)-\mathrm{f}(2x)=0.5 \Rightarrow \frac{x-2}{2(x+1)}-\frac{2x-3}{4x}=\tfrac{1}{2}$$

$\mathrm{LCM}[2(x+1),4x]=4x(x+1)$

$$\Rightarrow \frac{2x(x-2)-(2x-3)(x+1)}{4x(x+1)}=\tfrac{1}{2}$$

$\times$	$2x$	-3
x	$2x^2$	$-3x$
$+1$	$+2x$	-3

$$\begin{aligned}&\Rightarrow \frac{(2x^2-4x)-(2x^2-x-3)}{4x(x+1)}=\tfrac{1}{2}\\ &\Rightarrow \frac{2x^2-4x-2x^2+x+3}{4x(x+1)}=\tfrac{1}{2}\\ &\Rightarrow \frac{-3x+3}{4x(x+1)}=\tfrac{1}{2}\\ &\Rightarrow 2(-3x+3)=4x(x+1)\\ &\Rightarrow -6x+6=4x^2+4x\\ &\Rightarrow 4x^2+10x-6=0\\ &\Rightarrow 2(2x^2+5x-3)=0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & +5 \\ \text{multiply to:} & (+2)\times(-3) = -6\end{array}\right\} -1,\ +6$$

$$\begin{aligned}&\Rightarrow 2[2x^2+6x-x-3]=0\\&\Rightarrow 2[2x(x+3)-1(x+3)]=0\\&\Rightarrow 2(2x-1)(x+3)=0\\&\Rightarrow \underline{\underline{x=\tfrac{1}{2}\text{ or }x=-3}}.\end{aligned}$$