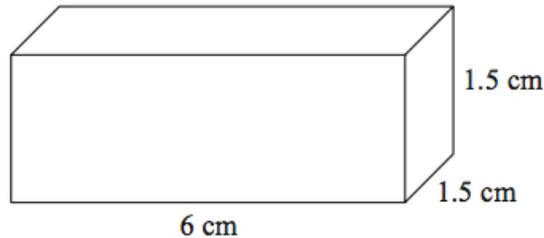


Dr Oliver Mathematics
GCSE Mathematics
2013 June Paper 2H: Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Here is a cuboid.

(3)



The cuboid is 6 cm by 1.5 cm by 1.5 cm.
Work out the total surface area of the cuboid.

Solution

$$\begin{aligned}\text{Surface area} &= 2[(6 \times 1.5) + (6 \times 1.5) + (1.5 \times 1.5)] \\ &= \underline{40.5 \text{ cm}^2}.\end{aligned}$$

2. Here is a list of ingredients for making 18 mince pies.

(4)

Ingredients for 18 mince pies

225 g of butter
350 g of flour
100 g of sugar
280 g of mincemeat
1 egg

Elaine wants to make 45 mince pies.

Elaine has 1 kg of butter, 1 kg of flour, 500 g of sugar, 600 g of mincemeat, and 6 eggs.

Does Elaine have enough of each ingredient to make 45 mince pies?
You must show clearly how you got your answer.

Solution

Butter:

$$\frac{45}{18} \times 225 = 562.5.$$

Flour:

$$\frac{45}{18} \times 350 = 875.$$

Sugar:

$$\frac{45}{18} \times 100 = 250.$$

Mincemeat:

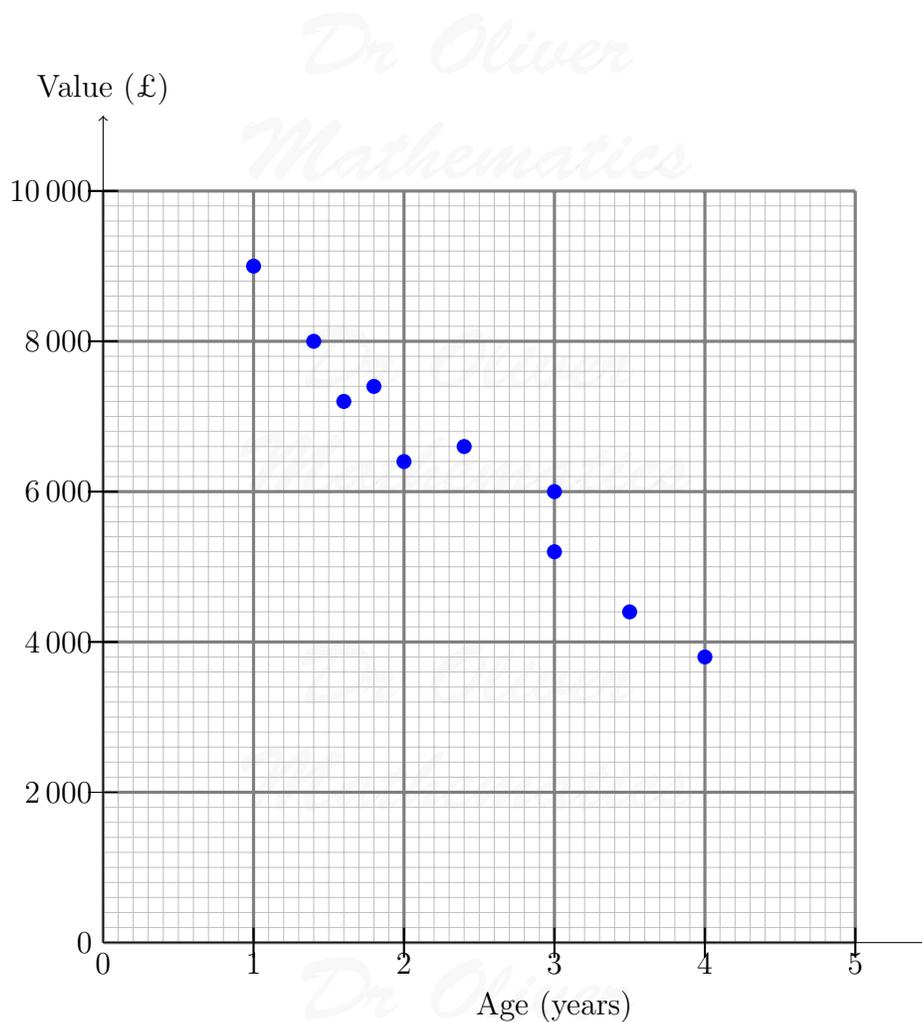
$$\frac{45}{18} \times 280 = 700.$$

Eggs:

$$\frac{45}{18} \times 1 = 2.5.$$

Hence, he does not have enough mincemeat.

3. The scatter graph shows some information about 10 cars, of the same type and make. The graph shows the age (years) and the value (£) of each car.



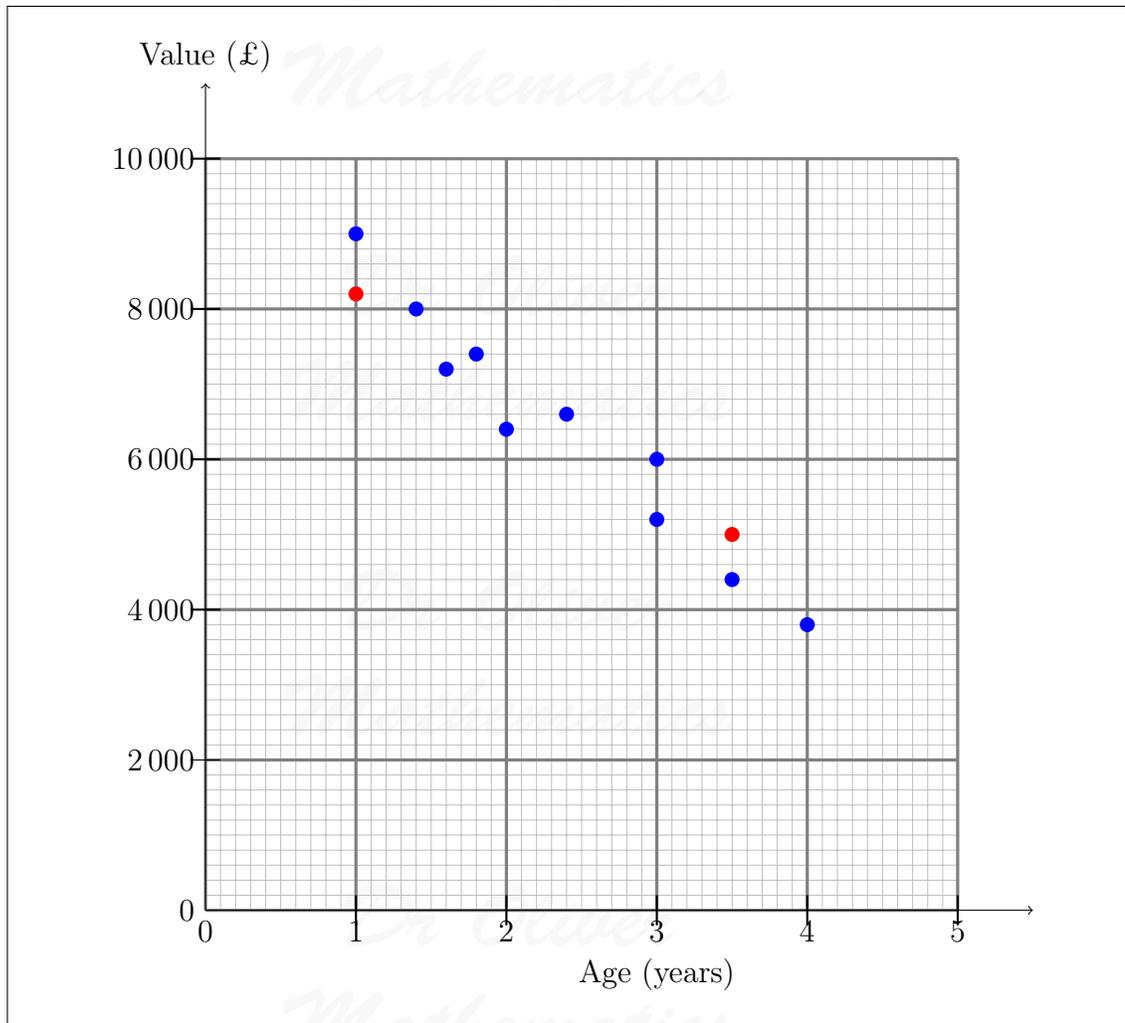
The table shows the age and the value of two other cars of the same type and make.

Age (years)	1	3.5
Value (£)	8 200	5 000

(a) On the scatter graph, plot the information from the table.

(1)

Solution



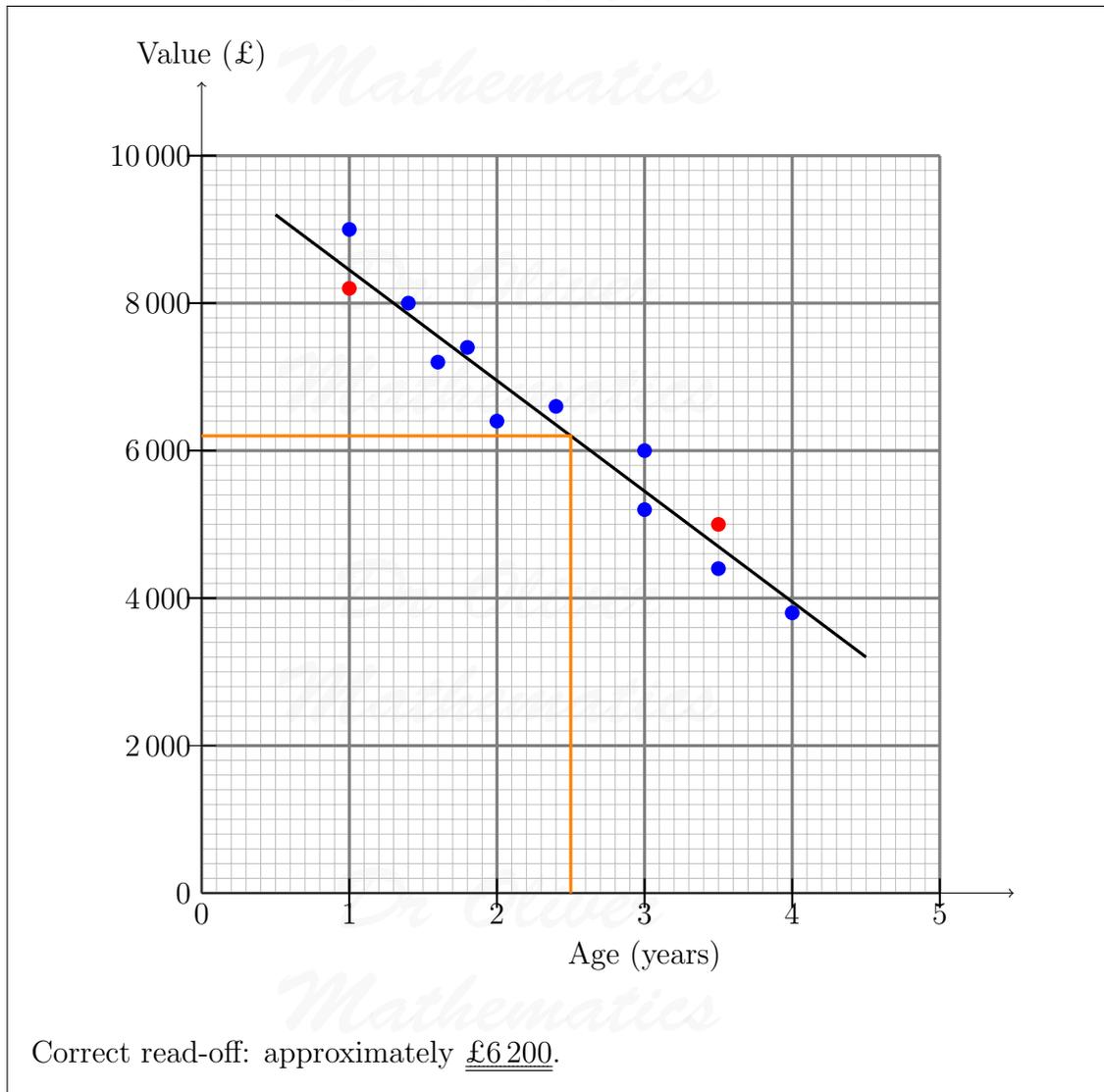
(b) Describe the relationship between the age and the value of the cars. (1)

Solution
Negative correlation, i.e., the older the car gets, the less that it is worth.

A car of the same type and make is $2\frac{1}{2}$ years old.

(c) Estimate the value of the car. (2)

Solution
 Draw a line of best fit onto the graph:



4. Rhiana plays a game. (3)
 The probability that she will lose the game is 0.32.
 The probability that she will draw the game is 0.05.
 Rhiana is going to play the game 200 times.
 Work out an estimate for the number of times Rhiana will win the game.

Solution

The estimate for the number of times Rhiana will win the game is

$$200 \times (1 - 0.32 - 0.05) = 200 \times 0.63$$

$$= \underline{126}.$$

5. Mason is doing a survey to find out how many magazines people buy. He uses this question on his questionnaire.

How many magazines do you buy?

<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
0 to 4	4 to 8	8 to 12

- (a) Write down **two** things wrong with this question. (2)

Solution

E.g., '4' is repeated, '8' is repeated, there is no place to write down 13 or more.

- (b) Write a better question for Mason to use on his questionnaire to find out how many magazines people buy. (2)

Solution

A suitable question with a time frame, e.g., "How many magazines did you buy last week/last month? Tick the appropriate box."

At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, 0, 1-3, 4-6, 7 or more.

Mason asks his friends at school to do his questionnaire. This may **not** be a good sample to use.

- (c) Give **one** reason why. (1)

Solution

E.g., small sample, they are friends of his, may all be males, biased, may all like same types of magazines, all the same age.

6. Tame Valley is a company that makes yoghurt. (4)
A machine fills trays of 20 pots with yoghurt.
In one hour, the machine fills a total of 15 000 pots.
Work out how many seconds the machine takes to fill each tray of 20 pots.

Solution

$$\frac{60 \times 60 \times 20}{15\,000} = \underline{\underline{4.8 \text{ s.}}}$$

7. Colin, Dave, and Emma share some money. (4)

Colin gets $\frac{3}{10}$ of the money.

Emma and Dave share the rest of the money in the ratio 3 : 2.

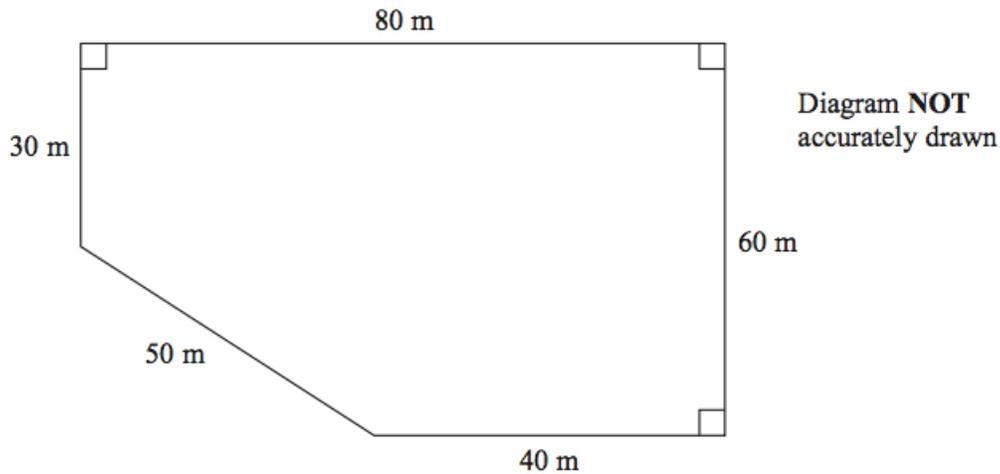
What is Dave's share of the money?

Solution

Emma and Dave get $1 - \frac{3}{10} = \frac{7}{10}$ of the money. Dave's share is

$$\frac{7}{10} \times \frac{2}{3+2} = \underline{\underline{\frac{7}{25}}}$$

8. The diagram shows the plan of a playground. (4)



Bill is going to cover the playground with tarmac.

It costs £2.56 to cover each square metre with tarmac.

Work out the total cost of the tarmac Bill needs.

Solution

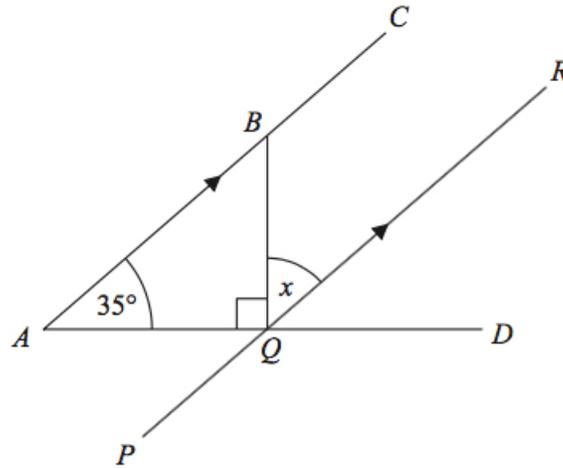
$$\begin{aligned} \text{Area} &= (40 \times 60) + \left[\frac{1}{2} \times (60 + 30) \times 40\right] \\ &= 2\,400 + 1\,200 \\ &= 4\,200 \text{ m}^2 \end{aligned}$$

and the total cost is

$$4\,200 \times 2.56 = \underline{\underline{\pounds 10\,752}}$$

9. ABC , PQR , and AQD are straight lines.

(4)



ABC is parallel to PQR .

Angle $BAQ = 35^\circ$.

Angle $BQA = 90^\circ$.

Work out the size of the angle marked x .

Give reasons for each stage of your working.

Solution

$\angle BAQ$ and $\angle AQR$ are (co-)interior angles so

$$35 + 90 + x = 180 \Rightarrow x + 125 = 180 \\ \Rightarrow \underline{x = 55}.$$

10. The equation

(4)

$$x^3 + 2x = 110$$

has a solution between 4 and 5.

Use a trial and improvement method to find this solution.

Give your answer correct to one decimal place.

You must show **ALL** your working.

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.

F(X)= and you type in $X^3 + 2X$; then you press $\boxed{=}$.

Start? and you enter 4; then you press [=].

End? and you enter 5; then you press [=].

Step? and enter 0.05 – 1 decimal place divided by 2; then you press [=].

x	$f(x)$	Comment
4.65	109.84	too low
4.7	113.22	too high

Clearly,

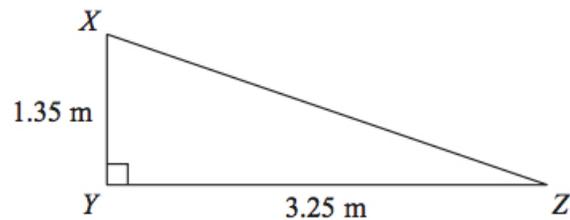
$$4.65 < x < 4.7$$

and the answer is

$$\underline{\underline{x = 4.7 \text{ (1 dp)}}}.$$

11. XYZ is a right-angled triangle.

(3)



Calculate the length of XZ .

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} XZ &= \sqrt{XY^2 + YZ^2} \\ &= \sqrt{1.35^2 + 3.25^2} \\ &= 3.519\,232\,871 \text{ (FCD)} \\ &= \underline{\underline{3.52 \text{ cm (3 sf)}}}. \end{aligned}$$

12. (a) Solve

$$3(x - 2) = x + 7.$$

(3)

Solution

$$\begin{aligned} 3(x - 2) = x + 7 &\Rightarrow 3x - 6 = x + 7 \\ &\Rightarrow 2x = 13 \\ &\Rightarrow x = \underline{\underline{6\frac{1}{2}}}. \end{aligned}$$

(b) Solve

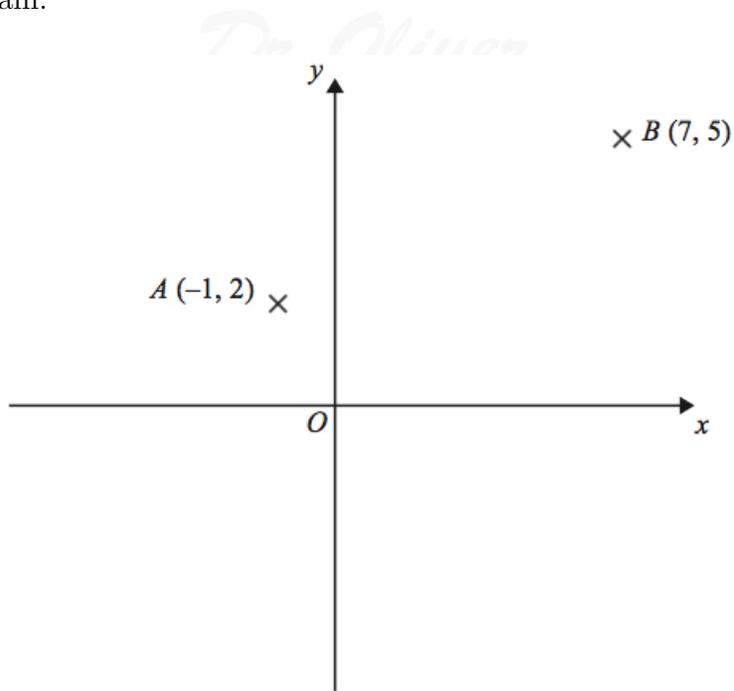
$$\frac{2 - y}{5} = 1.$$

(2)

Solution

$$\begin{aligned} \frac{2 - y}{5} = 1 &\Rightarrow 2 - y = 5 \\ &\Rightarrow \underline{\underline{y = -3}}. \end{aligned}$$

13. Here is a diagram.



A is the point $(-1, 2)$.

B is the point $(7, 5)$.

(a) Find the coordinates of the midpoint of AB .

(2)

Solution

$$\left(\frac{-1 + 7}{2}, \frac{2 + 5}{2} \right) = \underline{\underline{(3, 3\frac{1}{2})}}.$$

P is the point $(-4, 4)$.

Q is the point $(1, -5)$.

(b) Find the gradient of PQ .

(2)

Solution

$$\begin{aligned} \text{Gradient} &= \frac{4 - (-5)}{-4 - 1} \\ &= \underline{\underline{-\frac{9}{5}}}. \end{aligned}$$

14. Viv wants to invest £2 000 for 2 years in the same bank.

(4)

The International Bank

Compound Interest

4% for the first year

1% for each extra year

The Friendly Bank

Compound Interest

5% for the first year

0.5% for each extra year

At the end of 2 years, Viv wants to have as much money as possible.

Which bank should she invest her £2 000 in?

Solution

The International Bank:

$$2\,000 \times 1.04 \times 1.01 = 2\,100.8$$

The Friendly Bank:

$$2\,000 \times 1.05 \times 1.005 = 2\,110.5$$

So, Viv would be better off in The Friendly Bank.

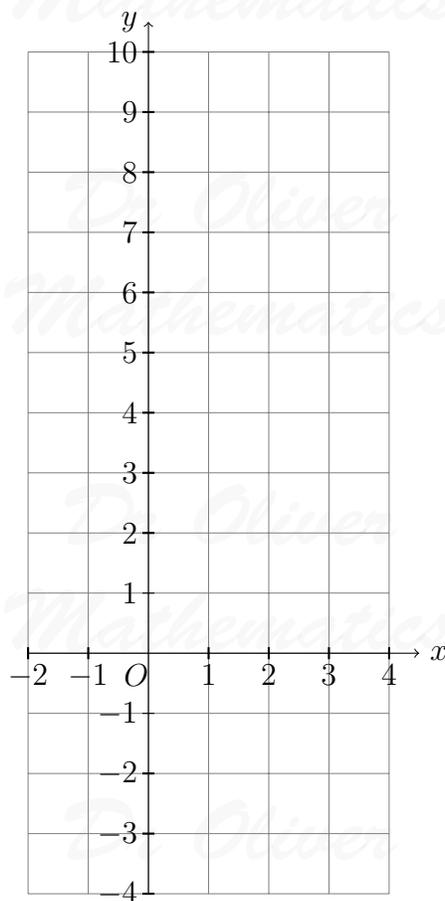
15. (a) Complete the table of values for $y = x^2 - 2x$. (2)

x	-2	-1	0	1	2	3	4
y		3	0			3	

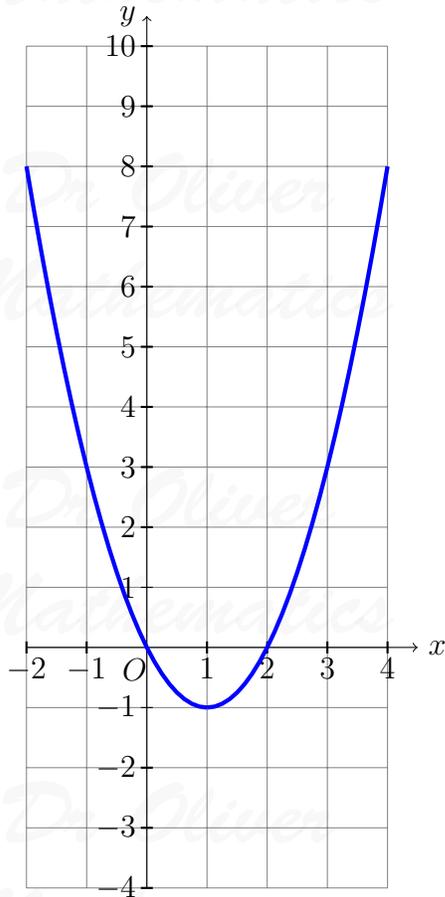
Solution

x	-2	-1	0	1	2	3	4
y	<u>8</u>	3	0	<u>-1</u>	<u>0</u>	3	<u>8</u>

- (b) On the grid, draw the graph of $y = x^2 - 2x$ for values of x from -2 to 4 . (2)



Solution



(c) Solve

$$x^2 - 2x - 2 = 1.$$

(2)

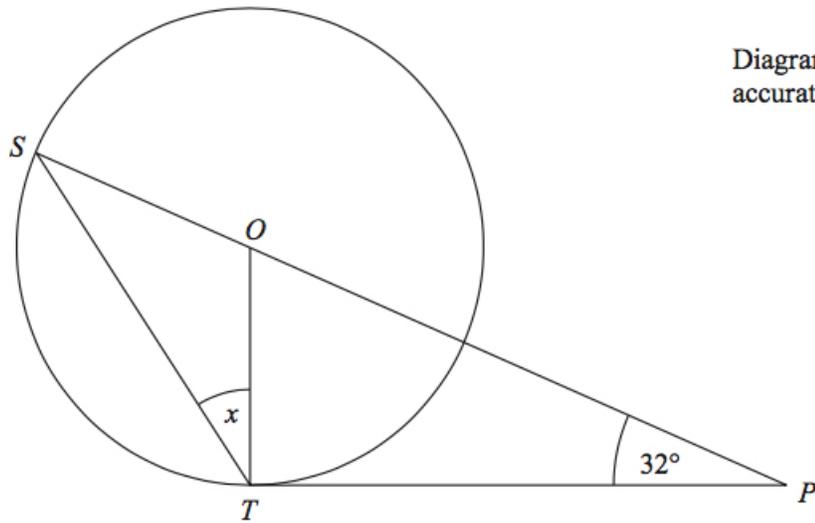
Solution

E.g., we will do this by completing the square:

$$\begin{aligned}x^2 - 2x - 2 = 1 &\Rightarrow x^2 - 2x = 3 \\ &\Rightarrow x^2 - 2x + 1 = 3 + 1 \\ &\Rightarrow (x - 1)^2 = 4 \\ &\Rightarrow x - 1 = -2 \text{ or } x - 1 = 2 \\ &\Rightarrow \underline{\underline{x = -1 \text{ or } x = 3}}\end{aligned}$$

16. S and T are points on the circumference of a circle, centre O .

(5)



PT is a tangent to the circle.
 SOP is a straight line.
 Angle $OPT = 32^\circ$.
 Work out the size of the angle marked x .
 Give reasons for your answer.

Solution

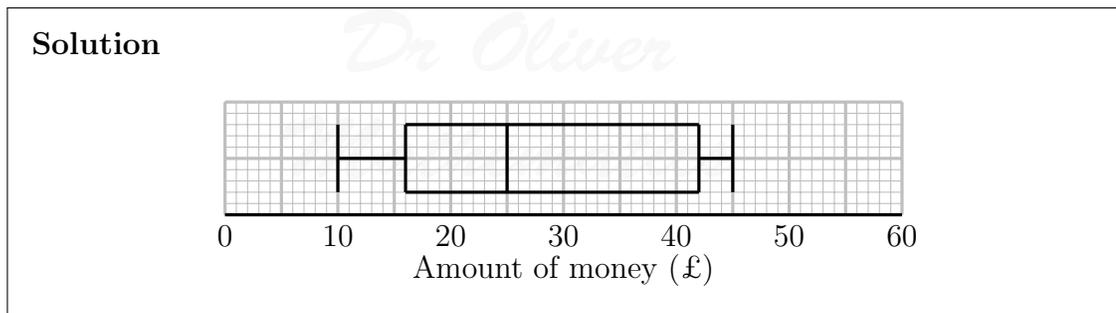
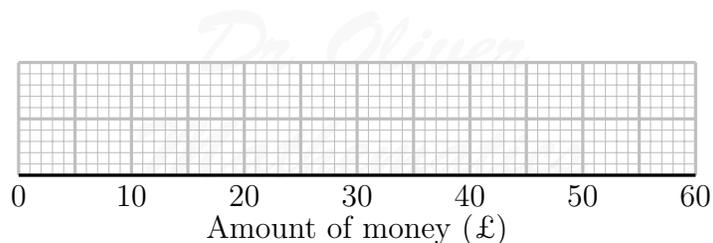
Angle $POT = 90 - 32 = 58^\circ$ (complementary angles)
 Angle $SOT = 180 - 58 = 122^\circ$ (supplementary angles)
 Angle $OTS = \frac{1}{2}(180 - 122) = \underline{\underline{29^\circ}}$ (base angles in an isosceles triangle)

17. Some girls did a sponsored swim to raise money for charity.
 The table shows information about the amounts of money (£) the girls raised.

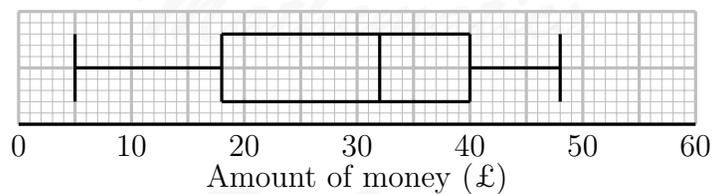
Least amount of money (£)	10
Greatest amount of money (£)	45
Median	25
Lower quartile	16
Upper quartile	42

- (a) On the grid, draw a box plot for the information in the table.

(2)



Some boys also did the sponsored swim.
 The box plot shows information about the amounts of money (£) the boys raised.



- (b) Compare the amounts of money the girls raised with the amounts of money the boys raised. (2)

Solution

Average
 Since the median for the boys' (32) is higher than the median for the girls' (25), the boys' raised more on average.

Spread
 Since the range for the boys' ($48 - 5 = 43$) is greater than the range for the girls' ($45 - 10 = 35$), the amount of money raised was more consistent in the girls'.

OR
 Since the IQR for the boys' ($40 - 18 = 22$) is smaller than the IQR for the girls' ($42 - 16 = 29$), the amount of money raised was more consistent in the boys'.

18. Make p the subject of the formula

(3)

$$y = 3p^2 - 4.$$

Solution

$$\begin{aligned}y = 3p^2 - 4 &\Rightarrow y + 4 = 3p^2 \\ &\Rightarrow p^2 = \frac{y + 4}{3} \\ &\Rightarrow p = \pm \sqrt{\frac{y + 4}{3}}.\end{aligned}$$

19. (a) Factorise $6 + 9x$.

(1)

Solution

$$6 + 9x = \underline{\underline{3(2 + 3x)}}.$$

(b) Factorise $y^2 - 16$.

(1)

Solution

$$\begin{aligned}\left. \begin{array}{l} \text{add to: } 0 \\ \text{multiply to: } -16 \end{array} \right\} +4, -4 \\ y^2 - 16 = \underline{\underline{(y - 4)(y + 4)}}.\end{aligned}$$

(c) Factorise $2p^2 - p - 10$.

(2)

Solution

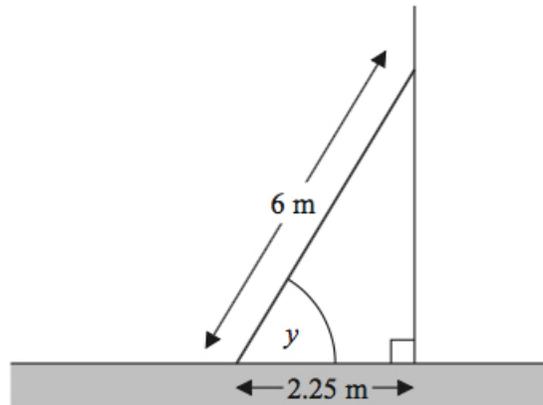
$$\left. \begin{array}{l} \text{add to: } -1 \\ \text{multiply to: } (+2) \times (-10) = -20 \end{array} \right\} -5, +4$$

Now, e.g.,

$$\begin{aligned}2p^2 - p - 10 &= 2p^2 - 5p + 4p - 10 \\ &= p(2p - 5) + 2(2p - 5) \\ &= \underline{\underline{(p + 2)(2p - 5)}}.\end{aligned}$$

20. The diagram shows a ladder leaning against a vertical wall.

(3)



The ladder stands on horizontal ground.
The length of the ladder is 6 m.
The bottom of the ladder is 2.25 m from the bottom of the wall.
A ladder is safe to use when the angle marked y is about 75° .
Is the ladder safe to use?
You must show all your working.

Solution

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos y^\circ = \frac{2.25}{6}$$
$$\Rightarrow y^\circ = 67.97568716 \text{ (FCD);}$$

hence, the ladder is not safe to use because it is not steep enough.

21. In Holborn School there are 460 students in Key Stage 3, 320 students in Key Stage 4, and 165 students in Key Stage 5.
Nimer is carrying out a survey.
He needs a sample of 100 students stratified by Key Stage.
Work out the number of students from Key Stage 3 there should be in the sample.

(2)

Solution

The total number of students is

$$460 + 320 + 165 = 945$$

and the number of students from Key Stage 3 there should be in the sample is

$$\frac{460}{945} \times 100 = 48.677\dots;$$

hence, the should be 49.

22. h is inversely proportional to the square of r .

(3)

When $r = 5$, $h = 3.4$.

Find the value of h when $r = 8$.

Solution

$$h \propto \frac{1}{r^2} \Rightarrow h = \frac{k}{r^2}$$

for some constant k . Now,

$$3.4 = \frac{k}{5^2} \Rightarrow k = 85$$

which means

$$h = \frac{85}{r^2}.$$

Finally,

$$h = \frac{85}{8^2} = 1\frac{21}{64}.$$

23. Dan does an experiment to find the value of π .

(4)

He measures the circumference and the diameter of a circle.

He measures the circumference, C , as 170 mm to the nearest millimetre.

He measures the diameter, d , as 54 mm to the nearest millimetre.

Dan uses

$$\pi = \frac{C}{d}$$

to find the value of π .

Calculate the upper bound and the lower bound for Dan's value of π .

Solution

$$169.5 \leq \text{circumference} < 170.5$$

and

$$53.5 \leq \text{diameter} < 54.5.$$

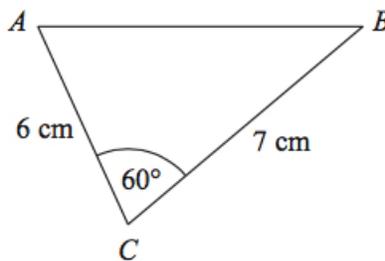
Now, the lower bound is

$$\frac{169.5}{54.5} = 3 \frac{12}{109} = \underline{\underline{3.110\ 091\ 743}} \text{ (FCD)}$$

and the upper bound is

$$\frac{170.5}{53.5} = 3 \frac{20}{107} = \underline{\underline{3.186\ 915\ 888}} \text{ (FCD)}.$$

24. ABC is a triangle.



- (a) Work out the area of triangle ABC . (2)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 6 \times 7 \times \sin 60^\circ \\ &= 18.186\ 533\ 48 \text{ (FCD)} \\ &= \underline{\underline{18.2 \text{ cm}^2}} \text{ (3 sf)}. \end{aligned}$$

- (b) Work out the length of the side AB . (3)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} AB &= \sqrt{6^2 + 7^2 - 2 \times 6 \times 7 \cos 60^\circ} \\ &= 6.557\ 438\ 524 \text{ (FCD)} \\ &= \underline{\underline{6.56 \text{ cm}}} \text{ (3 sf)}. \end{aligned}$$

25. Solve the simultaneous equations

(6)

$$\begin{aligned}x^2 + y^2 &= 9 \\x + y &= 2.\end{aligned}$$

Give your answers correct to 2 decimal places.

Solution

$$y = 2 - x \Rightarrow x^2 + (2 - x)^2 = 9$$

\times	$ $	2	$-x$
2	$ $	4	$-2x$
$-x$	$ $	$-2x$	$+x^2$

$$\begin{aligned}\Rightarrow x^2 + (4 - 4x + x^2) &= 9 \\ \Rightarrow 2x^2 - 4x - 5 &= 0\end{aligned}$$

$a = 2$, $b = -4$, and $c = -5$

$$\Rightarrow x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 2 \times (-5)}}{2 \times 2}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{56}}{4}$$

$$\Rightarrow x = -0.870\,828\,693\,4 \text{ or } x = 2.870\,828\,693 \text{ (FCD)}$$

$$\Rightarrow y = 2.870\,828\,693 \text{ or } y = -0.870\,828\,693\,4 \text{ (FCD);}$$

hence,

$$\underline{\underline{x = -0.87, y = 2.87 \text{ or } x = 2.87, y = -0.87 \text{ (2 dp)}}}.$$