Dr Oliver Mathematics
GCSE Mathematics
2006 June Paper 5H: Non-Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. $3x^2 = 108$.

   (a) Find the value of x. (2)

   Solution

   $$3x^2 = 108 \Rightarrow x^2 = 36$$
   $$\Rightarrow \underline{\underline{x = \pm 6}}.$$

   (b) Express108 as a product of its prime factors. (3)

   Solution

	108
2	54
2	27
3	9
3	3
3	1

   So
   $$108 = 2 \times 2 \times 3 \times 3 \times 3 = \underline{\underline{2^2 \times 3^3}}.$$

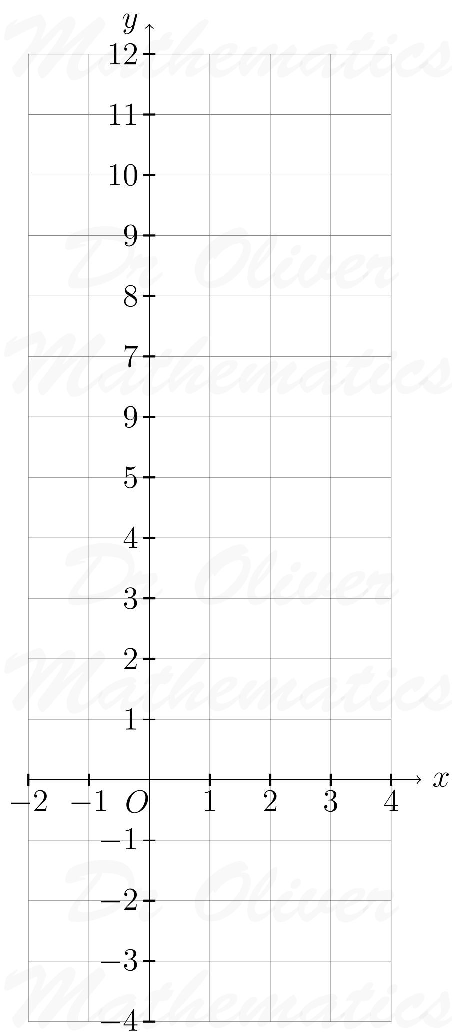
2. (a) Complete the table of values for $y = x^2 - 3x + 1$. (2)

x	-2	-1	0	1	2	3	4
y	11		1	-1			5

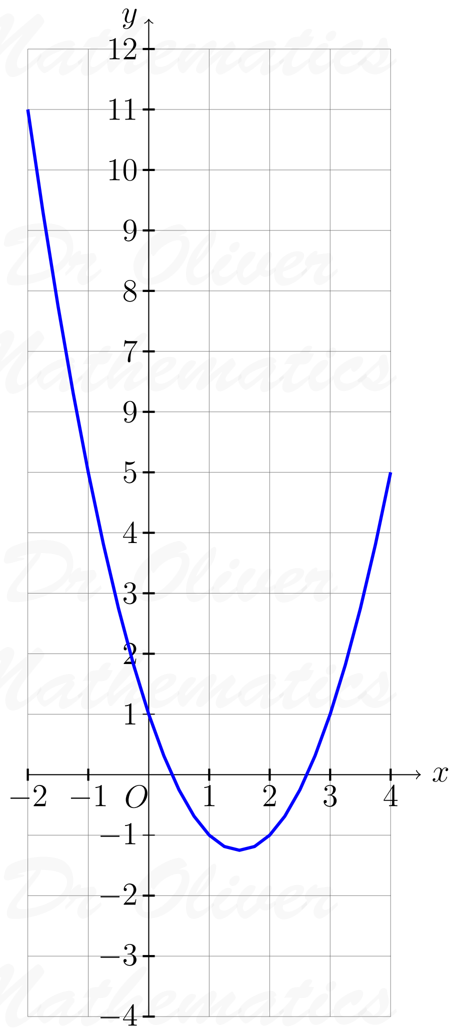
Solution

x	-2	-1	0	1	2	3	4
y	11	5	1	-1	-1	1	5

(b) On the grid, draw the graph of $y = x^2 - 3x + 1$. (2)

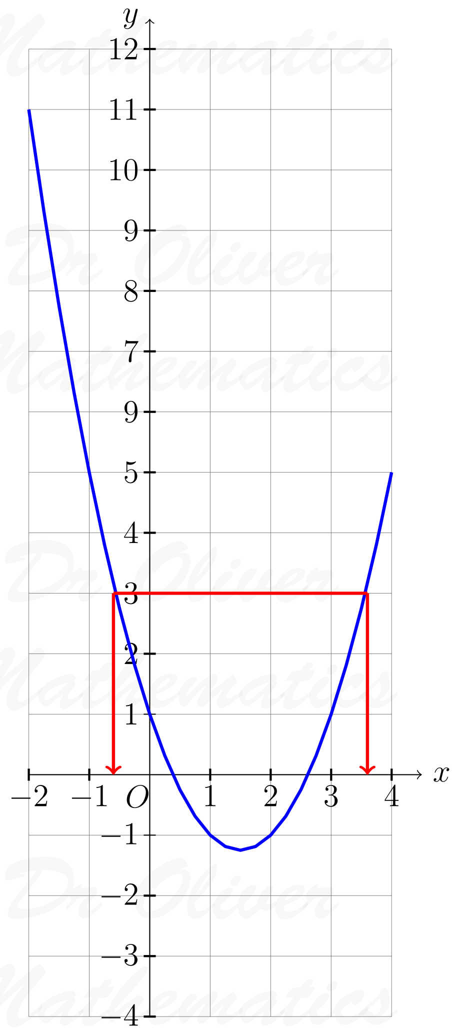


Solution



(c) Use your graph to estimate the values of x for which $y = 3$. (2)

Solution



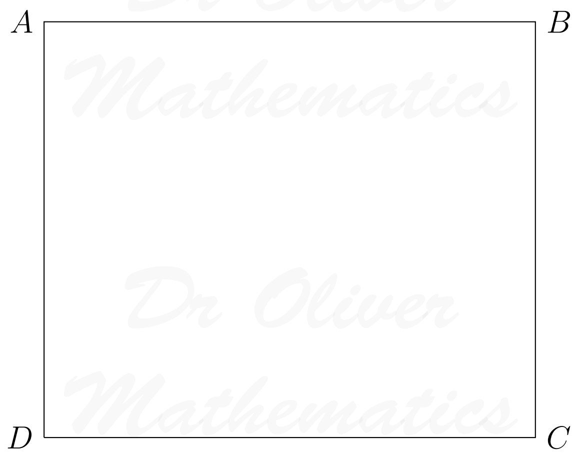
Correct read-off: approximately $\underline{\underline{x = -0.6}}$ and $\underline{\underline{x = 3.6}}$.

3. A silver chain has a volume of 5 cm^3. (2)
The density of silver is 10.5 grams per cm^3.
Work out the mass of the silver chain.

Solution

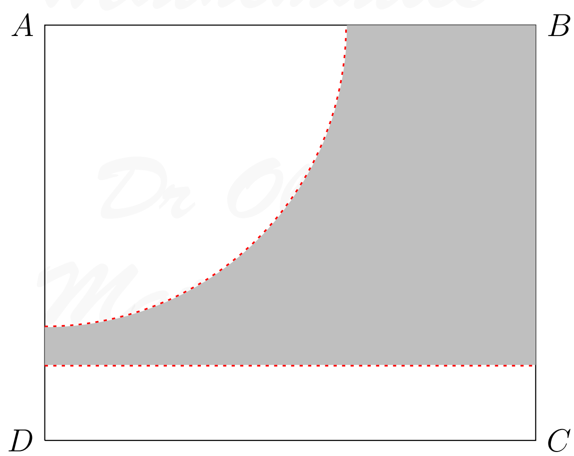
$$\text{Mass} = 10.5 \times 5 = \underline{\underline{52.5 \text{ grams}}}.$$

4. $ABCD$ is a rectangle. (4)



Shade the set of points inside the rectangle which are **both** more than 4 centimetres from the point A **and** more than 1 centimetre from the line DC.

Solution



5. Fred did a survey of the time, in seconds, people spent in a queue at a supermarket. Information about the times is shown in the table. (2)

Time (t seconds)	Frequency
$0 < t \leqslant 40$	8
$40 < t \leqslant 80$	12
$80 < t \leqslant 120$	14
$120 < t \leqslant 160$	16
$160 < t \leqslant 200$	10

A person is selected at random from the people in Fred's survey.
Work out an estimate for the probability that the person selected spent more than 120 seconds in the queue.

Solution

Time (t seconds)	Frequency
$0 < t \leqslant 40$	8
$40 < t \leqslant 80$	12
$80 < t \leqslant 120$	14
$120 < t \leqslant 160$	16
$160 < t \leqslant 200$	10
Total	60

Hence, the probability that the person selected spent more than 120 seconds in the queue is

$$\tfrac{26}{60} = \underline{\underline{\tfrac{13}{30}}}.$$

6. Work out an estimate for (3)

$$\frac{412 \times 5.904}{0.195}.$$

Solution

Use 1 significant figure:

$$\begin{aligned}\frac{412 \times 5.904}{0.195} &\approx \frac{400 \times 6}{0.2} \\ &= \frac{2\,400}{0.2} \\ &= \underline{\underline{12\,000}}.\end{aligned}$$

7. A gold necklace has a mass of 127 grams, correct to the nearest gram.

   (a) Write down the **least** possible mass of the necklace. (1)

Solution

126.5 grams.

(b) Write down the **greatest** possible mass of the necklace. (1)

Solution

127.5 grams.

8. A student wanted to find out how many pizzas adults ate. (2)
He used this question on a questionnaire.

How many pizzas have you eaten?

☐ A few ☐ A lot

This is not a good question.
Design a better question that the student can use to find out how many pizzas adults ate.
You should include some response boxes.

Solution

A suitable question with a time frame, e.g., "Did you eat pizza today/last week/last month? Tick the appropriate box."
At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, 0, 1-3, 4-6, 7 or more.

9. Write in standard form

(a) 456 000, (1)

Solution

$$456\,000 = \underline{\underline{4.56 \times 10^5}}.$$

(b) 0.000 34, (1)

Solution

$$0.000\,34 = \underline{\underline{3.4 \times 10^{-4}}}.$$

(c) 16×10^7. (1)

Solution

$$16 \times 10^7 = \underline{\underline{1.6 \times 10^8}}.$$

10. (a) Factorise (2)

$$x^2 + 6x + 8.$$

Solution

$$\left.\begin{array}{ll}\text{add to:} & +6 \\ \text{multiply to:} & +8\end{array}\right\} +2,\ +4$$

$$x^2 + 6x + 8 = \underline{\underline{(x+2)(x+4)}}.$$

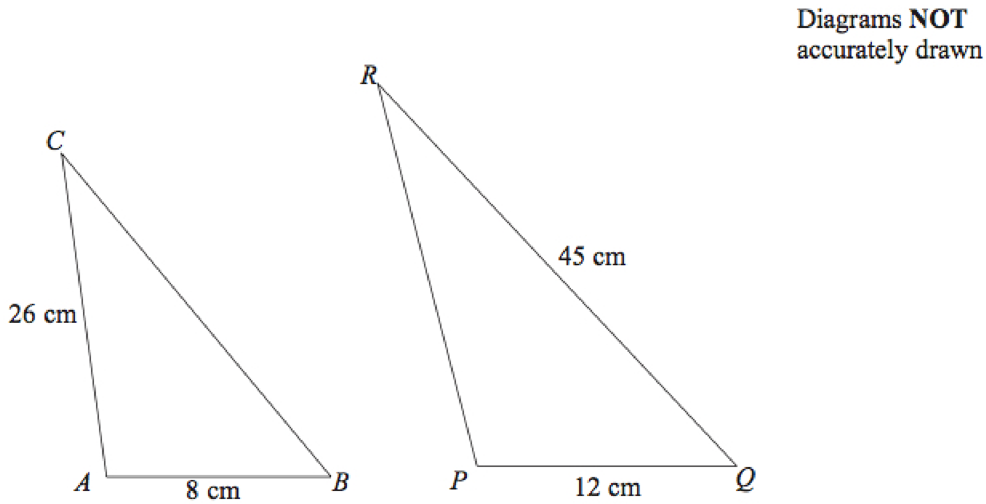
(b) Solve (1)

$$x^2 + 6x + 8 = 0.$$

Solution

$$\begin{aligned} x^2 + 6x + 8 = 0 &\Rightarrow (x+2)(x+4) = 0 \\ &\Rightarrow \underline{\underline{x = -2 \text{ or } x = -4}}. \end{aligned}$$

11. The two triangles ABC and PQR are mathematically similar.



Angle A = angle P.
Angle B = angle Q.
$AB = 8$ cm.
$AC = 26$ cm.
$PQ = 12$ cm.
$QR = 45$ cm.

(a) Work out the length of PR. (2)

Solution

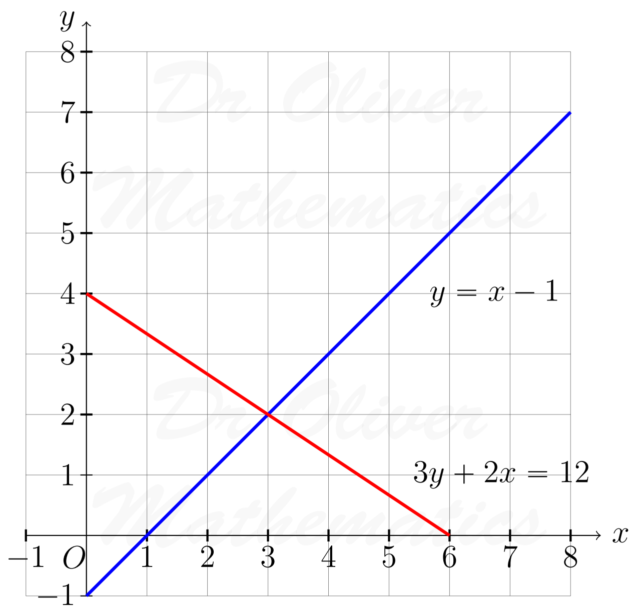
$$\frac{PR}{PQ} = \frac{AC}{AB} \Rightarrow \frac{PR}{12} = \frac{26}{8}$$
$$\Rightarrow PR = \frac{26 \times 12}{8}$$
$$\Rightarrow PR = \frac{26 \times 3}{2}$$
$$\Rightarrow PR = 13 \times 3$$
$$\Rightarrow \underline{\underline{PR = 39 \text{ cm}}}$$

(b) Work out the length of BC. (2)

Solution

$$\frac{BC}{AB} = \frac{QR}{PQ} \Rightarrow \frac{BC}{8} = \frac{45}{12}$$
$$\Rightarrow BC = \frac{45 \times 8}{12}$$
$$\Rightarrow BC = \frac{360}{12}$$
$$\Rightarrow \underline{\underline{BC = 30 \text{ cm}}}$$

12. The graphs of the straight lines with equations $3y + 2x = 12$ and $y = x - 1$ have been drawn on the grid.



(a) Use the graphs to solve the simultaneous equations (1)

$$3y + 2x = 12$$
$$y = x - 1.$$

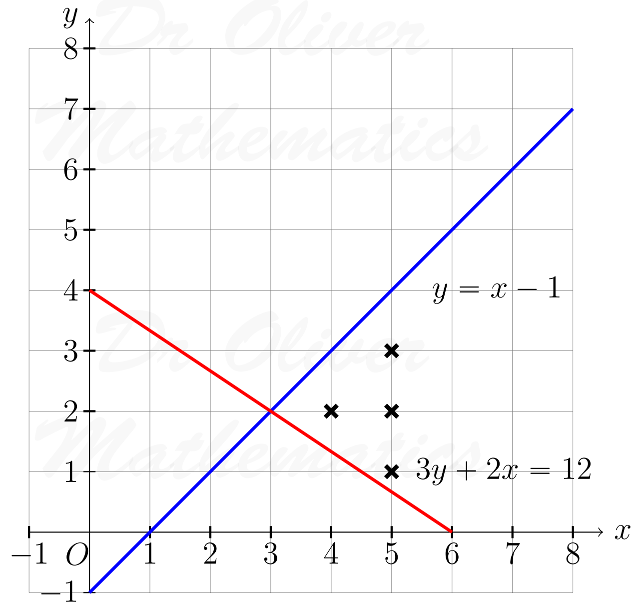
Solution

$\underline{\underline{x = 3, y = 2}}$.

(b) $3y + 2x > 12$. (3)
$y < x - 1$.

$x < 6$.
x and y are integers.
On the grid, mark with a cross (✓) each of the four points which satisfies all these 3 inequalities.

Solution



13. Hajra's weekly pay this year is £240.
This is 20% more than her weekly pay last year.
Bill says, "This means Hajra's weekly pay last year was £192."
Bill is wrong.

(a) Explain why. (1)

Solution

$\times$	100	90	2
1	100	90	2
0.2	20	18	0.4

Bill is wrong because

$$192 \times 1.2 = £230.40.$$

(b) Workout Hajra's weekly pay last year. (2)

Solution

$$\frac{240}{1.2} = \underline{\underline{\pounds 200}}.$$

14. A company tested 100 batteries.
The table shows information about the number of hours that the batteries lasted.

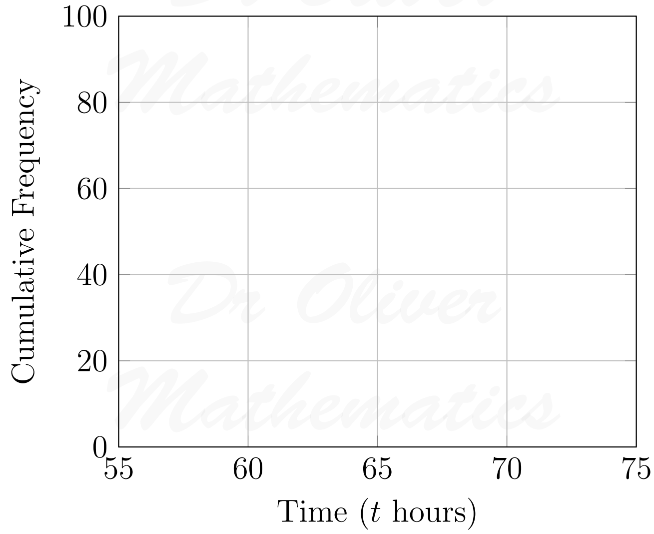
Time (t hours)	Frequency
$50 < t \leqslant 55$	12
$55 < t \leqslant 60$	21
$60 < t \leqslant 65$	36
$65 < t \leqslant 70$	23
$70 < t \leqslant 75$	8

(a) Complete the cumulative frequency table for this information. (1)

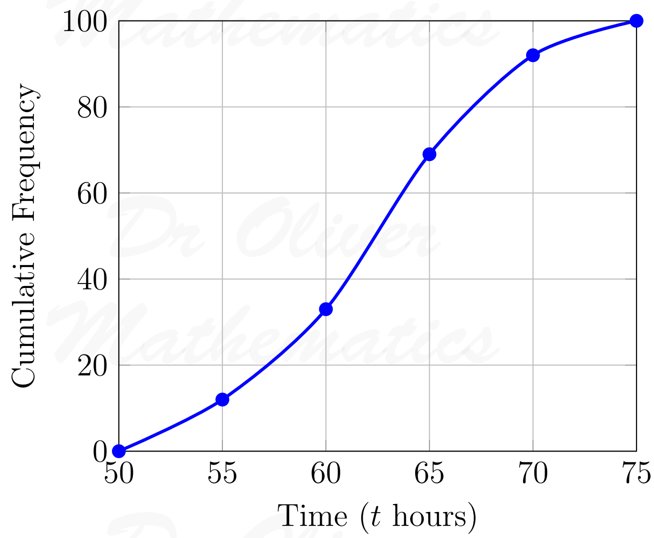
Solution

Time (t hours)	Cumulative Frequency
$50 < t \leqslant 55$	$\underline{\underline{12}}$
$50 < t \leqslant 60$	$12 + 21 = \underline{\underline{33}}$
$50 < t \leqslant 65$	$33 + 36 = \underline{\underline{69}}$
$50 < t \leqslant 70$	$69 + 23 = \underline{\underline{92}}$
$50 < t \leqslant 75$	$92 + 8 = \underline{\underline{100}}$

(b) On the grid, draw a cumulative frequency graph for your completed table. (2)

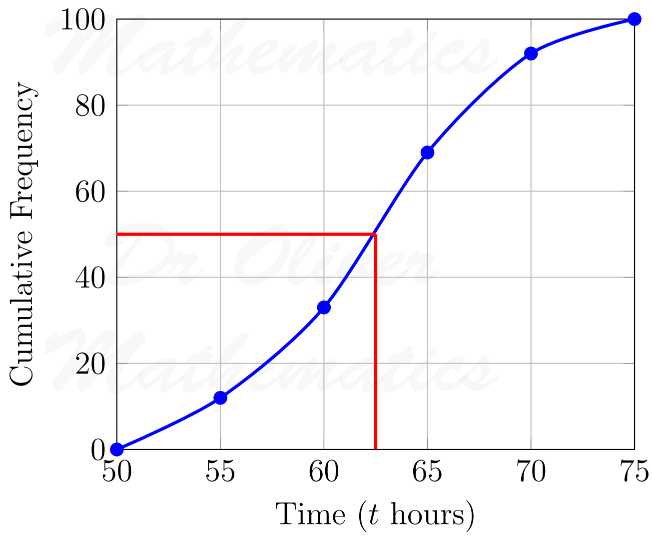


Solution



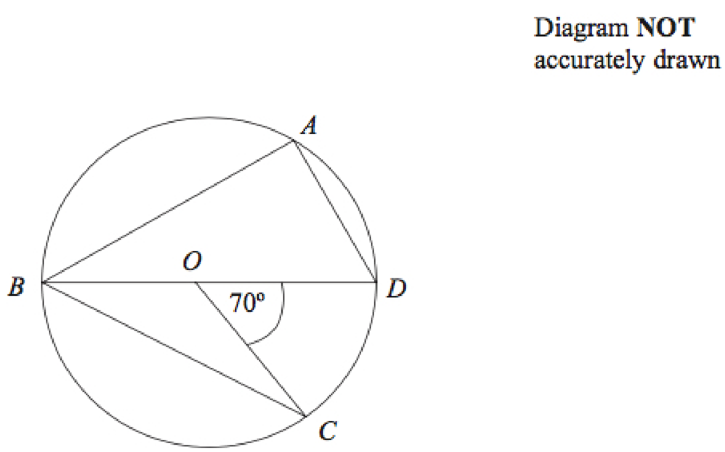
(c) Use your completed graph to find an estimate for the median time. You must state the units of your answer. (2)

Solution



Correct read-off from 50 or 50.5: approximately 62.5 hours.

15. A, B, C, and D are points on the circumference of a circle, centre O.



BOD is a straight line.
Angle $COD = 70°$.

(a) Find the size of angle BAD. (2)
Give a reason for your answer.

Solution

Angle BAD = 90° (angle in a semi-circle).

(b) Find the size of angle CBD. (2)
Give a reason for your answer.

> **Solution**
>
> Angle $BOD = 110°$ (supplementary angles).
> Angle $CBD = \underline{\underline{35°}}$ (angles in a triangle, base angles in a triangle).

16. The time, T seconds, it takes a water heater to boil some water is directly proportional to the mass of water, m kg, in the water heater.
When $m = 250$, $T = 600$.

(a) Find T when $m = 400$. (3)

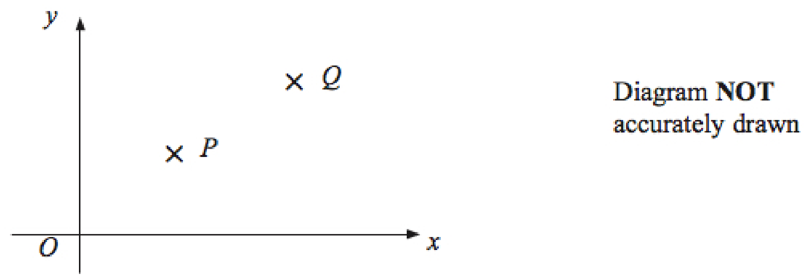
> **Solution**
>
> $$T \propto m \Rightarrow T = km$$
>
> for some k. Now,
>
> $$600 = k \times 250 \Rightarrow k = \tfrac{600}{250} = \tfrac{12}{5}$$
>
> and so
>
> $$T = \tfrac{12}{5}m.$$
>
> Finally,
>
> $$T = \tfrac{12}{5} \times 400 = \underline{\underline{960 \text{ seconds}}}.$$

The time, T seconds, it takes a water heater to boil a constant mass of water is inversely proportional to the power, P watts, of the water heater.
When $P = 1400$, $T = 360$.

(b) Find the value of T when $P = 900$. (3)

> **Solution**
>
> $$T \propto \frac{1}{P} \Rightarrow T = \frac{l}{P}$$
>
> for some l. Now,
>
> $$360 = \frac{l}{1\,400} \Rightarrow l = 504\,000$$
>
> and so
>
> $$T = \frac{504\,000}{P}.$$
>
> Finally,
>
> $$T = \frac{504\,000}{900} = \frac{560 \times 900}{900} = \underline{\underline{560 \text{ seconds}}}.$$

17. The diagram is a sketch.



P is the point $(2, 3)$.
Q is the point $(6, 6)$.

(a) Write down the vector $\overrightarrow{PQ}$. (2)

Write your answer as a column vector $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution

$$\begin{pmatrix} 6 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 4 \\ 3 \end{pmatrix}}}.$$

$PQRS$ is a parallelogram.
$\overrightarrow{PR} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$.

(b) Find the vector $\overrightarrow{QS}$. (2)

Write your answer as a column vector $\begin{pmatrix} x \\ y \end{pmatrix}$.

Solution

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$$

and

$$\overrightarrow{OS} = \overrightarrow{OR} + \overrightarrow{RS} = \begin{pmatrix} 6 \\ 10 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}.$$

Finally,

$$\overrightarrow{QS} = \overrightarrow{QO} + \overrightarrow{OS} = -\begin{pmatrix} 6 \\ 6 \end{pmatrix} + \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -4 \\ 1 \end{pmatrix}}}.$$

18. (a) Solve (2)

$$\frac{3}{x} + \frac{3}{2x} = 2.$$

Solution

Times by $2x$:

$$\begin{aligned}\frac{3}{x} + \frac{3}{2x} = 2 &\Rightarrow 6 + 3 = 4x \\ &\Rightarrow 4x = 9 \\ &\Rightarrow \underline{\underline{x = 2\tfrac{1}{4}}}.\end{aligned}$$

(b) Using your answer to part (a), or otherwise, solve (3)

$$\frac{3}{(y-1)^2} + \frac{3}{2(y-1)^2} = 2.$$

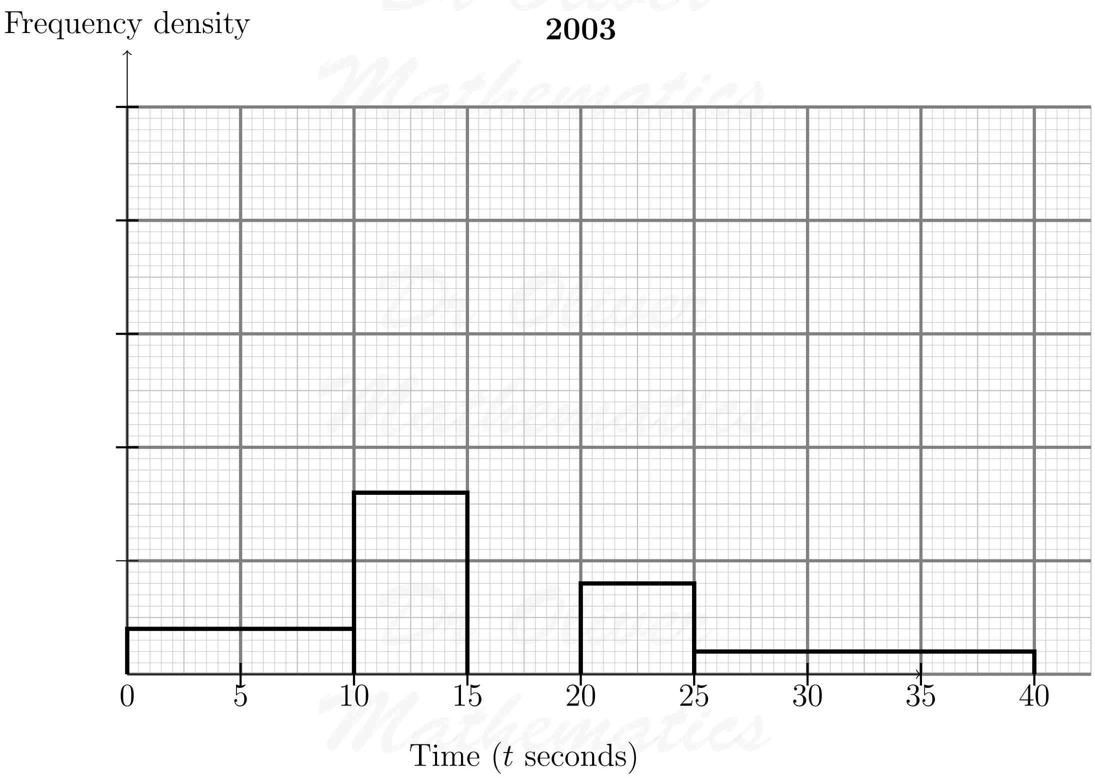
Solution

$$\begin{aligned}\frac{3}{(y-1)^2} + \frac{3}{2(y-1)^2} = 2 &\Rightarrow (y-1)^2 = 2\tfrac{1}{4} \\ &\Rightarrow y - 1 = \pm 1\tfrac{1}{2} \\ &\Rightarrow \underline{\underline{y = -\tfrac{1}{2} \text{ or } y = 2\tfrac{1}{2}}}.\end{aligned}$$

19. The table and histogram show information about the length of time it took 165 adults to connect to the internet.

Time (t seconds)	Frequency
$0 < t \leqslant 10$	20
$10 < t \leqslant 15$	
$15 < t \leqslant 17.5$	30
$17.5 < t \leqslant 20$	40
$20 < t \leqslant 25$	
$25 < t \leqslant 40$	

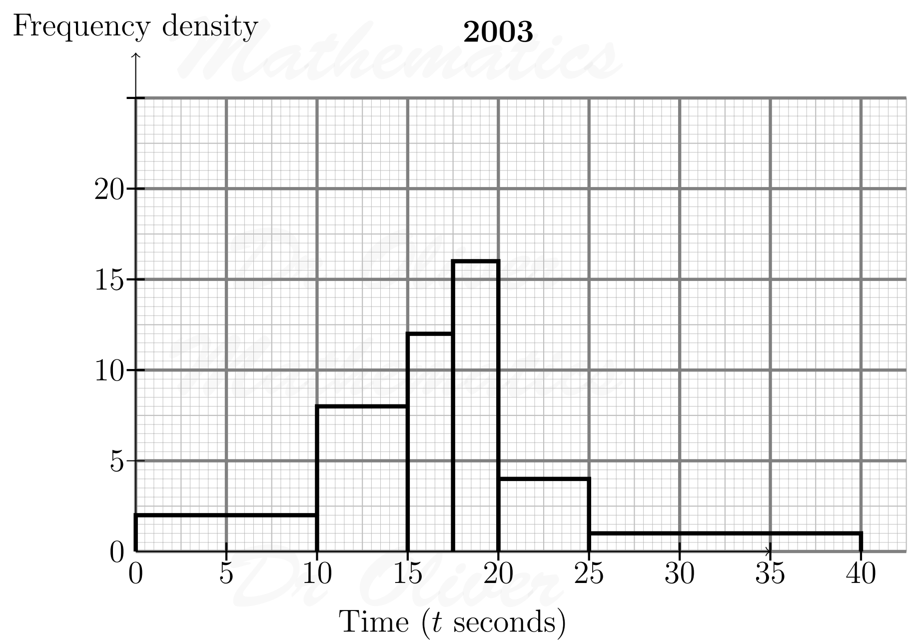
None of the adults took more than 40 seconds to connect to the internet.



(a) Use the table to complete the histogram. (2)

Solution

Time (t seconds)	Frequency	Width	Frequency Density
$0 < t \leqslant 10$	20	10	$\frac{20}{10} = 2$
$10 < t \leqslant 15$		5	$\frac{40}{5} = 8$
$15 < t \leqslant 17.5$	30	2.5	$\frac{30}{2.5} = \underline{\underline{\underline{12}}}$
$17.5 < t \leqslant 20$	40	2.5	$\frac{40}{2.5} = \underline{\underline{\underline{16}}}$
$20 < t \leqslant 25$		5	$\frac{20}{5} = 4$
$25 < t \leqslant 40$		15	$\frac{15}{15} = 1$

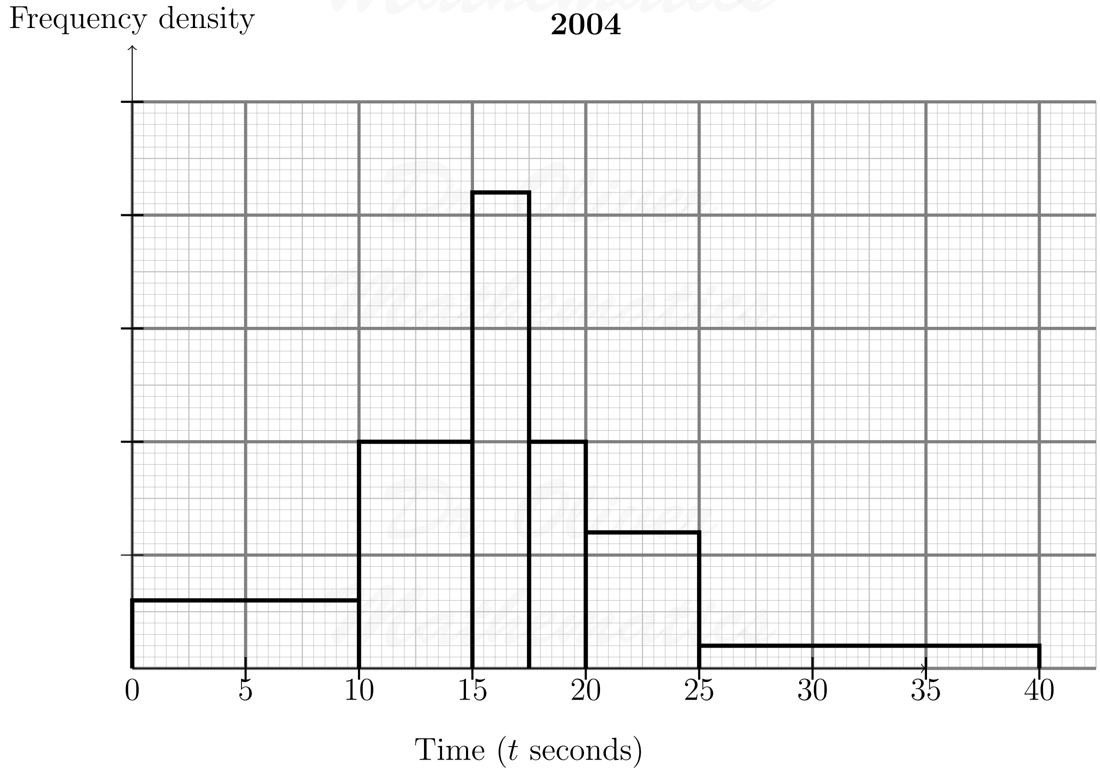


(b) Use the histogram to complete the table. (2)

Solution

Time (t seconds)	Frequency	Width	Frequency Density
$0 < t \leqslant 10$	20	10	$\frac{20}{10} = 2$
$10 < t \leqslant 15$	$\underline{\underline{40}}$	5	$\frac{40}{5} = 8$
$15 < t \leqslant 17.5$	30	2.5	$\frac{30}{2.5} = 12$
$17.5 < t \leqslant 20$	40	2.5	$\frac{40}{2.5} = 16$
$20 < t \leqslant 25$	$\underline{\underline{20}}$	5	$\frac{20}{5} = 4$
$25 < t \leqslant 40$	$\underline{\underline{15}}$	15	$\frac{15}{15} = 1$

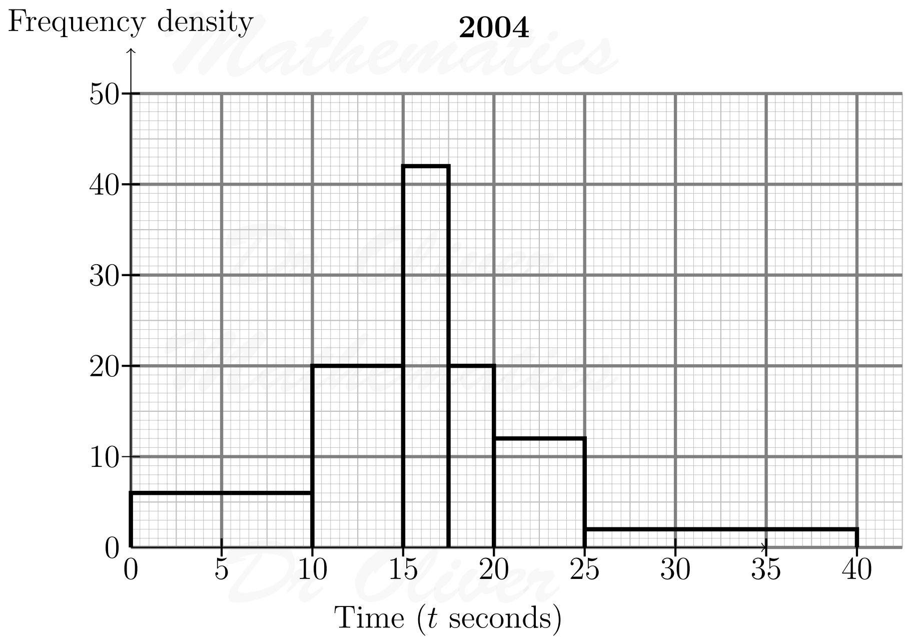
The histogram shows information about the time it took some children to connect to the internet.



None of the children took more than 40 seconds to connect to the internet.
110 children took up to 12.5 seconds to connect to the internet.

(c) Work out an estimate for the number of children who took 21 seconds or more to connect to the internet. (3)

Solution
So
$$110 \leftrightarrow (10 \times 0.6) + (2.5 \times 2) = 11$$
and so the histogram looks like this



An estimate for the number of children who took 21 seconds or more to connect to the internet is

$$(4 \times 12) + (15 \times 2) = 48 + 30 = \underline{\underline{78}}.$$

20. (a) Write down the value of $8^{\frac{1}{3}}$. (1)

Solution

$$8^{\frac{1}{3}} = \sqrt[3]{8} = \underline{\underline{2}}.$$

$8\sqrt{8}$ can be written in the form 8^k.

(b) Find the value of k. (1)

Solution

$$8\sqrt{8} = 8 \times 8^{\frac{1}{2}} = 8^{\frac{3}{2}}$$

and so $\underline{\underline{k = \frac{3}{2}}}$.

$8\sqrt{8}$ can also be expressed in the form $m\sqrt{2}$ where m is a positive integer.

(c) Express $8\sqrt{8}$ in the form $m\sqrt{2}$. (2)

Solution

$$\begin{aligned}8\sqrt{8} &= 2^3 \times 2\sqrt{2} \\ &= \underline{\underline{16\sqrt{2}}}.\end{aligned}$$

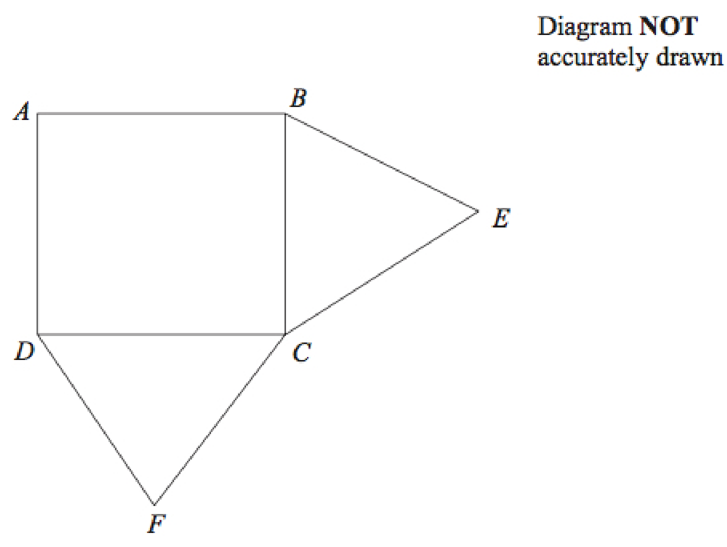
(d) Rationalise the denominator of $\frac{1}{8\sqrt{8}}$. (2)

Give your answer in the form $\frac{\sqrt{2}}{p}$ where p is a positive integer.

Solution

$$\begin{aligned}\frac{1}{8\sqrt{8}} &= \frac{1}{16\sqrt{2}} \\ &= \frac{1}{16\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \underline{\underline{\frac{\sqrt{2}}{32}}}.\end{aligned}$$

21. $ABCD$ is a square.



BEC and DCF are equilateral triangles.

(a) Prove that triangle ECD is congruent to triangle BCF. (3)

Solution

$BC = CD$ (square).
$CF = CE$ (equilateral triangle).
$\angle BCF = 90^\circ + 60^\circ = \angle DCE$.
So $\underline{\underline{\text{triangle } ECD \text{ is congruent to triangle } BCF}}$ (SAS).

G is the point such that $BEGF$ is a parallelogram.

(b) Prove that $ED = EG$. (2)

Solution

$BF = ED$ (congruent triangles) and $BF = EG$ (opposite sides of parallelogram).
Hene $\underline{\underline{ED = EG}}$.

22. (4)

$$P = \frac{n^2 + a}{n + a}.$$

Rearrange the formula to make a the subject.

Solution

$$\begin{aligned} P = \frac{n^2 + a}{n + a} &\Rightarrow P(n + a) = n^2 + a \\ &\Rightarrow nP + aP = n^2 + a \\ &\Rightarrow aP - a = n^2 - nP \\ &\Rightarrow a(P - 1) = n(n - P) \\ &\Rightarrow \underline{\underline{a = \frac{n(n - P)}{P - 1}}}. \end{aligned}$$

23. (a) Factorise (2)

$$2x^2 - 7x + 6.$$

Solution

$$\begin{array}{ll} \text{add to:} & -7 \\ \text{multiply to:} & (+2)\times(+6) = +12 \end{array} \Bigg\} -4,\ -3$$

$$\begin{aligned} 2x^2 - 7x + 6 &= 2x^2 - 4x - 3x + 6 \\ &= 2x(x-2) - 3(x-2) \\ &= \underline{\underline{(2x-3)(x-2)}}. \end{aligned}$$

(b) (i) Factorise fully (4)

$$(n^2 - a^2) - (n-a)^2.$$

Solution

$$\begin{aligned} (n^2 - a^2) - (n-a)^2 &= (n^2 - a^2) - (n^2 - 2an + a^2) \\ &= 2an - 2a^2 \\ &= \underline{\underline{2a(n-a)}}. \end{aligned}$$

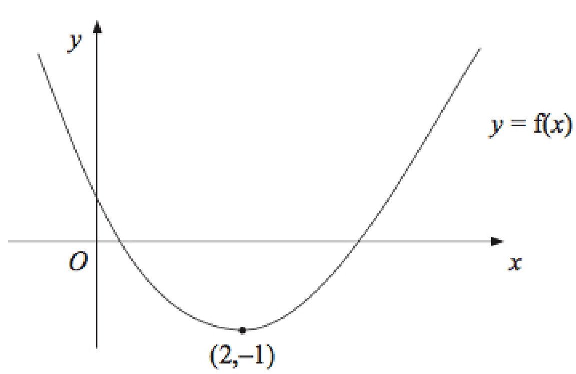
n and a are integers.

(ii) Explain why $(n^2 - a^2) - (n-a)^2$ is always an even integer.

Solution

Because $2 \times a(n-a) = 2b$, $b \in \mathbb{Z}$, is $\underline{\underline{\text{always an even integer}}}$.

24. The diagram shows part of the curve with equation $y = \mathrm{f}(x)$.



The minimum point of the curve is at $(2, -1)$.

(a) Write down the coordinates of the minimum point of the curve with equation (3)

(i) $y = \mathrm{f}(x + 2)$,

Solution
$\underline{\underline{(0, -1)}}$.

(ii) $y = 3\,\mathrm{f}(x)$,

Solution
$\underline{\underline{(2, -3)}}$.

(iii) $y = \mathrm{f}(2x)$.

Solution
$\underline{\underline{(1, -1)}}$.

The curve $y = \mathrm{f}(x)$ is reflected in the y-axis.

(b) Find the equation of the curve following this transformation. (1)

Solution
$\underline{\underline{y = \mathrm{f}(-x)}}$.

The curve with equation $y = \mathrm{f}(x)$ has been transformed to give the curve with equation $y = \mathrm{f}(x) + 2$.

(c) Describe the transformation. (1)

Solution
$\underline{\underline{\text{Translation}}}$, by $\underline{\underline{\begin{pmatrix} 0 \\ 2 \end{pmatrix}}}$.