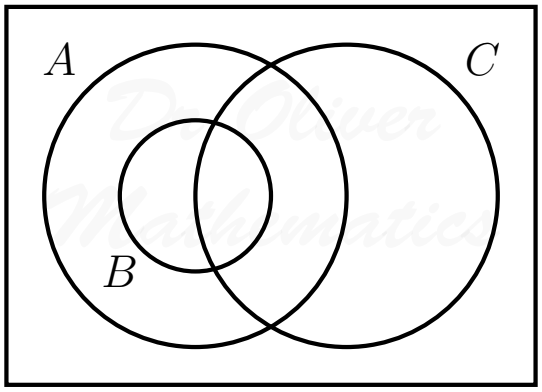
Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2007 June Paper 1: Calculator
2 hours

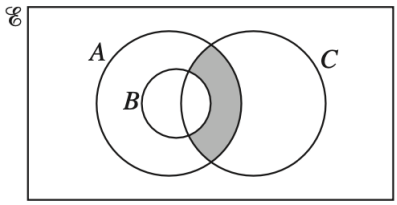
The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. (a) The diagram above shows a universal set $\mathscr{E}$ and the three sets A, B, and C.

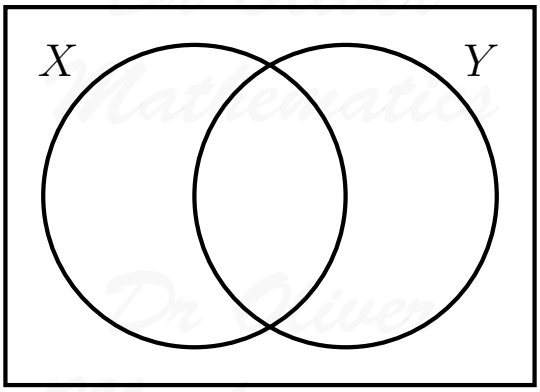


(i) Copy the above diagram and shade the region representing $(A \cup C') \cup B$. (1)

(ii) Express, in set notation, the set represented by the shaded region in the diagram below. (1)



(b) The diagram shows a universal set $\mathscr{E}$ and the sets X and Y. (2)



Show, by means of two diagrams, that the set $(X \cup Y)'$ is not the same as the set $X' \cup Y'$.

2. Find the equation of the normal to the curve (5)

$$y = \frac{2x + 4}{x - 2}$$

at the point where $x = 4$.

3. The straight line (6)

$$3x = 2y + 18$$

intersects the curve

$$2x^2 - 23x + 2y + 50 = 0$$

at the points A and B.

Given that A lies below the x-axis and that the point P lies on AB such that

$$AP : PB = 1 : 2,$$

find the coordinates of P.

4. (a) Find the first three terms, in ascending powers of u, in the expansion of (2)

$$(2 + u)^5.$$

(b) By replacing u with (4)

$$2x - 5x^2,$$

find the coefficient of x^2 in the expansion of

$$(2 + 2x - 5x^2)^5.$$

5. A curve has the equation

$$y = \sqrt{x} + \frac{9}{\sqrt{x}}.$$

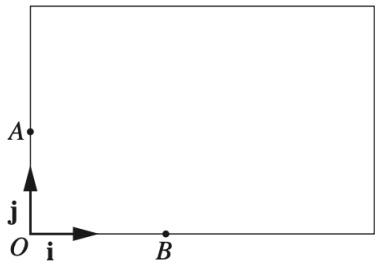
(a) Find expressions for (4)

$$\frac{\mathrm{d}y}{\mathrm{d}x} \text{ and } \frac{\mathrm{d}^2y}{\mathrm{d}x^2}.$$

(b) Show that the curve has a stationary value when $x = 9$. (1)

(c) Find the nature of this stationary value. (2)

6. The diagram shows a large rectangular television screen in which one corner is taken as the origin O and $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along two of the edges.



In a game, an alien spacecraft appears at the point A with position vector $12\mathbf{j}$ cm and moves across the screen with velocity $(40\mathbf{i} + 15\mathbf{j})$ cm per second.

A player fires a missile from a point B; the missile is fired 0.5 seconds after the spacecraft appears on the screen.

The point B has position vector $46\mathbf{i}$ cm and the velocity of the missile is $(k\mathbf{i} + 30\mathbf{j})$ cm per second, where k is a constant.

Given that the missile hits the spacecraft,

(a) show that the spacecraft moved across the screen for 1.8 seconds before impact, (4)

(b) find the value of k. (3)

7. (a) Use the substitution $u = 5^x$ to solve the equation (5)

$$5^{x+1} = 8 + 4(5^{-x}).$$

(b) Given that (3)

$$\log(p - q) = \log p - \log q,$$

express p in terms of q.

8. (a) Solve, for $0 \leqslant x \leqslant 2$, the equation (3)

$$1 + 5\cos 3x = 0,$$

giving your answer in radians correct to 2 decimal places.

(b) Find all the angles between 0° and 360° such that (5)

$$\sec y + 5\tan y = 3\cos y.$$

9. The table below shows experimental values of the variables x and y.

x	0.100	0.125	0.160	0.200	0.400
y	0.050	0.064	0.085	0.111	0.286

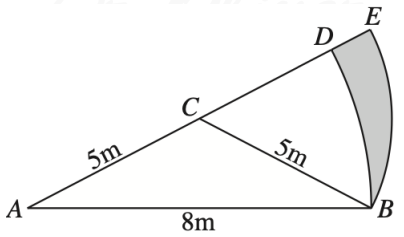
(a) On graph paper draw the graph of $\frac{1}{y}$ against $\frac{1}{x}$. (3)

Hence,

(b) express y in terms of x, (4)

(c) find the value of x for which $y = 0.15$. (2)

10. The diagram shows an isosceles triangle ABC in which $AB = 8$ m and $BC = CA = 5$ m.



- $ABDA$ is a sector of the circle, centre A and radius 8 m.
- $CBEC$ is a sector of the circle, centre C and radius 5 m.

(a) Show that angle BCE is 1.287 radians, correct to 3 decimal places. (2)

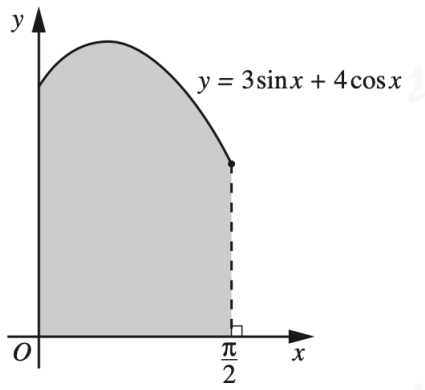
(b) Find the perimeter of the shaded region. (4)

(c) Find the area of the shaded region. (4)

EITHER

11. The graph shows part of the curve

$$y = 3\sin x + 4\cos x \text{ for } 0 \leqslant x \leqslant \tfrac{1}{2}\pi \text{ radians.}$$



(a) Find the coordinates of the maximum point of the curve. (5)

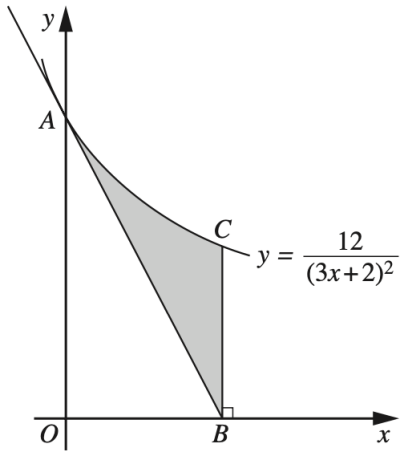
(b) Find the area of the shaded region. (5)

OR

12. The diagram, which is not drawn to scale, shows part of the curve

$$y = \frac{12}{(3x+2)^2},$$

intersecting the y-axis at A.



The tangent to the curve at A meets the x-axis at B.
The point C lies on the curve and BC is parallel to the y-axis.

(a) Find the x-coordinate of B. (4)

(b) Find the area of the shaded region. (6)