Dr Oliver Mathematics
GCSE Mathematics
2010 November Paper 3H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. A box contains milk chocolates and dark chocolates only. (2)
   The number of milk chocolates to the number of dark chocolates is in the ratio 2 : 1.
   There are 24 milk chocolates.
   Work out the total number of chocolates.

   Solution

   $2 + 1 = 3$ and so the total number of chocolates is

   $$\tfrac{3}{2} \times 24 = \underline{\underline{36}}.$$

2. (a) Simplify (1)
   $$p \times p \times p \times p.$$

   Solution
   $$p \times p \times p \times p = \underline{\underline{p^4}}.$$
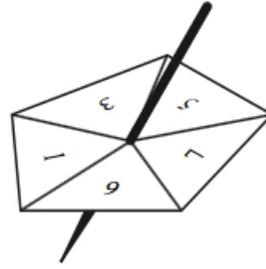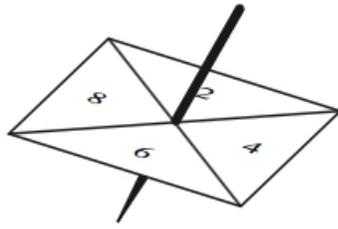
   (b) Simplify (1)
   $$2c \times 3d.$$

   Solution
   $$2c \times 3d = \underline{\underline{6cd}}.$$

3. Louise spins a four-sided spinner and a five-sided spinner.

The four-sided spinner is labelled 2, 4, 6, and 8.
The five-sided spinner is labelled 1, 3, 5, 7, and 9.
Louise adds the score on the four-sided spinner to the score on the five-sided spinner.
She records the possible total scores in a table.

+	2	4	6	8
1	3	5	7	9
3	5	7	9	11
5	7	9	11	13
7	9	11		
9	11	13		

(a) Complete the table of possible total scores. (1)

> **Solution**
>
> | + | 2 | 4 | 6 | 8 |
> |---|---|---|---|---|
> | 1 | 3 | 5 | 7 | 9 |
> | 3 | 5 | 7 | 9 | 11 |
> | 5 | 7 | 9 | 11 | 13 |
> | 7 | 9 | 11 | $\underline{\underline{13}}$ | $\underline{\underline{15}}$ |
> | 9 | 11 | 13 | $\underline{\underline{15}}$ | $\underline{\underline{17}}$ |

(b) Write down all the ways in which Louise can get a total score of 11. (2)
One way has been done for you: $(2, 9)$.

> **Solution**
>
> $$(4, 7), (6, 5), \text{ and } (8, 3).$$

Both spinners are fair.

(c) Find the probability that Louise's total score is less than 6. (2)

Solution
$$\text{Probability} = \underline{\underline{\tfrac{3}{20}}}.$$

4. Here are the first five terms of an arithmetic sequence:

$$2 \quad 6 \quad 10 \quad 14 \quad 18.$$

(a) Find, in terms of n, an expression for the nth term of this sequence. (2)

Solution
Let the
$$n\text{th term} = an + b.$$

$$2 \qquad\qquad 6 \qquad\qquad 10 \qquad\qquad 14$$
$$4 \qquad\qquad 4 \qquad\qquad 4$$

$$a + b \qquad 2a + b \qquad 3a + b \qquad 4a + b$$
$$a \qquad\qquad a \qquad\qquad a$$

We compare terms:
$$a = 4$$
and
$$a + b = 2 \Rightarrow 4 + b = 2$$
$$\Rightarrow b = -2.$$

Hence,
$$n\text{th term} = \underline{\underline{4n - 2}}.$$

(b) An expression for the nth term of another sequence is $10 - n^2$. (2)
   (i) Find the third term of this sequence.

Solution
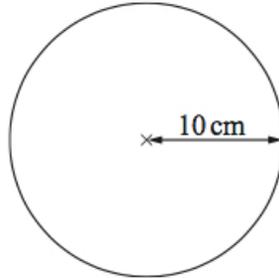$$3\text{rd term} = 10 - 3^2 = 10 - 9 = \underline{\underline{1}}.$$

   (ii) Find the fifth term of this sequence.

3

Solution

$$\text{5th term} = 10 - 5^2 = 10 - 25 = \underline{\underline{-15}}.$$

5. The radius of a circle is 10 cm. (2)

Work out the area of this circle.
Use $\pi = 3.14$.

Solution

$$3.14 \times 10^2 = 3.14 \times 100$$
$$= \underline{\underline{314 \text{ cm}^2}}.$$

6. Work out an estimate for (2)
$$\frac{3\,870}{236 \times 4.85}.$$

Solution

Round to 1 significant figure:

$$\frac{3\,870}{236 \times 4.85} \approx \frac{4\,000}{200 \times 5}$$
$$= \frac{4\,000}{1\,000}$$
$$= \underline{\underline{4}}.$$

4

7. Paul drives 175 miles to a meeting. (3)
   His company pays him 37 p for each mile.
   Work out how much the company pays Paul.

Solution

$\times$	100	70	5
30	3 000	2 100	150
7	700	490	35

He is paid

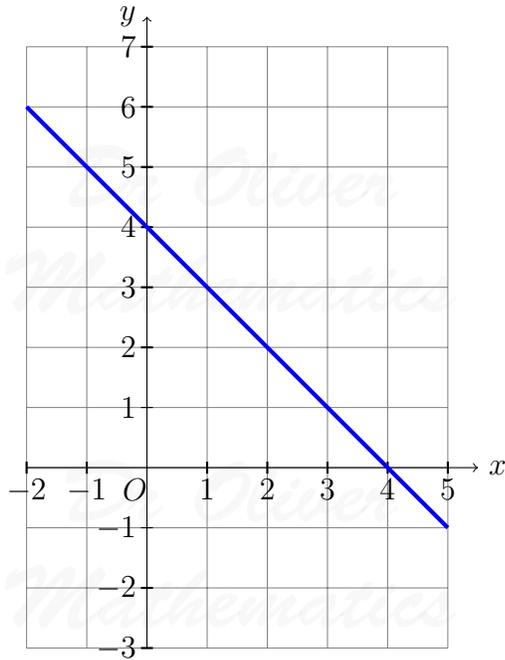$$3\,000 + 2\,100 + 150 + 700 + 490 + 35 = 6\,475 \text{ p}$$
$$= \underline{\underline{£64.75}}.$$

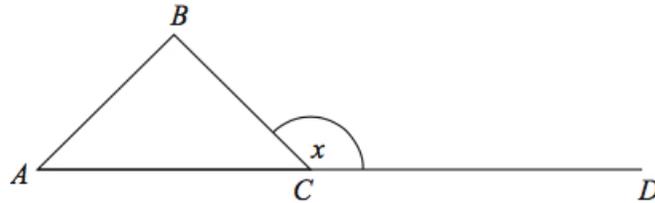8. On the grid draw the graph of $x + y = 4$ for values of x from -2 to 5. (3)
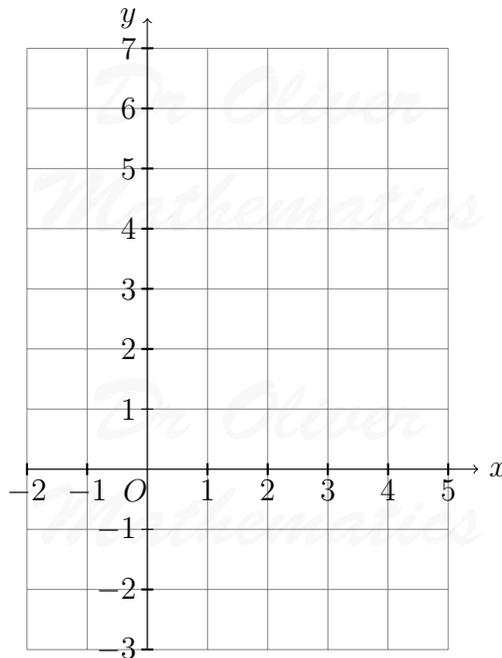
Solution

9. ABC is an equilateral triangle.

ACD is a straight line.

(a) Work out the size of the angle marked x. (2)

Solution

$60 + 60 = \underline{\underline{120°}}$.

(b) Give a reason for your answer. (1)

Solution

The exterior angle of a triangle is equal to the sum of the interior angles at the two other vertices.

10. Chris plays golf.
    Here are 15 of his scores.

69	78	82	86	77
83	91	77	92	80
74	81	83	77	72

(a) Draw an ordered stem and leaf diagram to show this information. (3)
    You must include a key.

Solution

$$
\begin{array}{c|cccccc}
9 & 1 & 2 \\
\hline
8 & 0 & 1 & 2 & 3 & 3 & 6 \\
\hline
7 & 2 & 4 & 7 & 7 & 7 & 8 \\
\hline
6 & 9
\end{array}
$$

Key: 6|9 means 69 strokes

(b) Write down the mode. (1)

Solution
77.

11. Lizzie bought a van. (6)
    The total cost of the van was £6 000 plus VAT at $17\frac{1}{2}\%$.
    Lizzie paid £3 000 when she got the van.
    She paid the rest of the total cost of the van in 10 equal monthly payments.
    Work out the amount of each monthly payment.

Solution

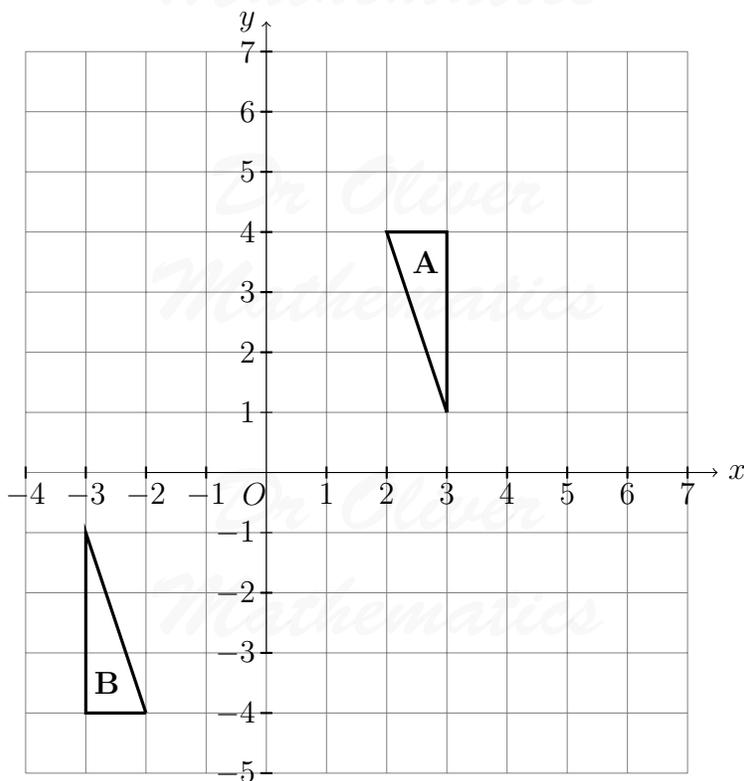|	
10%	600
5%	300
$2\frac{1}{2}\%$	150
Total	£1 050

She pays £3 000 and the remainder is

$$7\,050 - 3\,000 = 4\,050.$$
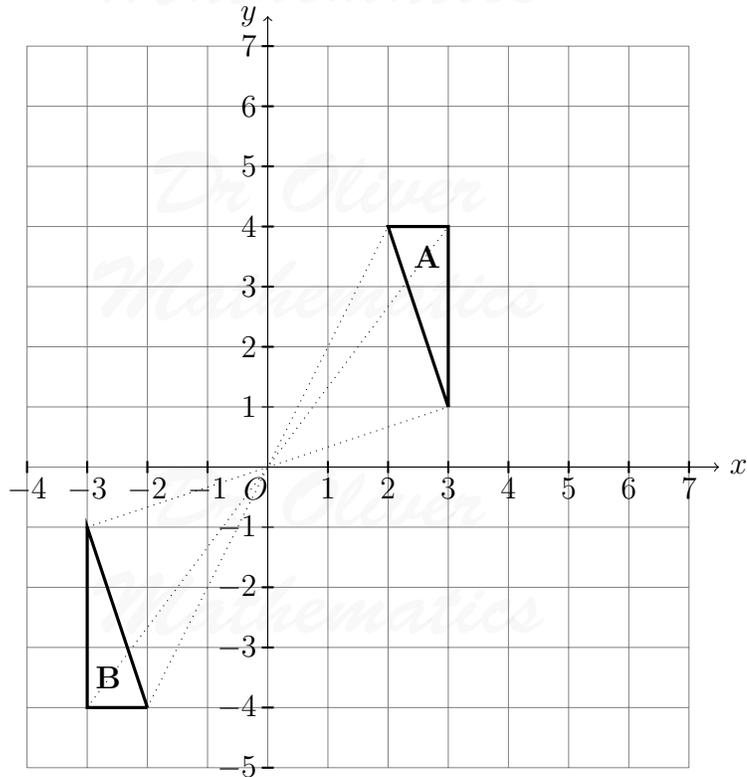
Hence, the amount of each monthly payment is

$$\frac{4\,050}{10} = \underline{\underline{£405}}.$$

12. Triangle **A** and triangle **B** are drawn on the grid.

(a) Describe fully the single transformation which maps triangle **A** onto triangle **B**. (3)
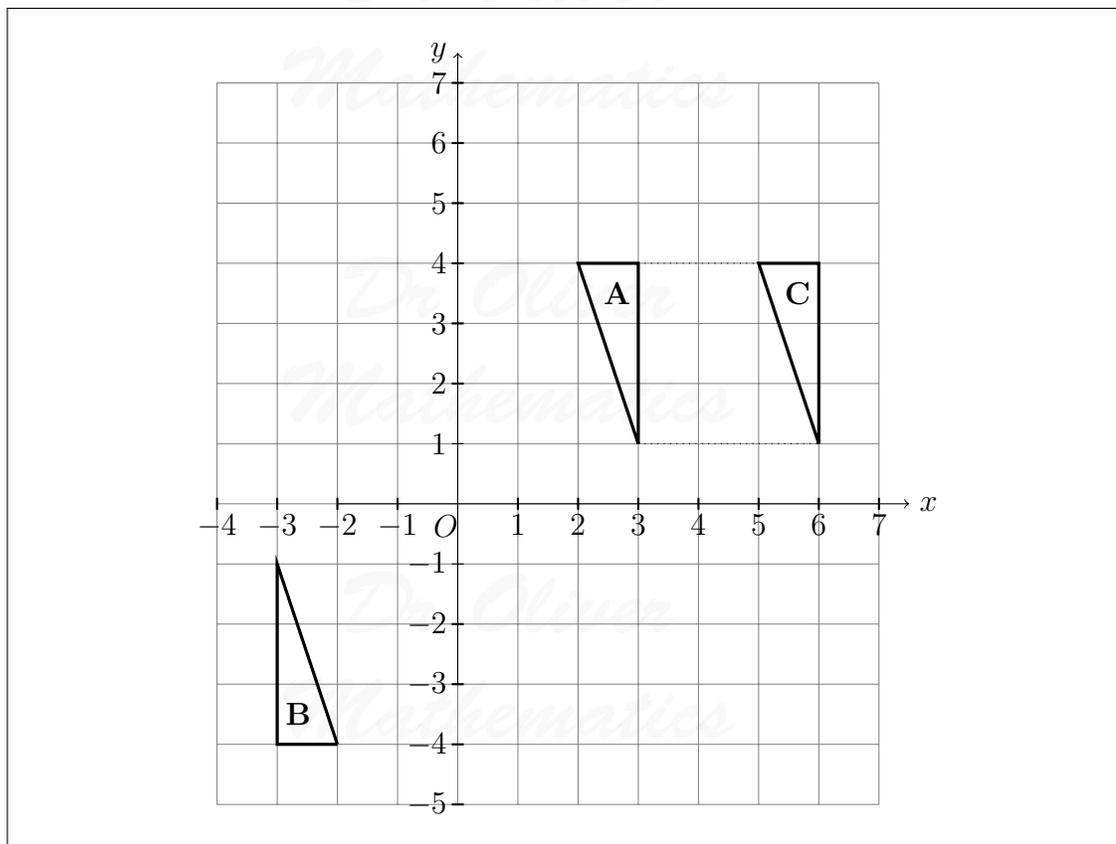
Solution

It is a <u>rotation</u>, centre <u>$(0,0)$</u>, <u>180°</u>.

(b) Translate triangle **A** by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.   (1)

Label the new triangle **C**.

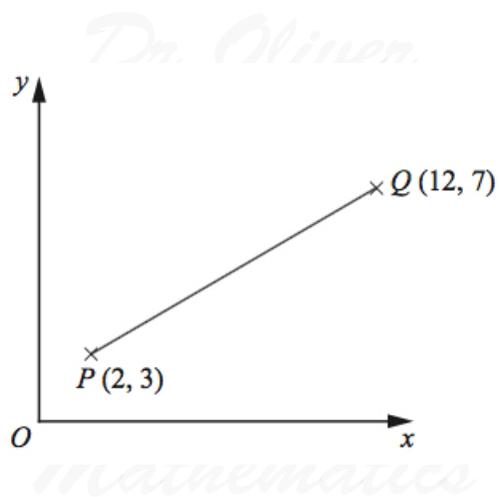**Solution**

9

13. Make v the subject of the formula (2)

$$t = \frac{v}{5} + 2.$$

Solution

$$t = \frac{v}{5} + 2 \Rightarrow \frac{v}{5} = t - 2$$
$$\Rightarrow \underline{\underline{v = 5(t - 2)}}.$$

14. P is the point with coordinates $(2, 3)$. (2)
    Q is the point with coordinates $(12, 7)$.

Work out the coordinates of the midpoint of the line PQ.

Solution

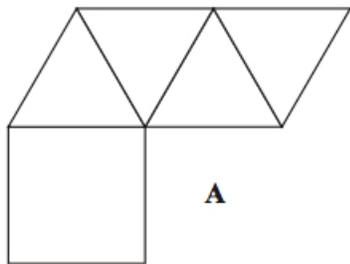$$\left(\frac{2+12}{2}, \frac{3+7}{2}\right) = \underline{\underline{(7,5)}}.$$

15. Here are 5 diagrams. (2)

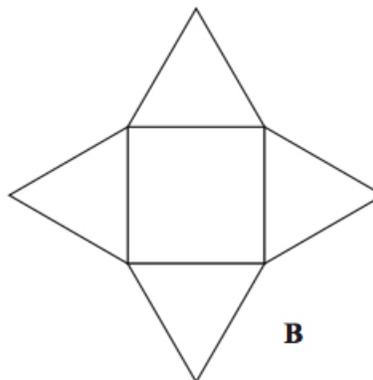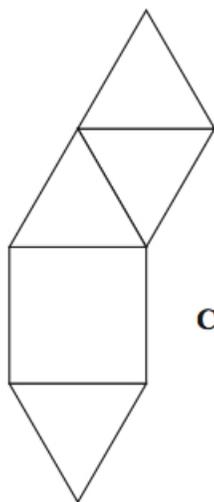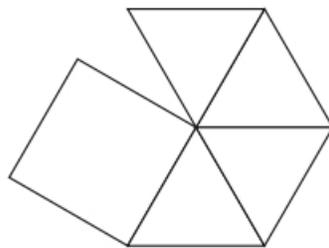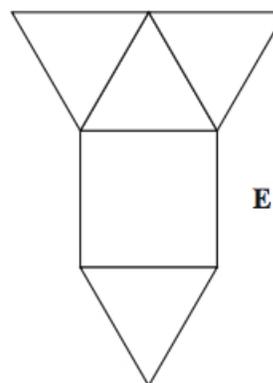Two of these diagrams show a net for a square-based pyramid.
Write down the letter of each of these two diagrams.

Solution

B and E.

16. (a) Expand and simplify (2)
$$3(x + 5) + 2(5x - 6).$$

Solution
$$3(x + 5) + 2(5x - 6) = 3x + 15 + 10x - 12$$
$$= 13x + 3.$$

(b) Simplify (1)

$$\frac{2x + 4}{2}.$$

Solution

$$\frac{2x + 4}{2} = \underline{\underline{x + 2}}.$$

(c) Factorise (1)

$$5x + 10.$$

Solution

$$5x + 10 = \underline{\underline{5(x + 2)}}.$$

(d) Factorise fully (2)

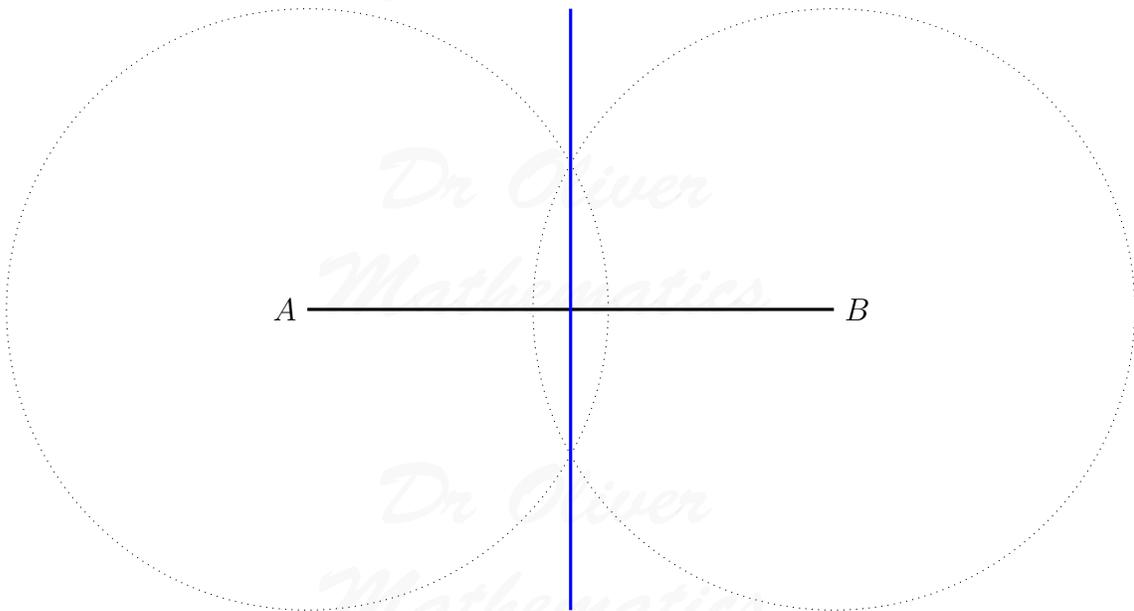$$x^2y + xy^2.$$

Solution

$$x^2y + xy^2 = \underline{\underline{xy(x + y)}}.$$

17. Use ruler and compasses to construct the perpendicular bisector of the line AB. (2)
You must show all your construction lines.

$A \;\rule{3cm}{0.4pt}\; B$

13

18. (a) Work out (3)

$$2\tfrac{17}{20} - 1\tfrac{2}{5}.$$

Solution

$$2\tfrac{17}{20} - 1\tfrac{2}{5} = 1\tfrac{17}{20} - \tfrac{8}{20}$$
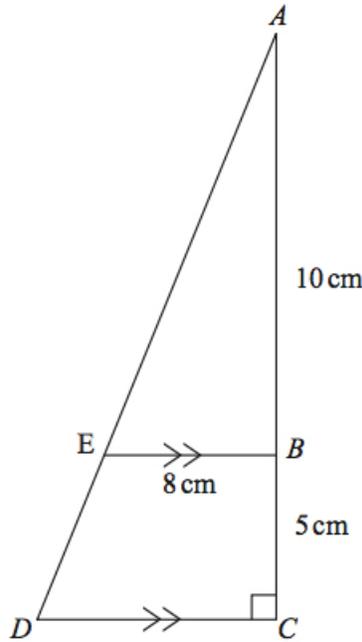$$= \underline{\underline{1\tfrac{9}{20}}}.$$

(b) Work out (3)

$$2\tfrac{2}{3} \times 1\tfrac{3}{4}.$$

Solution

$$2\tfrac{2}{3} \times 1\tfrac{3}{4} = \tfrac{8}{3} \times \tfrac{7}{4}$$
$$= \tfrac{56}{12}$$
$$= 4\tfrac{8}{12}$$
$$= 4\tfrac{2}{3}.$$

19. ABC and AED are straight lines.

EB is parallel to DC.
Angle $ACD = 90°$.
$AB = 10$ cm.
$BC = 5$ cm.
$EB = 8$ cm.

(a) Work out the length of DC. (2)

Solution

$$DC = \frac{EB}{BA} \times CA$$
$$= \frac{8}{10} \times 15$$
$$= \underline{\underline{12 \text{ cm}}}.$$
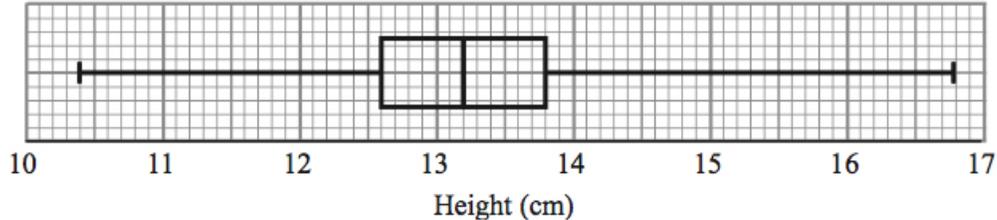
(b) Work out the area of the trapezium $EBCD$. (2)

Solution

$$\text{Area} = \tfrac{1}{2}h(a+b)$$
$$= \tfrac{1}{2} \times 5 \times (8 + 12)$$
$$= \tfrac{5}{2} \times 20$$
$$= \underline{\underline{50 \text{ cm}^2}}.$$

20. Mr Green measured the height, in cm, of each tomato plant in his greenhouse.
He used the results to draw the box plot shown below.

Height (cm)

(a) Write down the median height. (1)

Solution
$\underline{\underline{13.2 \text{ cm}}}$.

(b) Work out the interquartile range. (2)

Solution

$$\text{IQR} = 13.8 - 12.6$$
$$= \underline{\underline{1.2 \text{ cm}}}.$$

(c) Explain why the interquartile range may be a better measure of spread than the range. (1)

> **Solution**
>
> E.g., the IQR ignores <u>extreme values</u>.

21. Solve the simultaneous equations (4)

$$6x + 2y = -3$$
$$4x - 3y = 11.$$

> **Solution**
>
> $$6x + 2y = -3 \quad (1)$$
> $$4x - 3y = 11 \quad (2)$$
>
> $$2 \times (1): \quad 12x + 4y = -6 \quad (3)$$
> $$3 \times (2): \quad 12x - 9y = 33 \quad (4)$$
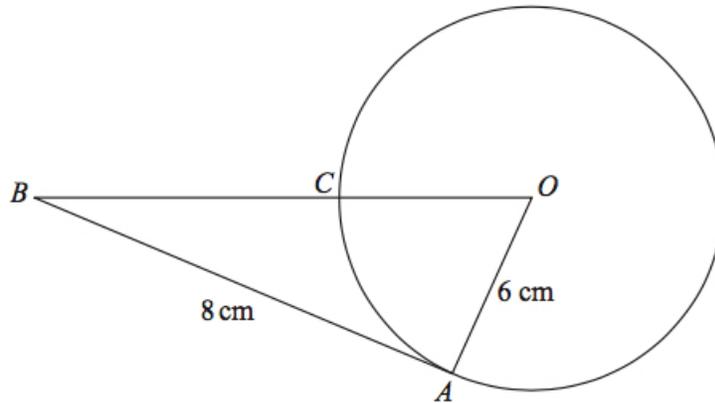>
> Now, $(3) - (4)$:
>
> $$13y = -39 \Rightarrow \underline{\underline{y = -3}}$$
> $$\Rightarrow 6x - 6 = -3$$
> $$\Rightarrow 6x = 3$$
> $$\Rightarrow \underline{\underline{x = \tfrac{1}{2}}}.$$

22. In the diagram, O is the centre of the circle.

17

A and C are points on the circumference of the circle.
BCO is a straight line.
BA is a tangent to the circle.
$AB = 8$ cm.
$OA = 6$ cm.

(a) Explain why angle OAB is a right angle. (1)

> **Solution**
>
> It is a tangent between AB and a radius OA and, hence, $\underline{\underline{\angle OAB = 90°}}$.

(b) Work out the length of BC. (3)

> **Solution**
>
> $$OB = \sqrt{AB^2 + OA^2}$$
> $$= \sqrt{8^2 + 6^2}$$
> $$= 10$$
>
> and
>
> $$BC = 10 - 6 = \underline{\underline{4 \text{ cm}}}.$$

23. (a) Expand and simplify (2)
$$(x - 3)(x + 5).$$

Solution

×	x	-3
x	x^2	$-3x$
$+5$	$+5x$	-15

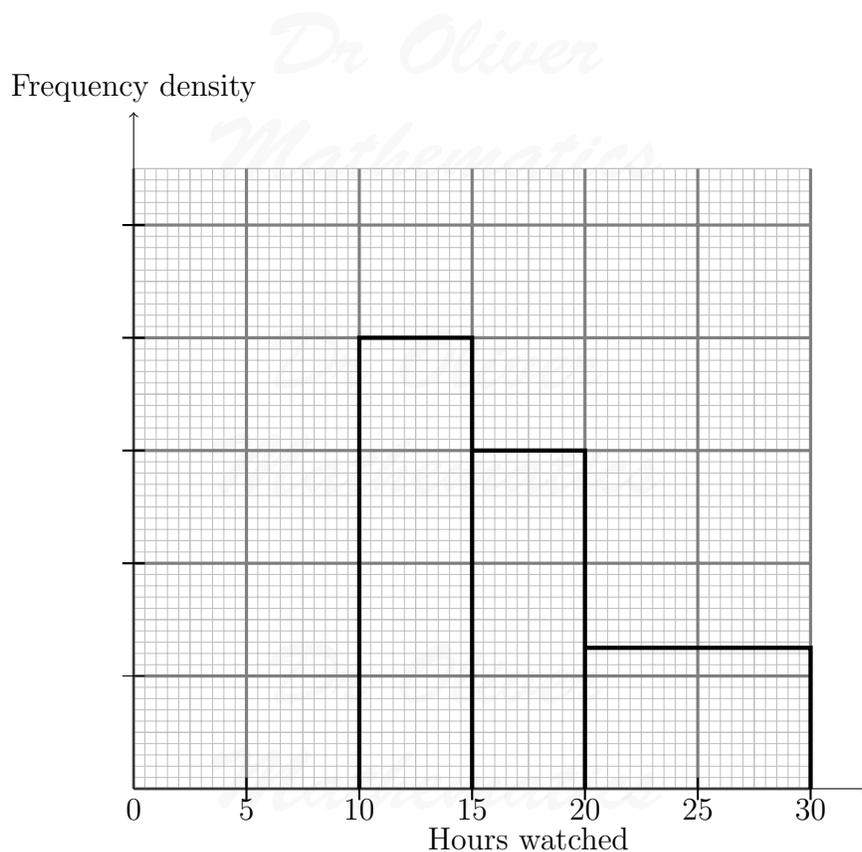$$(x-3)(x+5) = \underline{\underline{x^2 + 2x - 15}}.$$

(b) Solve (3)

$$x^2 + 8x - 9 = 0.$$

Solution

$$\left.\begin{array}{ll}\text{add to:} & +8 \\ \text{multiply to:} & -9\end{array}\right\} -1, +9$$

$$x^2 + 8x - 9 = 0 \Rightarrow (x+9)(x-1) = 0$$
$$\Rightarrow x + 9 = 0 \text{ or } x - 1 = 0$$
$$\Rightarrow \underline{\underline{x = -9 \text{ or } x = 1}}.$$

24. Tom asked the students in his class how many hours they watched television last week. The incomplete histogram was drawn using his results.

Frequency density

Hours watched

Eight students watched television for between 10 and 15 hours.
Six students watched television for between 0 and 10 hours.

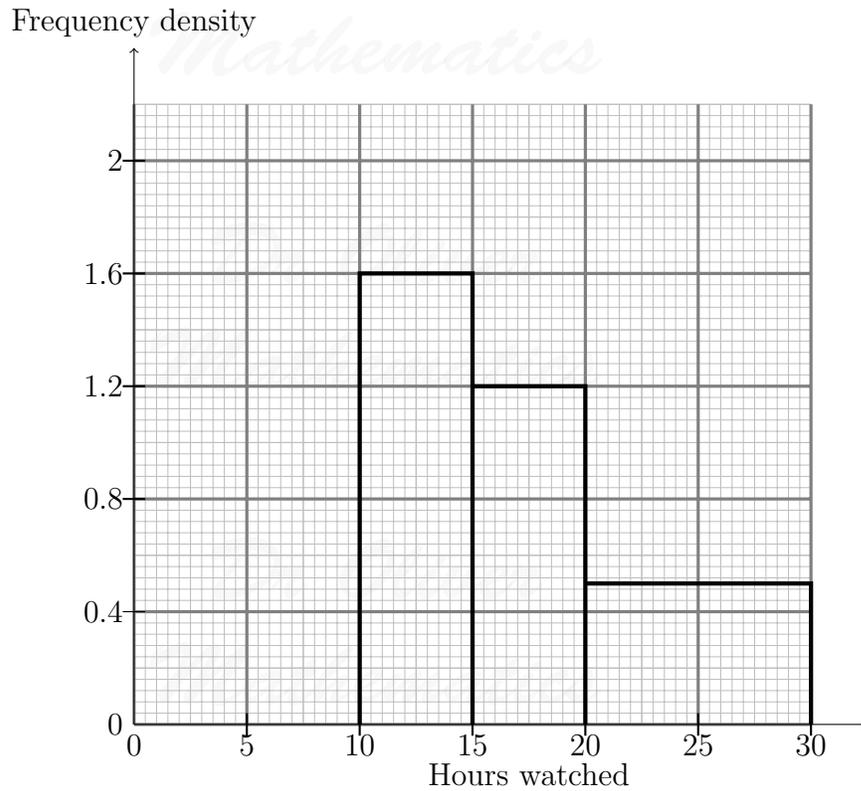(a) Use this information to complete the histogram. (2)

Solution
We make a table:

Distance (d km)	Frequency	Width	Frequency density
$0 \leqslant h < 10$		10	
$10 \leqslant h < 15$	8	5	$\frac{8}{5} = 1.6$
$15 \leqslant h < 20$		5	
$20 \leqslant h < 30$		10	

and we get the following histogram:

Frequency density

Hours watched

Now, we complete the table

Distance (d km)	Frequency	Width	Frequency density
$0 \leqslant h < 10$	6	10	$\frac{6}{10} = 0.6$
$10 \leqslant h < 15$	8	5	$\frac{8}{5} = 1.6$
$15 \leqslant h < 20$	6	5	$\frac{6}{5} = 1.2$
$20 \leqslant h < 30$	5	10	$\frac{5}{10} = 0.5$

and we complete the histogram
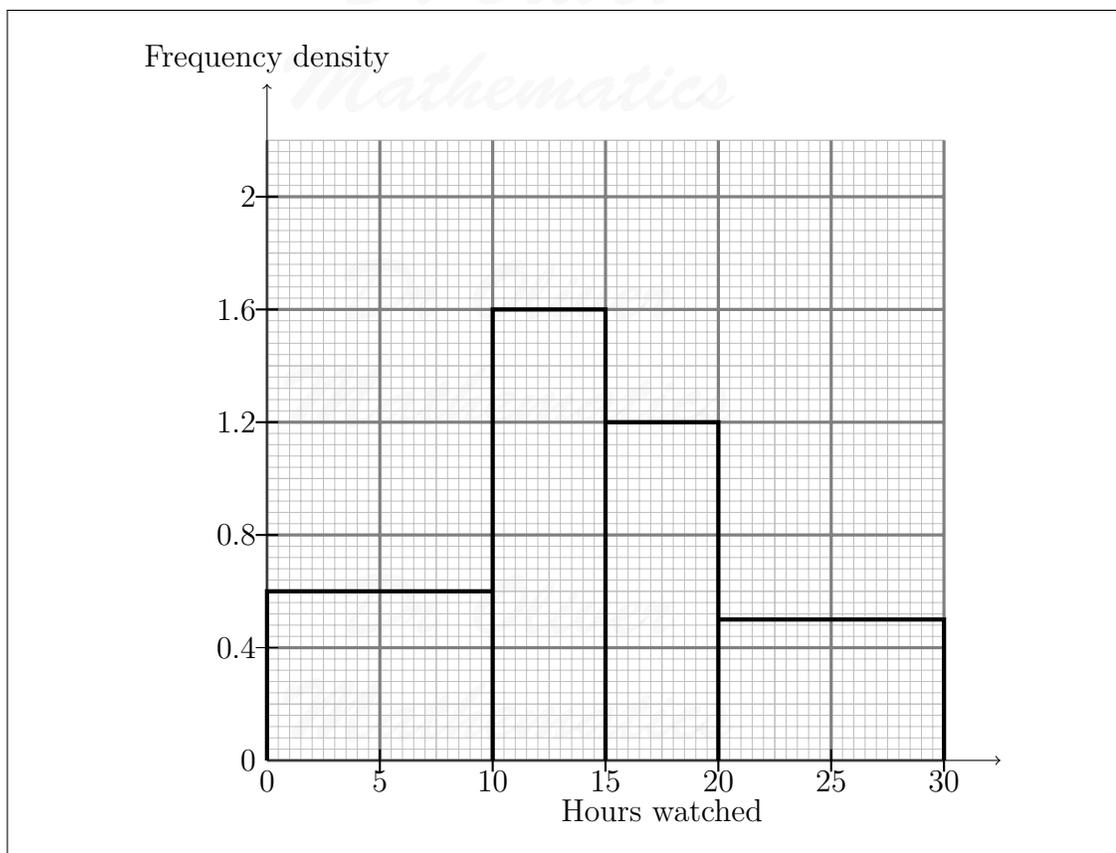
No students watched television for more than 30 hours.

(b) Work out how many students Tom asked. (2)

> **Solution**
> He asked
> $$6 + 8 + 6 + 5 = \underline{\underline{25 \text{ students}}}.$$

25. The table shows information about the ages, in years, of 1 000 teenagers. (2)

Age (years)	13	14	15	16	17	18	19
Number of teenagers	158	180	165	141	131	115	110

Simone takes a sample of 50 of these teenagers, stratified by age.
Calculate the number of 14 year olds she should have in her sample.

Solution

$$\frac{180}{1\,000} \times 50 = \underline{\underline{9 \text{ students}}}.$$

26. P is inversely proportional to V.
    When $V = 8$, $P = 5$.

    (a) Find a formula for P in terms of V. (3)

    Solution

    $$P \propto \frac{1}{V} \Rightarrow P = \frac{k}{V}$$

    for some constant k. Now,

    $$5 = \frac{k}{8} \Rightarrow k = 40$$

    which means
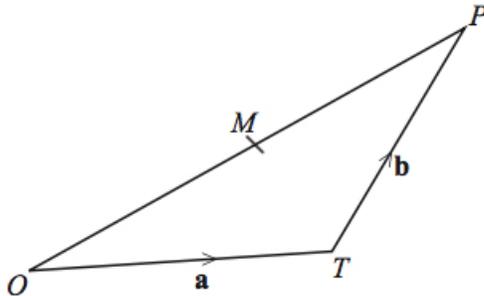
    $$P = \frac{40}{V}.$$

    (b) Calculate the value of P when $V = 2$. (1)

    Solution

    $$P = \frac{40}{2} = \underline{\underline{20}}.$$

27. OPT is a triangle.

M is the midpoint of OP.
$\overrightarrow{OT} = \mathbf{a}$.
$\overrightarrow{TP} = \mathbf{b}$.

(a) Express $\overrightarrow{OM}$ in terms of **a** and **b**. (2)

Solution

$$\overrightarrow{OM} = \tfrac{1}{2}\overrightarrow{OP}$$
$$= \tfrac{1}{2}(\overrightarrow{OT} + \overrightarrow{TP})$$
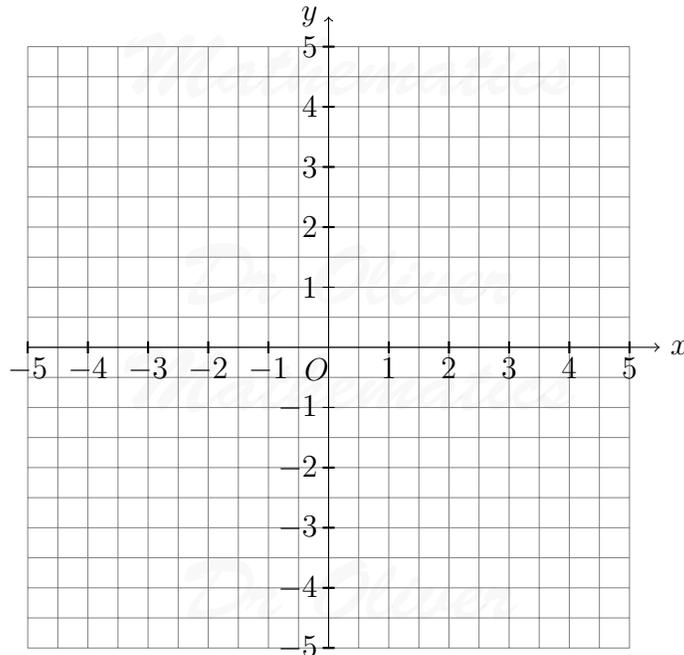$$= \tfrac{1}{2}(\mathbf{a} + \mathbf{b}).$$

(b) Express $\overrightarrow{TM}$ in terms of **a** and **b**. (2)
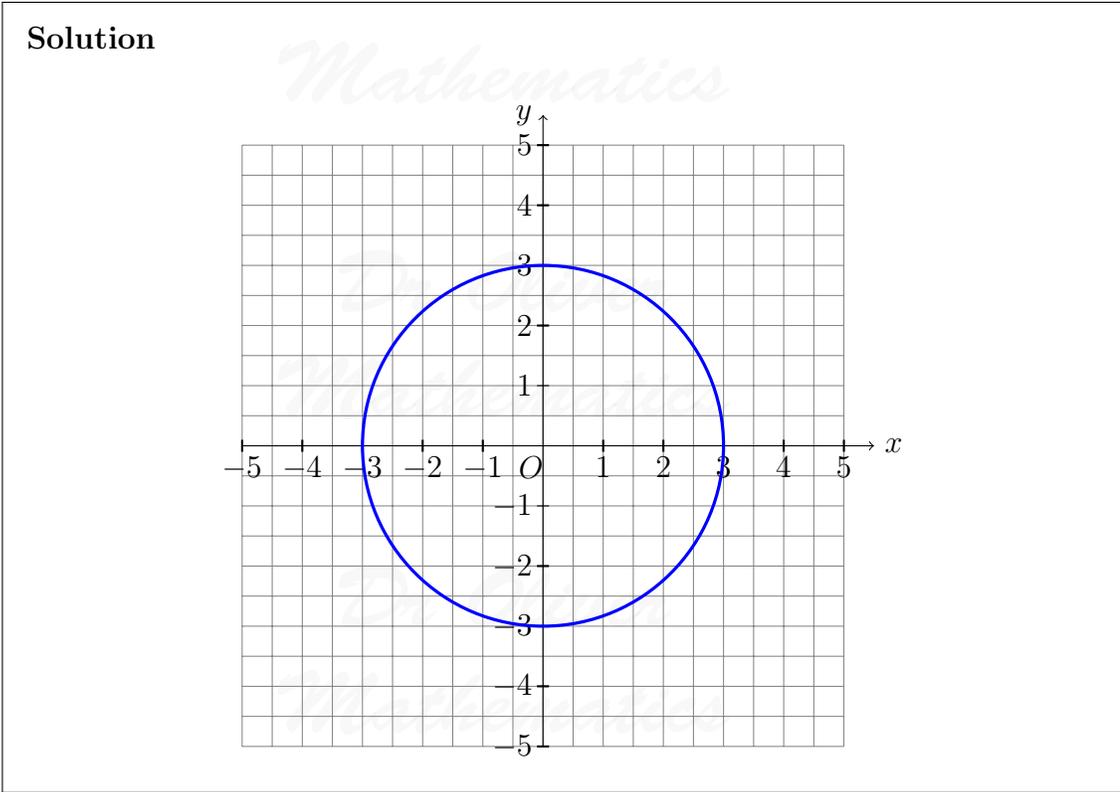Give your answer in its simplest form.

Solution

$$\overrightarrow{TM} = \overrightarrow{TO} + \overrightarrow{OM}$$
$$= -\mathbf{a} + \tfrac{1}{2}(\mathbf{a} + \mathbf{b})$$
$$= \tfrac{1}{2}(\mathbf{b} - \mathbf{a}).$$

28. (a) Construct the graph of (2)

$$x^2 + y^2 = 9.$$

24

Solution

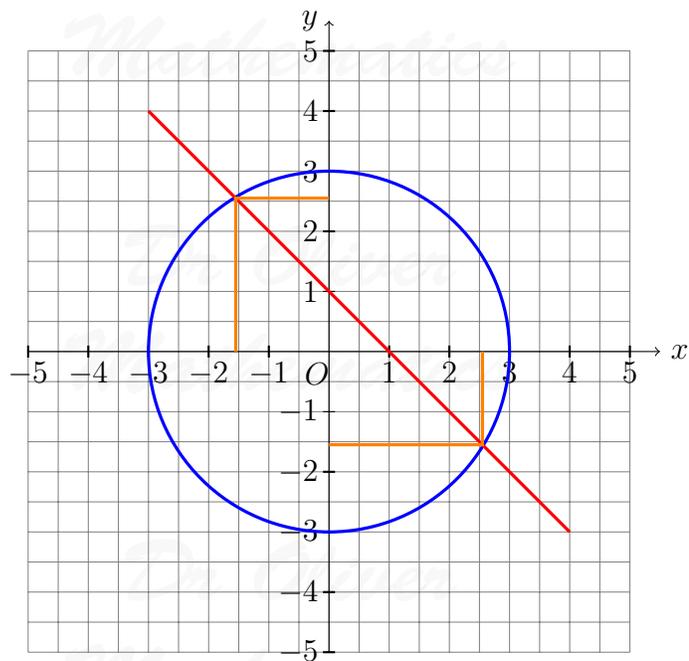(b) By drawing the line $x + y = 1$ on the grid, solve the equations (3)

$$x^2 + y^2 = 9$$
$$x + y = 1.$$

Solution

$\underline{\underline{x = 2.55, y = -1.55}}$ or $\underline{\underline{x = -1.55, y = 2.55}}$.